Towards a Theory of Part

My aim in this paper is to outline a general framework for dealing with questions of part-whole. Familiar as this topic may be, my treatment of it is very different from more conventional approaches. For instead of dealing with the single notion of mereological part or sum, I have attempted to provide a comprehensive and unified account of the different ways in which one object can be a part of another. Thus mereology, as it is usually conceived, becomes a small branch of a much larger subject.[1]

My discussion has been intentionally restricted in a number of ways. In the first place, my principal concern has been with the notion of absolute rather than relative part. We may talk of one object being a part of another relative to a time or circumstances (as when we say that the tire was once a part of the car or that or that the execution of Marie Antoinette was as a matter of contingent fact a part of the French Revolution) or in a way that is not relative to a time or the circumstances (as when we say that this pint of milk is a part of the quart or that the letter 'c' is part of the word 'cat'). Many philosophers have supposed that the two notions are broadly analogous and that what goes for one will tend to go for the other.[2] I believe this view to be mistaken and a source of endless error. But it is not my aim to discuss either the notion of relative part or its connection with the absolute notion.[3]

In the second place, I have focused on the 'pure' theory of part-whole rather than its application to our actual ontology. Once given a theory of part-whole, there arises the question of how it applies to the objects with which we are already familiar. This question becomes especially delicate and intricate on my own approach since, although we may recognize that such and such a familiar object is a part or whole, it may not be clear, according to the theory, what kind of whole or part it is. But despite the considerable interest of this question, my focus has been on the abstract development of the theory itself and not on its application to ontology.

Finally, I have only provided the merest sketch of the framework (on which I hope say more elsewhere). Many points are not developed and some not even stated. I have, in particular, said relatively little about the technical foundations of the subject, which are mathematically quite distinctive, or about some of the broader philosophical issues to which they give rise. I have given a rough map of the terrain rather than a guided tour, but I hope I have done enough to bring out the interest of the approach and to make clear how a more systematic and philosophically informed account might proceed.

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[2]As in T. Sider, *Four Dimensionalism* (Oxford: Clarendon Press, 2001), for example.

[3]The matter is briefly discussed in K. Fine, *Things and Their Parts*,Midwest Studies in Philosophy XXIII (1999), 61-74.

§1 The Intuitive Notion of Part

One object may be a part of another - some thunder a part of a storm, for example, or the shell a part of a nut. When one object is a part of another, there is a sense in which it is *in* the other - not in the sense of being *enclosed by* the other, as when a marble is in an urn, but more in the sense of being *integral to* the other. When parts are in question, it is also appropriate to talk of a given object being *composed of* or *built up from* the objects that it contains. Thus a storm may be composed of various occurrences of lightning and thunder, while an urn is not composed - even in part - of the marbles that it contains.

We may perhaps make the sense of containment characteristic of part especially vivid by considering what happens to an object when a part of it is replaced. For, as a general rule, the object within which the replacement has been made will *change* - either in the radical sense of being different from what it was or in the less radical sense of being different from how it was. And conversely, it is only parts (or objects containing parts) whose replacement can result in change. Thus if my kidney, which is part of me, is replaced with another kidney, then I will have changed with respect to how I was, while if the hat I am now wearing, which is not a part of me, is replaced with another hat, then I will not thereby have changed with respect either to what or to how I was.

Philosophers have often supposed the notion of part only has proper application to material things or the like and that its application to abstract objects such as sets or properties is somehow improper and not sanctioned by ordinary use. But I suspect that this is something of a philosopher's myth. We happily talk of a sentence being composed of words and of the words being composed of letters - and not just the sentence and work tokens, mind, but also the types. And similarly, a symphony (and not just its performance) will be composed of movements, a play of acts, a proof of steps. I wonder how many of these philosophers have said such things as 'this paper is in three parts'. When they have, then I very much doubt that they would have any inclination, as ordinary speakers of the language, to add 'but not of course in a strict or literal sense of the term'; and the intended reference here is not primarily - or perhaps not at all - to the tokens of the paper but to the type of which they are the tokens. The evidence concerning our ordinary talk of part is mixed and complicated, but it does not seem especially to favor taking material things to be the only true relata of the relation.

§2 Pluralism

According to the pluralist about part-whole, there are different ways in which one object can be a part of another. Thus he may well think that the way in which a pint of milk is part of a quart is different from the way in which the letter (type) 'c' is part of the word (type) 'cat' and different from the way in which a member is part of a set.

There is a way in which pluralism may be trivially false. For one may simply stipulate that 'part' is to have such and such a sense - the sense which it is assumed to have in standard mereology, for example; and so the truth of monism will be guaranteed as long as there is only one way, in this sense, for an object to be a part of another. But my interest is in the intuitive notion of part, not in some stipulated sense, and, although this notion may be subject to further clarification, there remains a genuine question as to whether any reasonable clarification of it will admit of different ways for one object to be a part of another.

There is also a way in which pluralism may be trivially true. For one way to be a part is

to be a small part and another way is to be a large part; and everyone can agree that there are small parts and large parts. But being a small part or a large part are what one might call *derivative* senses of part, they are to be understood in terms of more basic senses of part; and in considering the question of pluralism, the derivative senses of part should be set aside.

Let us say that a way of being a part is *basic* if it is not definable in terms of other ways of being a part. A basic way of being a part may not be basic in an absolute sense, since it may be possible to define it in terms of other mereological notions and, indeed, later I shall suggest that this is so. But there should be no definition of it in terms of other ways of being a *part*. Our question, then, is whether there are different basic ways in which one object may, intuitively, be a part of another.

Now on the face of it, there would appear to be a wide variety of basic ways in which one object can be a part of another. The letter 'n' would appear to be a part of the expression 'no', for example, and a particular pint of milk part of a particular quart; and if these two relations of part are not themselves basic (perhaps through being restricted to expressions or quantities), there would appear to be basic relations of part that hold between 'n' and 'no' or the pint and the quart. It is also plausible that the way in which 'n' is a part of 'no' is different from the way in which the pint is a part of the quart. For if the two ways were the same, then how could it be that two pints were only capable of composing a single quart, while the two letters 'n' and 'o' were capable of composing two expressions, 'no' and 'on'? For the same reason, the way in which the letter 'n' is a part of the expression 'no' would appear to be different from the way in which it is a part of the set of letters {'n', 'o'}; and the way in which 'n' is a part of {'n', 'o'} would appear to be different from the way in which a pint is a part of a quart, since if four quarts compose a gallon the pints which compose the quarts will compose the gallon in the same way in which they compose the quarts whereas, if four sets compose a further set the members of the sets will not compose the further set in the same way in which they compose the component sets. Thus we would now appear to have three different basic ways in which one object can be a part of another (pint/gallon, letter/word, and member/set); and once these cases have been granted, it is plausible that there will be many more.

Although pluralism would appear to be the more plausible view, it is not the view that has been most widely held. The majority of philosophers currently working in metaphysics have been monists.[4] They have supposed that there is but one (basic) way for a given object to be a part of another; and they have thought that this one way is the relation of part-whole explored in classical mereology, according to which a whole is a mere sum, or 'aggregate' or 'fusion', formed from its parts without regard for how they might fit together or be structured within a more comprehensive whole.

Of the many putative counter-examples to the monist position, philosophers have paid most serious attention to the case of set-membership; and so let us focus on this one case as being perhaps typical of the rest. There are three successively stronger claims that should be

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[4]Though there have been exceptions. They include: R. Grossman, *Ontological Reduction* (Bloomington: Indiana University Press, 1973); P. Simons, *Parts: A Study in Ontology* (Oxford: Oxford University Press, 1987); D. Armstrong, *A World of States of Affairs* (Cambridge: Cambridge University Press, 1997); and, more recently, M. Johnston, *Hylomorphism*, Journal of Philosophy, CIII.12 (2006), 614-30 and K. Koslicki, *The Structure of Objects* (Oxford: Oxford University Press, 2008).

established if the case of sets is to pose a threat to monism: (i) a member of a set is a part of a set; (ii) it is a part of the set in a basic way; and (iii) the basic way in which it is a part of the set is different from the way in which something is a mere part of a sum. (ii) may reasonably be taken to hold given (i); and (iii) may be taken to hold for the reason already given - for the set $\{\{x, y\}, \{y, z\}\}$, say, which is composed of the sets $\{x, y\}$ and $\{y, z\}$ will not be composed of x, y and z, whereas the fusion $(x + y) + (y + z)$ composed of the fusions $(x + y)$ and $(y + z)$ *will* be composed of x, y and z . So the success of the counter-example turns on (i). Is a member of a set a part of the set?

There is a strong prima facie case in favor of taking the members of a set to be parts. For we do indeed talk of a set containing its members and of its being composed or being built up from its members; and as I have suggested, such talk is not to be dismissed simply on the grounds that sets are not material things.

Two main arguments have been given on the other side (apart from a mere distaste for non-material wholes). According to the first, the relation of part to whole is transitive but, since the relation of member to set is not transitive, the members of a set cannot be its parts. But this is to confuse two claims - that the relation of member to set is a relation of part and that the members of a set are parts. The first may indeed be denied on the grounds that the relation of member to set is not transitive but that is still compatible with each case of membership being a case of part. Indeed, it may well be thought that the way in which a member is a part of a set is given, not by the membership relation itself, but by the ancestral of the membership relation, where this is the relation that holds between x and y when x is a member of y or a member of a member of y or a member of a member of a member of y, and so on. The way in which a member is a part of a set will then indeed be transitive and the relation of member to set will merely correspond to the special case in which the object is *directly* a part of the whole. (I should note that membership, not being transitive, is not properly a way of being a part and so the fact that ancestral membership is definable in terms of membership does not impugn the fact that ancestral membership is a basic way of being a part).

The other argument is that it is only in a metaphorical or non-literal sense that we may talk of a set containing or being built up from its members. It may be conceded that there is some kind of analogy between the members of a set and the bricks that make up a wall but that should not mislead us, so the objection goes, into thinking that we have a relationship of part in both cases.

I would not wish to deny that there may be metaphorical or non-literal ways of talking of part-whole. Thus it is sometimes said that the conclusion of a valid argument is contained in the premises or that the mother or father is 'in' the child. But, of course, the conclusion is not literally a part of the premises and nor is the mother or father literally a part of the child; and not only is this intuitively evident, it is also revealed by the fact that the analogies upon which the part-whole talk is based will only extend so far. Thus we cannot say that the premises are composed or built up from various conclusions or that the child is composed of or built up from his mother and father; and nor can we meaningfully talk of replacing the given conclusion in the premises with another conclusion or replacing the mother in the child with someone else.

However, in the case of set-membership, there would appear to be nothing that might plausibly be taken to indicate that the talk of part-whole is not to be taken literally. A set is indeed composed of or built up of its members and we should add that we may meaningfully talk - and in the intended way - of *replacing* one member of a set with another. Thus Aristotle in the

set {Plato, Aristotle} may be replaced with Socrates to obtain the set {Plato, Socrates}, with the given set becoming a different set from what it was. In the case of sets, our conception of members as parts seems to extend all the way.

But perhaps what is most significant is that by assuming that members and the like are parts we can achieve a degree of generality and unity in the theory of part-whole that could not otherwise be attained. Diverse phenomena can be explained and connected under the common rubric of part-whole; and although the full force of this point will only later become apparent, it is hardly good methodological practice to adopt the standard monist position at the start of enquiry and thereby deprive ourselves of the possibility of discovering whether a broader conception of part-whole can indeed be sustained. We should see if we can coherently treat members and the like as parts and only give up the assumption that they are if we fail.

§3 Operationalism

The adoption of pluralism has significant implications for the study of mereology. For the single way of being a part of classical mereology must now give way to a number of different ways of being a part. These should then be characterized and the connections between them explored. The adoption of pluralism will also have significant implications for metaphysics at large. For if the part-theoretic structure of familiar things need not be the aggregative structure of classical mereology, then we need to investigate what it might be and how it might be realized. There is therefore the possibility of discovering new kinds of part-theoretic structure and of providing novel analyses of the part-theoretic structure of familiar things.

I am not here concerned to pursue the second line of investigation but I do wish to provide some indication of how a more general theory of part-whole might be developed. In formulating the principles of mereology, it has been usual to take the relation of part-whole or some associated relation (such as overlap) as primitive. But I believe that, in formulating a more general theory, it is important to take the operation of composition as primitive rather than the more familar relation of part-whole. In the case of classical mereology, the operation of composition will take some objects into the sum or fusion of those objects, while, in the set-theoretic case, it will take some objects into the set of those objects; and, in general, the operation of composition will be the characteristic means (summation, set-builder etc.) by which a given kind of whole is formed from its parts.

Even if we set philosophical considerations on one side, there are compelling logical reasons for favoring the operation over the relation. For it is always possible to define the relation in terms of the operation but not always possible to define the operation in terms of the relation. In the set-theoretic case, for example, the operation is the set-builder and the relation is ancestral membership. But it may be demonstrated to be in principle impossible to define the set-builder (or, equivalently, membership) in terms of ancestral membership.[5] Thus someone who took the relation of part-whole to be primitive in this case would deprive themselves of the means of talking of composition.

Indeed, even in the case of classical mereology, the standard definitions of summation in terms of part-whole will only be correct under certain existential assumptions. For suppose we

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[5]The obvious definition will not work. For if one were to define x to be a member of y if x is an ancestral member of y but not an ancestral member of an ancestral member of y, then this would incorrectly exclude x from being a member of {x, {x}}. A general proof is outlined in K. Fine, *Aristotle on Matter*, Mind 101.401 (1992), 35-57, footnote 16.

define y to be the sum (or fusion) of x_1, x_2, ... just in case (i) each of x_1, x_2, ... is a part of y and (ii) y is a part of z whenever each of x_1, x_2, ... is a part of z. Consider now an ontology consisting of three atoms a, b and c and a universal element d:

$$d$$
$$/ \quad | \quad \backslash$$
$$a \quad b \quad c$$
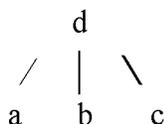
The definition would then predict that d is the sum of a and b, of b and c, and of a, b and c when, intuitively, it is only the sum of a, b and c. Or again, suppose we define y to be the sum of x_1, x_2, ... just in an object z overlaps with y iff it overlaps with at least one of x_1, x_2, ... Consider now an ontology consisting of two objects, an atom a and a non-atom a^+ of which it is a proper part:

$$a^+$$
$$|$$
$$a$$

Then this definition would predict that a^+ is the fusion of a (alone) when, intuitively, this is not so. The whole subject of the implicit existential assumptions of classical mereology calls for much more discussion than I can give here, but it should be clear from these two examples that, when these assumptions are in dispute, then the usual definitions of fusion will not work and it will not be adequate to base the formulation of the theory on the standard relational primitives of part-whole or overlap.

Let me briefly mention three further advantages that derive from adopting the operational approach, although others will become apparent in the course of the discussion. In set theory and standard mereology, there is a common notion of a 'null' object - where this is the null set in the case of set theory and the null sum in the case of mereology. Under the operational approach, it is possible to provide a uniform definition of the 'null' object, for this is the whole, if any, which results from applying the compositional operator Σ to zero objects. But on the usual relational approach, it will be impossible to identify the null set, since its behavior with respect to membership will be indistinguishable from that of an arbitrary non-set, and it will be impossible to identify the null sum (should it exist), since it will behave in the same way as an atom that is a part of everything else.

Or again, there is an intuitive distinction between wholes which are like sets in being hierarchically organized and those which are like sums in being 'flat' or without an internal division into levels. The distinction, under the operational approach, can be seen to turn on whether repeated applications of the operation are capable of yielding something new. Thus, to use our previous example, $\{\{x, y\}, \{y, z\}\}$ cannot be obtained by a single application of the set-builder to x, y and z, whereas the corresponding sum $(x + y) + (y + z)$ can be so obtained. However, it is not at all clear what this distinction might come to under the relational approach. That there are proper parts of proper parts is certainly not enough, since a sum may contain proper parts of proper parts; and merely given the 'graph' of the part-whole relation, it is hard to see what else might reveal the presence of different levels within a given whole.

Or again, we would sometimes like to distinguish between the different *occurrences* of a

part within a whole. The multi-set [x, x], for example, contains two occurrences of x while the multi-set [x] contains only one. We can draw this distinction between the two multi-sets on the operational approach, since [x, x] results from applying the multi-set builder to x and x while [x] results from applying the multi-set builder to x alone. Under the relational approach, by contrast, we can only say that x is a part of each multi-set and there is no basis upon which the different occurrences might be distinguished.

In developing the operational approach, it will be important to take a permissive attitude to the 'logical form' of the compositional operations. We should allow them to be variably polyadic in the sense of applying to any number of objects (including possibly none or one or infinitely many). We will want the set-builder, $\Sigma_{\in}$, for example, to apply to any number of objects x_1, x_2, ... (with $\Sigma_{\in}(x_1, x_2, ...) = \{x_1, x_2, ...\}$). We should also allow the operations to be partial, i.e. to be not always defined. Thus it has often been supposed that the summation operation, Σ_m, has no application to zero objects and that the operation for forming a predicative proposition will have no application to the Eiffel Tower and the Tower of London (since neither can play a predicative role). The operations should also be capable of being sensitive to the order of the arguments to which they apply. The result of applying the operation for forming sequences to Socrates and Plato, for example, should be different from the result of applying it to Plato and Socrates.

Once given a compositional operation, a corresponding relation of part may be defined in two steps. We say first that x is a *component* of y if y is the result of applying Σ to x or to x and some other objects. In other words, y should be of the form $\Sigma(x_1, x_2, ...)$, where at least one of x_1, x_2, ... is x. Thus when Σ is mereological summation, the components of an object will be mere parts and, where Σ is the set-builder, the components of an object will be its members. We may then define x to be a *part* of y if there is a sequence of objects x_1, x_2, ... x_n, n > 0, for which x = x_1, y = x_n and x_i is a component of x_{i+1} for i = 1, 2, ..., n-1. The parts of an object are the object itself, or its components, or the components of the components, and so on. So whereas components will yield their wholes under a single application of the compositional operation, parts will yield their wholes under successive applications of the operation (in addition to a single application or no application at all).[6]

The relation of part is plausibly taken to conform to the following three principles:
Reflexivity Each object is a part of itself,
Transitivity If x is a part of y and y of z then x is a part of x,
Anti-symmetry x is a part of y and y of x only when x = y.
These principles, or something like them, are usually taken to be axioms. But an interesting feature of the operational approach is that they can now be derived through the definition of part.

Reflexivity and Transitivity follow directly from the definition, without making any assumptions about the underlying behavior of the compositional form. For the one-term chain x = x_1 (with n = 1) shows that x is a part of x. And in regard to Transitivity, let us suppose that x

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[6]It might be thought that some forms of composition are compositional in only some of their argument-places. The operation for forming a state from a relation R, some individuals x_1, x_2, ..., x_n, and the location L, for example, might be taken to be compositional in R and x_1, x_2, ..., x_n though not in L, so that the relation and the individuals would be parts of the resulting state while the location is not. In such cases, the definitions of component and part should be restricted to the argument-places in question.

is a part of y and y of z. There will then be an appropriate chain $x = x_1, x_2, ..., x_n = y$ of components connecting x to y and an appropriate chain $y = y_1, y_2, ..., y_m = z$ of components connecting y to z; and so their combination $x_1, x_2, ..., x_n, y_2, ..., y_m$ will be an appropriate chain of components connecting x to z.

Anti-symmetry follows from the definition of part with the help of a corresponding assumption concerning the behavior of the compositional form:

Anti-Cyclicity If $x = \Sigma(..., \Sigma(..., x, ...), ...)$, then $x = \Sigma(..., x, ...)$.
In other words, if x can be built up from x itself, then any intermediate whole $\Sigma(..., x, ...)$ involved in the construction must itself be x (here $\Sigma(..., x, ...)$ can occur at any depth within $\Sigma(..., \Sigma(..., x, ...), ...)$). In many cases, this assumption may be derived from more fundamental assumptions. When Σ is the set-builder, for example, we may appeal to the absence of infinitely descending membership chains to show that the antecedent $x = \Sigma(..., \Sigma(..., x, ...), ...)$ is never satisfied.

Although Reflexivity and Transitivity follow directly from the definition, there are cognate notions of part under the operational approach for which this is not so. For suppose that we take x to be a part of y if there is a sequence of objects $x_1, x_2, ... x_n$, $n > 1$, for which $x = x_1$, $y = x_n$ and x_i is a component of x_{i+1} for $i = 1, 2, ..., n-1$ (the case $n = 1$ is no longer allowed). With this notion of part in play, perhaps more natural than the one we previously adopted, it is a substantive question whether or not the relation is reflexive in any given case. For we are asking whether an object can be built up from itself by means of some positive application of the given form of composition; and this will depend upon how the form of composition behaves. Thus when the underlying operation is summation, each object will be a part of itself since the unit sum of any object is the object itself but, when the underlying operation is the set-builder, no object will be a part of itself since no object is ever an ancestral member of itself.

Similarly, if part is understood as component, it will be a substantive question whether or not the relation is transitive. In the case of sums, for example, it will be transitive since components of components are components while, in the case of sets, it will fail to be transitive since components of components (i.e. members of members) will sometimes fail to be components (members). It is a shortcoming of the relational approach that it makes no room for such distinctions.

§4 Types of Principle

In developing a general theory of part-whole, we will wish to state the principles by which the various forms of composition are governed. If a given form of composition is definable in terms of more basic forms, we may derive the principles for the one from the principles for the others. It therefore suffices to state the principles for the most basic forms of composition, those not definable from others.

I believe that the principles governing the basic forms of composition will conform to a general template. Variations in the principles for the different forms of composition will then arise from variations in how the template is to be filled in. The template will comprise two broad categories of principle - the *formal* and the *material* (though not quite in the sense of Husserl). From among the formal principles, we may distinguish between those that provide conditions of application for the operation and those that provide identity conditions; and from among the material principles, we may distinguish between those that provide conditions for the presence of a whole (in space and time or at a world) and those that specify the descriptive character of the

whole. The presence conditions, in their turn, may concern either the existence of the whole or its extension. The general classification of principles within the template is therefore as follows:

Principles

¬   °

Formal   Material

¬   °   ¬   °

Application Identity   Presence   Character

¬   °   ¬   °

Existence Extension   ...
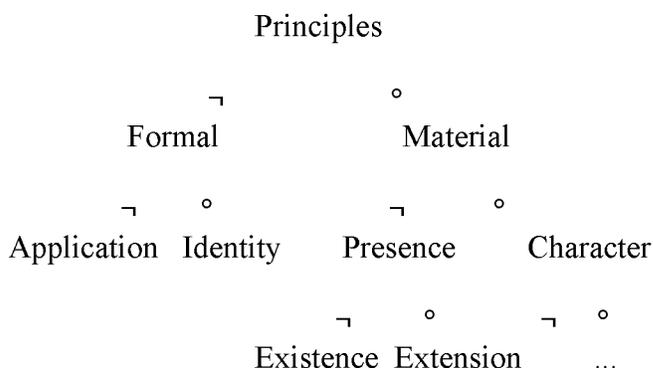
Let us discuss each kind of principle in turn. The formal principles are the counterpart, within the operational approach, to the standard axioms of mereology. They can usually be stated within a purely logical vocabulary that has been enriched with whatever mereological primitive is in question. These principles are of two kinds. The first concerns the conditions under which there are wholes of a given sort - which, on the operational approach, is a matter of stating when the result of applying the compositional operation to various objects will be defined. The second concerns the conditions of identity for wholes - which, on the operational approach, is a matter of stating when a whole formed in one way by means of the compositional operation is the same as a given object or a whole that has been formed in some other way. In the case of summation, for example, it might be thought that the result of applying the sum operation to no objects is undefined (so that the null sum will not exist) and that the result of applying the sum operation to a single object is that very object.

The material principles concern the conditions under which the wholes will possess certain material features (those that are neither logical nor mereological). Two classes of material features may be distinguished: those that relate to the presence of the whole in space or time or the world; and those that concern its more descriptive character, such as its color or weight. Principles of the latter sort are usually ignored in the more formal development of mereology though they are often critical to how the subject is to be applied or put to philosophical use.

My own view is that there are two fundamentally different ways in which an object might be present in space or time; it may *exist* in space or time; or it may be *extended* (or *located*) in space or time. Thus a material thing will exist in time but be extended in space while an event will be extended in both space and time. This means that we will need to state separate presence conditions for the existence and extension of a whole.[7] But someone who does not accept the distinction may simply provide conditions for a single uniform notion of presence.

The character conditions tell us what the wholes are like. In the case of mass, for example, they may tell us that the mass of a sum $\Sigma_m(x_1, x_2, ...)$ at a given time is the sum of the

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[7] For a defense and development of this view see K. Fine's *Compounds and Aggregates*, Nous 28.2 (1994),137-158 and *In Defense of Three-Dimensionalism*, Journal of Philosophy CIII.12 (2006), 699-714.

masses of the components $x_1, x_2, \ldots$ at that time, as long as the components are spatially disjoint; or they may tell us that the sum $\Sigma_m(x_1, x_2, \ldots)$ is gold just in case each of the components $x_1, x_2, \ldots$ is gold.

The character conditions will tend to have a much more ad hoc character than the other conditions that we have considered. The color of a house, for example, is the color of its siding, the color of an egg the color of its shell, the color of a pencil the color of its lead. In the case of the 'intrinsic' character of a thing - such as its mass or color - the character of the whole will be some sort of function of the character of the parts. But the function in question will vary from case to case.

I am inclined to regard these various principles as definitive of the form of composition in question. It will lie in the nature of any form of composition to conform to various principles of this sort.[8] Moreover, such principles will be exhaustive of its nature (beyond its being a basic form of composition); and any two basic forms of composition will differ in the principles by which they are governed. Thus once given the principles, the form of composition will be uniquely determined.

In what follows, I shall focus on the identity principles. The other principles are certainly of interest and call for an extended discussion in their own right. But in the context of the present paper, we can only go so far in showing how the various different forms of mereology might be developed; and it is the identity principles that correspond most closely to the standard axioms of mereology.

§5 Mere Sums

Let us begin by showing how classical mereology can be made to conform to the above template. This will shed some further light on what might appear to be an excessively familiar subject, as well as providing an illustration of how the general framework is to be applied. With this case in hand, we will then be in a position to apply the framework to a wide range of other cases.

There are two aspects of the notion of whole that have often been implicit in the recent development of mereology. The first, more formal, aspect is that a whole is a 'mere sum'. It is nothing over and above its parts - or perhaps we should say, more cautiously, that it is nothing over and above its parts except in so far as it one object rather than many. The second, more material, aspect is that wholes are 'four-dimensional' objects, equally extended in time as they are in space. The two aspects are often taken together but it is possible to accept the one, treating wholes as mere sums, without accepting the other, and treating them as four-dimensional objects.

I myself favor a three-dimensional account of mere sums (taking them to be 'compounds' rather than 'aggregates'). But in keeping with our general focus on identity, let us put the issue of 'dimensionality' on one side. Our question, then, is: given the conception of wholes as mere sums, when should two wholes be taken to be the same?

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[8]The character conditions may be an exception in this regard. For it is more plausible to suppose in the case of particular material properties (such as having a certain color) that it lies in the nature of the material properties, rather than the forms of composition, that the appropriate conditions should hold. Thus the conditions in this case should be regarded as definitive of how the material properties are to be extended to the wholes rather than as definitive of the form of composition by which the wholes are formed.

Although the intuitive conception of a mere sum might appear to be hopelessly vague, I believe that it enables us to completely determine what the identity conditions for sums should be. An identity condition will, in general, take the form of an identity statement flanked by terms in Σ, where these are constructed from variables and a symbol 'Σ' for the form of composition in question. Thus each variable x, y, z, ... will be a term (that might figure on the left or right hand side of such an identity); and $\Sigma(r, s, t, ...)$ will be a term when r, s, t, ... are terms.[9]

Call an identity condition s = t *regular* if the variables appearing in s and in t are the same. Thus $\Sigma(x, y) = \Sigma(y, x)$ is regular while $\Sigma(x, y) = x$ is not. We now have the following compendious statement of the identity conditions for sums:

Summative Identity s = t whenever 's = t' is a regular identity.

Thus since the conditions $\Sigma(x) = x$, $\Sigma(x, y) = \Sigma(y, x)$ and $\Sigma(x, \Sigma(y, z)) = \Sigma(\Sigma(x, z), y)$ are all regular, it will follow from the principle that they all obtain.

This principle gives formal expression to the idea that wholes built up from the same parts should be the same; and this is something that appears to be constitutive of our intuitive conception of a mere sum as nothing over and above its parts.[10] For consider a typical case of a regular identity, say $\Sigma(\Sigma(x, y), \Sigma(y, z)) = \Sigma(x, y, z)$. According to our intuitive conception of a mere sum, there is nothing more to the whole $\Sigma(\Sigma(x, y), \Sigma(y, z))$ than its parts $\Sigma(x, y), \Sigma(y, z)$ and nothing more to the wholes $\Sigma(x, y), \Sigma(y, z)$ than *their* parts x, y and y, z respectively. Presumably, it follows from this that there is nothing more to $\Sigma(\Sigma(x, y), \Sigma(y, z))$ than x, y and z and that, likewise, there is nothing more to the whole $\Sigma(x, y, z)$ than its parts x, y, and z. But if there is nothing more to either whole than the parts x, y and z, then it presumably follows that the two wholes should be the same. Thus philosophical reflection on the notion of mere sum is able to provide us with a simple and natural characterization of classical mereology.[11]

But simple as this characterization may be, the set of identities that it appeals to is highly redundant. From $\Sigma(x) = x$, for example, it will follow that $\Sigma(\Sigma(x)) = \Sigma(x)$ and hence that $\Sigma(\Sigma(x)) = x$; and there is therefore no need to appeal to these other identities. A more compact formulation may be obtained by providing an analysis of the different grounds upon which a regular identity may hold. We thereby arrive at the following four principles:

Absorption $\Sigma(..., x, x, ..., ..., y, y, ..., ...) = \Sigma(..., x, ..., y, ...)$;

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[9]Although this is not a theme I shall develop, the terms by which parts and wholes are designated on the operational approach provide a very convenient tool for investigating their formal properties.

[10]This idea provides the basis for Goodman's version of nominalism, 'no distinction of entity without a distinction of content' (in N. Goodman, *A World of Individuals*, from *The Problem of Universals*, Notre Dame, Indiana: University of Notre Dame Press (1956), 13-31, and in N. Goodman, *On Relations which Generate*, Philosophical Studies, vol.8 (1957), 65-66). However, his attempt to provide a precise formulation of the principle is handicapped by the fact that he works within the relational framework and a superior formulation (not resting on the existence of mereological atoms) may be obtained by adopting the operational framework.

[11]We may also show that the set of regular identities is 'Post-complete'; for an irregular Σ-identity can only be added to them on pain of being able to derive the existence of exactly one object. Thus the principles for identifying wholes under the conception of them as mere sums is maximal. Nothing else can be said without reducing the theory to triviality.

Collapse $\sum(x) = x$;

Leveling $\sum(..., \sum(x, y, z, ...), ..., \sum(u, v, w, ...), ...) = \sum(..., x, y, z, ..., ..., u, v, w, ..., ...)$;

Permutation $\sum(x, y, z, ...) = \sum(y, z, x, ...)$ (and similarly for all other permutations).

According to Absorption, the repetition of components is irrelevant to the identity of the whole; according to Collapse, the whole composed of a single component is identical to that very component; according to Leveling, the embedding of components is irrelevant to the identity of the whole; and according to Permutation, the order of the components is irrelevant to the identity of the whole. These four principles follow from Summative Identity; and, conversely, Summative Identity follows from them. Thus together they constitute an analysis of the notion of mere sum.[12]

§6 More-than-mere Sums

The previous principles - Absorption, Collapse, Leveling and Permutation - point to different features of composition; and this suggests that there may be forms of composition that satisfy some of these principles but not others. Indeed, I think it will be found that for many subsets of these principles, there will naturally correspond a form of composition - some familiar and some not so familiar - that satisfies each of the principles of the subset and fails to satisfy the others.

Let us go through the cases in turn, using the letters A, C, L and P as a mnemonic for the respective principles. C̶L̶AP̶, for example, will be used for a form of composition that satisfies C and A but not L or P. There are four cases that will serve as familiar points of reference:

CLAP: Sums.

C̶L̶AP: Sets. Repetition and order of components are irrelevant.

CLA̶P̶: Strings. The form of composition is concatenation and concatenating two strings, say xy and uv, is the same as concatenating their components x, y, u and v.

C̶L̶A̶P̶ : Sequences. The form of composition is the 'sequence-builder', where sequencing two sequences (xy) and (uv) to obtain ((xy)(uv)) is to be distinguished from sequencing x, y, u and v to obtain (xyuv) (in contrast to the case of strings).

These four cases are determined by the natural association between C and L and between A and P, under which either one of each pair is always accompanied by the other. Thus the possibilities may be set out in the following chart:

	L	L̶
P	sums	sets
P̶	strings	sequences

where the 'horizontals' (sums and sets or strings and sequences) and the 'verticals' (sums and strings or sets and sequences) represent axes of similarity and where the 'diagonals' (sums and sequences or strings and sets) are most opposed.

From each of the associated pairs (C, L) and (A, P), L or P is the 'dominant' or more

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[12]The above formulation presupposes that the sum $\sum(x, y, z, ...)$ is always defined. We may avoid making this presupposition by stating the principles as 'weak' identities, whose truth only requires that the terms on the right and left be co-designative when one or the other of them is defined.

significant element; and so we may say that a form of composition is *sum-like* if it conforms to LP, *set-like* if it conforms to 𝐿P, *string-like* if it conforms to L𝐏; and *sequence-like* if it conforms to 𝐿𝐏. Thus within each of these categories there are three possible variants on the core member. In the case of sets, for example, the variants will include:

        C𝐿AP: Quinean sets - like sets but with the singleton identical to its sole component;

        𝒞𝐿AP: Multi-sets - like sets but sensitive to multiple occurrences of the same component.

        C𝐿𝐴P: Quinean multi-sets - like multi-sets but with the singleton identical to is sole
                component.

And similarly for the other categories.[13]

    The sense in which these various forms of composition will *fail* to satisfy the relevant principles calls for further comment. It is not enough that the principle *sometimes* fails. For then a single case in which {x} ≠ x fails would enable the set-builder to fail to satisfy Collapse. And it would be going too far to say that the principle *always* fails. After all, the string xy will be the same as the string yx when x and y are the same even though concatenation fails to satisfy Permutation. Rather the only identities that should hold are the ones that can be shown to hold on the basis of the defining principles. Thus it is because there is no way to establish {x} = x on the basis of Absorption and Permutation, no matter how the object x might be designated, that {x} = x will always fail; and, as a rule, different designations of objects will designate different objects unless the designations can be shown to be co-designative on the basis of the given identity principles.[14]

    The above approach provides a single and elegant account of summation and the various other operations; it provides a unified treatment of these operations; and it points to the existence of a number of operations that might otherwise have gone unnoticed. We thereby have what I hope is a convincing demonstration of the power and beauty of the method, quite apart from the particular advantages - such as the ability to define a null object or the distinction between flat and hierarchical forms of composition - that were mentioned before.

    We should note that there would appear to be no good reason to require that the defining principles for the various operations should be limited to the particular principles (C, L, A and P) that we used in characterizing sums; for *any* set of regular identities would appear to be equally well suited to defining a basic form of composition, so long as they conform to Anti-cyclicity. Indeed, I would conjecture that any such set of principles will in fact correspond to a form of composition and a corresponding form of whole. How the resulting forms of composition and whole might be organized is an interesting question, but it should be apparent that the approach will lead to an infinitude of different forms of composition, each differing from one another in how exactly the identity of the resulting wholes is to be determined.

    Thus from the present perspective, the operation of summation is one tiny star in a vast mereological firmament and there is no reason to think of it as possessing better mereological credentials than any of the others. It is indeed distinguished by the fact that it is blind to all

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[13]Though from among the twelve possible variants, two (ACL𝐏 and A𝒞L𝐏) should not be allowed since they are in violation of Anti-cyclicity.

[14]Or to put it algebraically, the intended model for the principles should be isomorphic to a 'word algebra' over the 'generators' or given elements. This is a place in which the syntax of the theory can be seen as a guide to its intended interpretation.

aspects of the whole other than the parts from which it was formed. But it is hard to see why sensitivity to structure in the other operations should somehow be an impediment to their ability to form wholes.

Vast as the ensuing ontology of forms of composition may be, they do not by any means exhaust all the forms of composition that there are. They do not include the operation of predication, for example, whereby the subject-predicate proposition that x F's is formed from a predicative component F and a subject component x; they do not include the various logical operations, such as conjunction and negation, whereby complex propositions may be formed from simpler propositions; and they do not include the various kinds of structural whole in which the various components of a whole are taken by its very nature to be related in some way. It seems that, in addition to the more 'structural' forms of composition which we previously considered there are more 'substantive' forms of composition, in which various non-structural connections between the parts also play a role. But the hope is that, even in these other cases, we should be able to characterize the various form of composition by means of an appropriate choice of principles from our 'template'.

§7 Derived Part

It is plausible to suppose that every form of composition is either basic or derived from basic forms of composition and that every relation of part is either basic or derived from basic relations of parts. But what are these derivative forms of composition or relations of part and how are they to be derived from the basic forms and relations?

There are two obvious routes by which derivative relations of part may arise. One is through Subsumption and is illustrated by the case of *small part*. When x is a small part of y, then this is something that holds in virtue of (or partly in virtue of) the more basic relationship of x being a part of y.

The other route is through Chaining. Suppose, for the sake of illustration, that a pint of milk is a part of a gallon of milk in one way and that Socrates is a part of singleton Socrates in another way. There would then be a way in which the pint of milk is a part of the singleton gallon of milk. But this is a relationship of part that would hold in virtue of the pint being a part of the gallon (in one way) and the gallon being a part of the singleton gallon (in another way). The given relationship of part is mediated, so to speak, though these other relationships of part.[15]

Similarly, in the case of composition, Subsumption or Chaining provide the two obvious routes by which one form of composition might be derived from others. Thus the operation of summation might be restricted to material things or to events; or we might restrict the application of the set-builder to one or two objects and then chain it to obtain the operation that yields the standard set-theoretic representation {{x}, {x, y}} of the ordered pair from the component terms x and y.

Are these the only means by which the non-basic relations of part or forms of composition can be derived from the basic relations or forms? I believe that the answer to this question is 'yes' in the case of part but, somewhat surprisingly, is 'no' in the case of composition. There are, it seems, other, more 'creative', ways in which the basic forms of composition can give rise to new forms of composition. Consider the case of sets. The union

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[15]Since any relation of part must be transitive, it is only the *ancestral* of the restriction or the chaining that can strictly be said to be a relation of part.

$^\wedge(x_1, x_2, ...) = x_1 \cup x_2 \cup ...$ of the sets $x_1, x_2, ...$ is intuitively a way of composing a whole from its parts. But the application of this operation is not something that holds via the subsumption or chaining of other more basic operations. When $y = {}^\wedge(x_1, x_2, ...)$, it is not as if y is the set of x_1, x_2, ... and nor do there appear to be other more basic compositional operations by which y might be obtained from $x_1, x_2, ...$. Similarly for the union of multi-sets (whereby [x, x, y] $\cup$ [x, y], say, is [x, x, x, y, y]) or for the juxtaposition of sequences (whereby <a, b> * <c, d, e>, say, is <a, b, c, d, e>). They intuitively provide us with a way of composing a whole from its parts, even though their application cannot be subsumed under the multi-set-builder or sequence-builder or obtained, in some other way, through the subsumption or chaining of more basic operations.

Within the operational framework, we can provide a general account of how these new forms of composition arise. Using the case of sets as an example, we have the following definition of the operation of set-theoretic union in terms of the set-builder:

$$^\wedge(\{x_{11}, x_{12}, ... \}, \{x_{21}, x_{22}, ... \}, ...) = \{x_{11}, x_{12}, ... x_{21}, x_{22}, ..., ... \}.$$

In other words, the union operation, in application to given sets, will result in the set which is built up from the objects from which the given sets were built up.

This suggests a general recipe for defining a 'horizontal' operation $^\wedge\Sigma$ in terms of a given 'vertical' operation Σ. For we may set:

$$^\wedge\Sigma(\Sigma(x_{11}, x_{12}, ...), \Sigma(x_{21}, x_{22}, ...), ...) = \Sigma(x_{11}, x_{12}, ... x_{21}, x_{22}, ..., ...).[16]$$

If, for example, Σ is the sequence-builder < ... >, then the corresponding operation $^\wedge{}_{<...>}$ of juxtaposition will be defined by:

$$^\wedge{}_{<...>}(<x_{11}, x_{12}, ... >, <x_{21}, x_{22}, ... >, ...) = <x_{11}, x_{12}, ... x_{21}, x_{22}, ..., ... >.$$

Where Σ is hierarchical, $^\wedge\Sigma$ will be flat; and where Σ is already flat, as with summation, then Σ and $^\wedge\Sigma$, in application to objects of the form $\Sigma(...)$, will yield the same result. Thus $^\wedge\Sigma$ constitutes a method for 'flattening' a hierarchical operation.

I am inclined to believe that this method for deriving forms of composition is of general application and that the method, in conjunction with Subsumption and Chaining, is capable of providing all the means by which one form of composition may be derived from others, although I have no proof that other means of deriving new forms of composition are not available.

The new forms of composition will give rise in the usual way to corresponding relations of part-whole. Thus corresponding to set-theoretic union is the relation of set-inclusion, corresponding to multi-set union is the relation of multi-set-inclusion, and corresponding to juxtaposition is the relation of subsequence. Even though the horizontal operation will be definable in terms of the vertical operation (set-theoretic union in terms of the set-builder, for example), it will not in general be possible to define the corresponding horizontal relation of part

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[16]Of course, the acceptability of this definition depends upon its not being sensitive to the way the Σ-wholes are represented on the left: if $\Sigma(x_{11}, x_{12}, ...) = \Sigma(y_{11}, y_{12}, ...)$, $\Sigma(x_{21}, x_{22}, ...) = \Sigma(y_{21}, y_{22}, ...)$, ... hold, then $\Sigma(x_{11}, x_{12}, ... x_{21}, x_{22}, ..., ...) = \Sigma(y_{11}, y_{12}, ..., y_{21}, y_{22}, ..., ...)$ should hold. I have assumed, for simplicity, that the relevant Σ-terms are always defined. The account is readily modified to take care of the cases in which this assumption fails; and there are some other variants on the method that I also have not discussed.

in terms of the corresponding vertical relation (set-theoretic union in terms of the ancestral of membership). We should therefore recognize that there are basic relations of part that do not correspond to basic forms of composition; and this provides yet another reason for adopting the operational approach, since it is only at the level of the operations that the relationships between these different notions of part can be made clear.

It has often been thought that if sets have parts then one must make a choice between taking their parts to be members or subsets, that they cannot be both. There is perhaps something to be said for thinking that there can only be one 'canonical' or basic way of constructing a whole from its parts. But this thought should not be taken to exclude the possibility that there might be other 'non-canonical' or 'derived' ways of constructing a whole from its parts. And if I am right, then this possibility is realized in the case of sets. The canonical way of constructing a set is from its members. But this is perfectly compatible with constructing the set from its subsets and taking both its members and its subsets to be parts.

§8 Hybrid and General Part

Given the specific relations of part, we may derive various *hybrid* relations of part. Suppose, for example, that we are given the relations of set-theoretic and mereological part - which we may designate as ε-part and m-part. We may then take one object to be an ε,m-part of another if it is an ε-part or an m-part or an m-part of an ε-part or an ε-part of an m-part, or an m-part of an ε-part of an m-part, and so on. More generally, if K is a family of specific ways of being a part, we may take an object to be a K-*part* of another if x and y can be linked by relationships of k-part for k in K.

The hybrid relationships of part often appear to have an artificial character, as if there were something illegitimate in passing from the one way of being a part to the other. Thus even though a soldier is part of a regiment and his head is part of the soldier, it is odd to say that his head is part of the regiment. Likewise, even though a gallon of milk g may be part of the singleton of the gallon and a pint of milk may be a part of the gallon, it is odd to say that the pint is a part of the singleton.

The oddity of these cases might be used as an additional test for distinguishing between the different specific ways of being a part. For if the pint were a part of the gallon in the same way in which the gallon was a part of the singleton, there should be no oddity in saying that the pint was a part of the singleton. However, I do not think that the oddity of these cases should lead one to *deny* that the relevant objects are parts. For if the pint is in the gallon, in the sense of 'in' appropriate to part, and the gallon is in the singleton, then it is hard to see how the pint could fail to be in the singleton.

To each family of specific ways of being a part will also correspond a family of compositional forms; and just as we may attempt to characterize the principles by which the specific compositional forms are governed, we should also attempt to characterize the principles by which they interact. How for example do sets interact with sequences or sets with sums or all three interact with one another? The critical question here is one of inter-species identity. When is a whole of one kind identical to a whole of some other kind?

My inclination is to suppose, as before, that the only identities which hold are the ones which can be shown to hold on the basis of the defining principles for the operations in question. So, since the operation for forming sums is subject to Collapse, the unit sum of a set must be identical to that very set but, since there is nothing in the identity principles governing sums and

sets that would force the sum of two or more sets to be identical to any set, no sum of two or more sets will ever be a set.[17] This conclusion is contrary to the view of D. Lewis, *Parts of Classes* (Oxford: Blackwell, 1991), who takes each set to be the sum of its singleton subsets. But Lewis's view, in my opinion, rests upon conflating the derived form of composition for sets with the mereological operation of sum and has no intuitive support.

Among the hybrid relations of part, of special interest is the relation of K-part where K is the family of *all* the specific ways of being a part. This is the relation of K-part that holds between two objects when they may be linked by relationships of k-part without restriction on k. We might call it the *general* relation of part and it is a relation that holds between x and y whenever x is in any way whatever a part of y.

The general relation, through its transitive application, provides us with a highly artificial sense of part; and it is not clear that we would ordinarily have a use for a notion of such wide generality, since our interest is usually in some specific relation of part or in some small family of closely connected relations of part. However, the problem of establishing that the general relation is antisymmetric is of enormous technical and philosophical interest.

It seems clear, in the first place, that the general relation should be anti-symmetric (thinking of it now as the result of chaining the various specific relations of part and not necessarily as a relation of part in its own right). For suppose that x is a proper part of y. There would then appear to be a broad sense of 'more' in which it is correct to say that there is more to y than x. Suppose now that x_1 is a proper part of x_2 in some specific way, x_2 a proper part of x_3 in some specific way, ..., and x_{n-1} a proper part of x_n in some specific way. There should then be more to x_2 than x_1, more to x_3 than x_2, ..., and more to x_n than x_{n-1} and hence more to x_n than x_1. Hence x_1 and x_n in such a sequence cannot be the same - which is just what is required for the general relation to be anti-symmetric.

However, anti-symmetry is not simply one condition among others that the general relation of part should satisfy. It provides a key test for our having a coherent conception of part in the first place. For under the pluralist approach, we have wished to maintain that there are many different ways in which one object can be part of another (through membership, subset, mere part etc.). But what assurance can we have that our judgements in all of these cases are informed by a single coherent conception of part? A key - perhaps *the* key - test of coherence is that the resulting general relation of part should be anti-symmetric. For it would be too much of a coincidence, so to speak, if anti-symmetry held even though there was no single coherent conception of part in virtue of which it could be seen to hold.

Despite its importance, it is far from trivial to establish that the test is met. Indeed, as we have already seen, it is not even straightforward, once we adopt the operational approach, to show that the basic relations of part-whole are antisymmetric, since this must be shown by appeal to the underlying properties of the compositional form. But even if the basic relations of part are assumed to be anti-symmetric, there is still no assurance or obvious way to establish that the general relation of part will be anti-symmetric. For what is to stop one relation of part-whole taking us from x to a distinct object y and some other relation or relations of part-whole taking us back from y to x?

We may illustrate in miniature the nature of the difficulty by considering the case of membership and subset. It is straightforward to show that each of these relations is anti-

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[17]Again, this idea may be made precise by appeal to the appropriate word algebra.

symmetric, the first because we never have infinitely descending membership chains, with $x_2 \in x_1$, $x_3 \in x_2$, ..., and the second because mutual subsets will have the same members and will therefore be the same sets. But what of the hybrid relation, in which we may arbitrarily 'chain' membership with subset? Is it anti-symmetric? In fact it is, although it is not altogether straightforward to establish that this is so.[18]

This is but one case; and what we require of the general theory of part is a general demonstration that the chaining of the different ways of being a part can lead us from one object, through other objects, back to itself.

§9 Priority and Generation

One object may be (*ontologically*) *prior* to another in the sense that it is possible to provide an explanation of the identity of the one object, to explain *what it is*, in terms of the other object. Thus it is plausible to suppose that the members of a set (within the usual cumulative hierarchy) are prior to the set, since one may account for the identity of the set by saying that it is the set with these members; and it is plausible to suppose that the two pints of milk that make up a quart of milk are prior to the quart, since one may account for the identity of the quart by saying that it is the quantity of milk that is made up of those two pints.

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[18]Here is one possible method of proof. Let $TC(x)$ be the transitive closure of x. Show: (i) $x \in y \supset TC(x) \subset TC(y)$; and (ii) $x \subseteq y \supset TC(x) \subseteq TC(y)$. Consider now a sequence $x_1, x_2, ..., x_n$, in which $x_i \in x_{i+1}$ or $x_i \subseteq x_{i+1}$ for $i = 1, 2, ..., n-1$. Then either the latter case always holds, in which case $x_1 \subseteq x_n$ and $x_1 \neq x_n$, or the former case sometimes holds, in which case it follows by (i) and (ii) that $TC(x_1) \subset TC(x_n)$ and so, again, $x_1 \neq x_n$.

In many cases, including those of current interest to us, an explanation of identity will proceed via the application of an appropriate operation. In other words, there will be an operation - taking objects into objects - and the identity of a given object will be explained to be the result of applying this operation to certain other objects. The given object *is* the result of applying the operation, not just in the innocuous sense of being identical to the result, but also in the philosophically significant sense of having its identity thereby explained. So, for example, we may account for the identity of the set {Socrates, Plato} by saying that it is the result of applying the set-builder to Socrates and Plato and we may account for the identity of the proposition that Socrates is wise by saying that it is the result of applying the operation of predication to Socrates and the property of being wise.[19]

We might say that the application $y = \Gamma(x_1, x_2, x_3, ...)$ of an operation Γ is *generative* if there is an explanation of the identity of y as $\Gamma(x_1, x_2, x_3, ...)$; and we might say that the operation Γ is itself *generative* if it permits a generative application. Thus both the set-builder and the operation of predication will be generative in this sense.

It should not be thought that every application of a generative operation must itself be generative. Consider the operation for forming sums. I take it that we can account for the identity of the sum of Socrates and Plato in terms of its being the result of applying this operation to Socrates and Plato. So the operation is indeed generative. However, we cannot account for the identity of Socrates in terms of his being the result of applying this operation to Socrates himself, since the purported explanation of identity is circular. As we shall see, there are other, less blatant ways, in which the application of compositional operations can lead to circularity.

We might take the *generative core* of a generative operation to be the result of restricting the operation to its generative applications. Given the generative core Γ^* of a generative operation Γ, we might define the notion of priority in broad analogy to the earlier definition of part. Thus we may say that x is a *prior component* of y if y is the result of applying Γ^* to some objects that include x - i.e. if y is of the form $\Gamma^*(x_1, x_2, ...)$, where x is a among the objects x_1, x_2, ... We may then define x to be *prior to* y if there is a sequence of objects $x_1, x_2, ... x_n$, $n > 1$, for which $x = x_1$, $y = x_n$ and x_i is a prior component of x_{i+1} for $i = 1, 2, ..., n-1$. Thus analogously to the case of part, prior components yield a posterior object under a single application of the generative operation, while priors also yield a posterior object under successive applications of the operation. Socrates, for example, will be a prior component of {Socrates} and {Socrates} will be a prior component of {{Socrates}, {Plato}}; and so Socrates will be prior to {{Socrates}, {Plato}}.

The above definitions are all relative to the given generative operation Γ. They therefore yield a notion of k-priority, where k is the kind of generative operation in question. But we may obtain a general notion of priority by allowing the generative operation to vary from step to step. Thus x will be prior to y in this general sense if there is a chain $x_1, x_2, ..., x_n$, beginning with $x = x_1$ and ending with $y = x_n$, in which each of the steps from x_k to x_{k+1} involves the generative application of a generative operation. And similarly when some more limited family of generative operations is in play.

It might be thought to be something of an embarrassment for the theory of priority that

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[19]These ideas are meant to relate to the essentialist framework of K. Fine's *Essence and Modality*, 'Philosophical Perspectives 8' (ed. J. Tomberlin) (1994), 1 – 16, and of his *Ontological Dependence*, Proceedings of the Aristotelian Society, v. 94 (1995), 269-290 though, in the present context, they are capable of standing on their own.

we have had to posit a generative core Γ^* in addition to the operation Γ itself. But I believe that it is in general possible to define the one in terms of the other. To this end, we suppose that certain objects are simply *given*. These are the objects whose identity does not require an explanation in terms of Γ. Thus, when Γ is the set-builder, they are the objects that are not sets and, when Γ is summation, they are the objects that are not sums or, rather, the objects that do not need to be *seen* as sums.

We now 'generate' objects in stages. At stage 0 are the givens; at stage 1, we add the objects that result from a single application of the generative operation Γ to the givens; at stage 2, we add the objects that result from a single application of Γ to the objects from stages 0 and 1; and, in general, at any ordinal stage α, we add the objects that result from a single application of Γ to objects from the earlier stages:

stage 0: $x_{00}, x_{01}, x_{02}, ...$ the givens;

stage 1: $x_{10}, x_{11}, x_{12}, ...$ objects obtained from stage 0 by a single application of Γ;

stage 2: $x_{20}, x_{21}, x_{22}, ...$ objects obtained from stages 0 and 1 by a single application of Γ;

...

...

The *level of* an object is the first stage at which it appears. Thus in the generation of sums from mereological atoms, the atoms will be of level 0 and all the sums will be of level 1. There will be no sum of higher level, since each sum can be obtained by summing atoms.

A *strict* generative application of Γ to the objects $x_1, x_2, ...$ can now be defined as one in which $y = \Gamma(x_1, x_2, ...)$ is of a higher level than each of $x_1, x_2, ...$. What is characteristic of strict generation is that it moves us up the ontological hierarchy, with a generated object being of a higher level than the objects from which it is generated. Given the notion of strict generation, we can then define the strict component priors and the strict priors in the usual way. Thus each member of a set will be strictly prior to the set and each (mereological) atom will be strictly prior to any sum of atoms that contains it as a proper part.

There is also a weak sense of generation that may be defined. Suppose that a, b and c are three distinct atoms. Then a, b and c will be strictly prior to a + b + c, as we have seen, but a + b will not be strictly prior to a + b + c, since a + b is at the same level as a + b + c. However, there would appear to be a weak sense in which a + b + c can be generated from a + b and c and in which a + b is thereby prior to a + b + c, even though a + b and a + b + c are at the same level.

This notion of priority must be defined with some care so as to avoid cycles. Let us say, in the first place, that $y = \Gamma(x_1, x_2, ...)$ is a *putative* generative application of Γ if y is of a higher or of the same level as each of $x_1, x_2, ...$. This gives us the notions of a putative prior component and of a putative prior in the usual way. We now say that the application $y = \Gamma(x_1, x_2, ...)$ of Γ is a *weak* generative application if it is a the putative generative application and if y is not putatively prior to any of $x_1, x_2, ...$. We can get from $x_1, x_2, ...$ to y without an ascent in level but not from y to any of $x_1, x_2, ...$. This then gives us the required notion of weak priority; and a + b, for example, will be weakly prior to a + b + c, since a + b + c = Σ_m(a + b, c) is putatively generative application of the summation operator Σ_m in which a + b + c is not putatively prior to either a + b or c.

§10 The Generation of Parts

Every basic compositional operation is plausibly taken to be generative. For how could

there be a basic way of forming wholes unless it were sometimes explanatory of the identity of the whole that was thereby formed? However, it is somewhat questionable whether any basic generative operation is also compositional and, indeed, it seems to me that some basic generative operations are in fact *de*-compositional. Far from serving to account for the identity of a whole in terms of its parts, they serve to account for the parts in terms of the whole.[20]

One example is that of a 'segment' or 'restriction' of a material thing. Let us suppose that the universe consists of physical atoms which are physically indivisible but of finite volume. We might then distinguish between the upper and lower parts of the atom (relative to its orientation at a given time); and it is plausible that the atoms are to be taken as givens, there being no explanation of their identity in more basic terms, while the identity of the upper and lower parts of an atom is to be explained in terms of their *being* the upper and lower parts of the atom. Thus the account of the part is in terms of the whole rather than the other way round.

Within the generative approach, this suggests that there is a generative operation, /, of segmentation which, in application to a material thing x and a spatio-temporal extension R, will result in the restriction x/R of the object x to R (assuming that there is such an object).[21] Segmentation generalizes the familiar idea of a time-slice or temporal part, the temporal parts of a thing being the special case in which the restriction is to an instantaneous slice of space-time.

Just as with the compositional operations, the present operation might be taken to be defined by various of the principles to which it conforms. Let us use |x| for the spatio-temporal extension of x and let us use $\cup$ and $\cap$ for the union and intersection of space-time regions. These principles might then be taken to comprise:

(i) there is an object x/R when $|x| \cap R$ is non-empty;

(ii) x = x/V (where V is the whole of space-time);

(iii) x/R = x/S if $|x| \cap R = |x| \cap S$;

(iv) $|x/R| = |x| \cap R$;

(v) $\Sigma_m(x/R_1, x/R_2, ...) = x/(R_1 \cup R_2 \cup ...)$.

Clause (i) is an application condition; it states that there will be a segment x/R if the restrictor R overlaps in extension with the parent object x. Clauses (ii) and (iii) are identity conditions: the first states that the 'degenerate' restriction of x to the whole of space-time is x itself; and the second states that two restrictions of an object are the same if they result in the same restriction on its extension. Clause (iv) is a presence condition; it states that the extension of a restriction is the intersection of the extension of the parent object with the restrictor.

Clause (v) states that the sum of the restrictions of a particular object is the restriction of the object to the union of its restrictors; and it provides us with a sense in which the operation of segmentation is 'inverse' to the operation of summation. In contrast to the earlier defining principles that we have considered, it is not free standing but makes reference to another

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[20]The idea that a whole may be prior to its parts has had a long philosophical history. See J. Schaffer, *Monism: The Priority of the Whole*, Philosophical Review 119.1 J. (2010), 31-76, for a recent attempt, somewhat different from my own, to rehabilitate the idea.

[21]If we distinguish between existence and extension and if Σ_m is accordingly taken to be the operation of compounding rather than of aggregation, then the segments x/R must be so defined that they exist at any time at which their parent object x exists. I avoid this complication in what follows and also take no account of the modal dimension of segments.

generative operation, that of summation; and I suspect that it generally true of generative operation for forming wholes from parts that they must always be understood as some kind of inverse of a complementary operation for forming parts from wholes. Parts in this generic sense are always prior to wholes even though particular wholes may sometimes be prior to their parts.

Our formal definition of priority enables us to see how it is that the parent object is prior to its restrictions. For if we start off with the givens (the atoms, say), then the operation of summation will only take us so far in generating the objects of the ontology. We may obtain sums of atoms, but not segments of atoms or sums of segments or segments of sums. It is only through the application of segmentation that these other objects can be introduced into the ontology; and the resulting segments must therefore be posterior (indeed, strictly posterior) to the larger 'wholes' from which they were obtained.

Another significant example of a decompositional operation is given by abstraction. Consider a complex proposition, such as the proposition that Socrates is wise and Socrates is a philosopher (Ws & Ps). Then the 'canonical' analysis of this into parts is as the conjunction of the proposition that Socrates is wise and the proposition that Socrates is a philosopher, where these propositions, in turn, are the result of predicating the respective properties of being wise and being a philosopher of Socrates. But there would appear to be another decomposition of the proposition into parts, under which it is the result of predicating the complex property of being a wise philosopher ($\lambda x(Wx \& Px)$) of Socrates. This decomposition does not perhaps give an *analysis* of the proposition, i.e. an account of what it is, but the complex property of being a wise philosopher does seem to be involved in the analysis of other propositions. The proposition that someone is a wise philosopher ($\exists x(Wx \& Px)$), for example, is plausibly taken to be the result of predicating second-order 'existence' of this property.

What account can we give of the identity of such properties within a generational ontology? One possibility is to treat the property of being a wise philosopher as the conjunction of the properties of being wise (λxWx) and of being a philosopher (λxPx). But such an approach, when applied across the board, would lead to an unwieldy parallel treatment of the various logical operations, such as conjunction or quantification, and might not even be workable in certain cases.[22] A more promising approach, to my mind, is to take the property of being a wise philosopher to be the result of abstracting the individual Socrates from the proposition that Socrates is wise and Socrates is a philosopher. Thus we *start* with the proposition that Socrates is wise and Socrates is a philosopher (or with some other individual in place of Socrates) and then take the property of being a wise philosopher to be the result of 'removing' or abstracting that individual from the proposition.

In general, we may suppose that there is a generative operation, Λ, of abstraction which, in application to a complex C and an individual x, will result in the abstract $[\Lambda xC]$ that results from removing the individual x from the complex C. As with segmentation, we may take Λ to be 'defined' by means of an appropriate set of principles. Let us use the notation 'C(x)' for the corresponding operation of concretion, whereby C(x) is the result of 'completing' C with x (predication corresponding to the case in which C is a property). We would then expect to have the two operations related by the following fundamental principle:

$$[\Lambda xC](x) = C,$$

according to which the result of re-concretizing the abstraction on a complex will be that very

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[22]Difficulties of this sort are discussed in K. Fine, *Semantic Relationism* (Oxford: Blackwell, 2007), section 1.4.

complex.[23] Given that concretion is a compositional operation, it will follow that the result of abstracting on a complex is a part of the complex; and so we can indeed say that the complex property of being a wise philosopher is a *part* of the proposition that Socrates is wise and Socrates is a philosopher, though a posterior rather than a prior part.

Again, our formal definition of priority enables us to see how it is that the complex properties or abstracts are posterior to the propositions or complexes to which they belong. For if we just start off with the basic ingredients from which propositions are constructed (simple properties, individuals etc), we will not be able to obtain complex properties or the quantified propositions that are constructed from them. It is only through the application of abstraction or the like that these other objects can be introduced into the ontology; and the resulting abstracts must therefore be (strictly) posterior to the complexes from which they were obtained and of which they are a part.

Although the operations of segmentation and abstraction appear to be completely different and to have application in completely different spheres, we see from our analysis that there is a very deep analogy between them. For just as segmentation 'manufactures' parts which can be put together by the operation of summation to give us back the object from which they were obtained, so abstraction will 'manufacture' parts which can be put together by the operation of concretion (or predication) to give us back the complexes (or propositions) from which they were obtained. And this suggests that there should be other pairs of operations which behave in a similar way and give rise to other forms of generated or 'manufactured' part.

The theory of generation also enables us to 'round out' the theory of part in a way that would not otherwise be possible; and let me conclude by briefly discussing two key respects in which this is so. In the first place, it is natural to suppose that wholes may be classified into kinds - into sums, sets, subject-predicate propositions etc. Now the obvious definition of a kind of whole on the operational approach is that an object will be of a given kind k if it is the result $\Sigma_k(x_1, x_2, \dots)$ of applying the associated compositional operation Σ_k to a number of objects x_1, x_2, ... But then any object must be classified as a sum, since the unit sum $\Sigma_m(x)$ of any object x is that very object; and this is clearly not our intention. To get round this difficulty, we might take a whole of kind k to be an object that is the result $\Sigma_k(x_1, x_2, \dots)$ of applying the operation Σ_k to a number of objects x_1, x_2, ..., one of which is not identical to x. But the null set would not then be a set-theoretic whole on this definition and, more seriously, each physical atom, from our earlier example in which it was decomposable into segments, would be a sum, even though we would not wish to classify it as a sum, i.e. as something that by its very nature was a sum.

We may appeal to the theory of generation to solve this difficulty. For a *genuine* whole of a given kind k may be taken to be an object that has an explanation of its identity in terms of the associated operator Σ_k. In other words, x should be identical to $\Sigma_k(x_1, x_2, \dots)$ for some (weak) *generative* application of Σ_k to x_1, x_2, ... This definition avoids the previous difficulties, for the null set would be obtained through a generative application $\Sigma_\in()$ of $\Sigma_\in$ to zero objects and so would be classified as a set-theoretic whole, while the unit sum $\Sigma_m(x)$ and the sum $\Sigma_m(a_1, a_2)$ of the segments a_1 and a_2 of an atom a would not be obtained through generative applications of

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[23]Despite the similarity in notation to 'λxA', 'Λ' in '$[\Lambda xC]$' is not a variable-binding operator but a straight binary operator, applying to the two terms 'x' and 'C'. Under a more suggestive notation, we might write $[\Lambda xC]$ as (C - x) and C(x) as (C + x). The principle then states that (C - x) + x = C.

Σ_m and so would not be classified as sums.[24]

In the second place, the admission of generated parts makes it much more difficult to show that the general relation of part is anti-symmetric. For wholes are not simply 'built up' from their parts, parts are also 'built down' from their wholes; and so we must now envisage such possibilities as an object y being both built up into the distinct object x and built down into the object x, thereby creating a violation of anti-symmetry. It must therefore be shown to be somehow integral to the way in which wholes are generated from parts and parts from wholes that such possibilities cannot arise; and so even though anti-symmetry is a thesis lying purely within the theory of part, it is only by appeal to the more general theory of generation that it can be shown to be true.

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[24] P. Van Inwagen, in *Can Mereological Sums Change Their Parts*, Journal of Philosophy CIII.12 (2006), 614-30, denies that there is a nontrivial kind, *sum*. But this is highly counter-intuitive and, if I am right, it rests on a misconstrual of the relationship between *sum* and *sum of*.