Some Puzzles of Ground

In recent years there has been a growing interest in the concept of ground – of one thing holding in virtue of another; and in developing an account of ground, a number of philosophers have laid down principles which they regard as unquestionably true of the concept.[1] The purpose of this note is to show that these principles are in conflict with seemingly impeccable principles of logic. Thus a choice must be made; either one or more of the metaphysical principles or one or more of the logical principles should be given up.

Some philosophers – and especially those already unsympathetic to *ground* – may think that the conflict reveals some underlying defect in the concept. For if acceptance of the concept of ground has such untoward consequences, then this can only be because the concept was no good in the first place. My own view – which I suggest towards the end of the paper – is quite different. It is not that considerations of ground should be ignored or even that the principles of ground should be given up in the light of their conflict with the principles of logic. Rather we need to achieve some kind of reflective equilibrium between the two sets of principles, one that does justice both to our logical intuitions and to our need for some account of their ground. Thus the conflict, far from serving to undermine the concept of ground, serves to show how important it is to arriving at a satisfactory view of what in logic, as in other areas of thought, can properly be taken to be hold.

The puzzle to which the conflict of principles gives rise bears some resemblance to the paradoxes of self-reference. It is not itself a paradox of self-reference: the puzzle, on the one side, makes no direct use of self-reference; and the paradox, on the other side, makes no direct appeal to the notion of ground. But considerations of ground are often used to motivate certain solutions to the paradoxes; and the puzzle makes clear the reasoning behind these considerations and brings out the critical role played by the notion of ground.[2]

§1 Informal Argument

Let me first give an informal presentation of the puzzle and then give a more formal presentation in which the underlying notions and assumptions are made explicit.

Here is an especially simple version of the puzzle. Surely, everything exists. Let f_0 be the fact that everything exists. Then everything exists partly in virtue of f_0's existing. But f_0 exists partly in virtue of everything existing, since f_0 is the fact that everything exists; and so everything exists partly in virtue of everything existing. Which is an absurdity, given that nothing can hold in virtue of, or partly in virtue of, itself.

The above reasoning involved facts. But there is a similar version of the puzzle for propositions. For every proposition is either true or not true. Let p_0 be the proposition that every proposition is true or not true. Then every proposition is true or not true partly in virtue of p_0 being true or not true. But p_0 is true or not true partly in virtue of p_0 being true if p_0 is true or partly in virtue of p_0 not being true if p_0 is not true. But p_0 is true; and so p_0 is true or not true

[1] They include Audi [2010], Batchelor [2009], Correia [2005], Correia[2009], Gideon [2010], Schaffer [2010] and Schneider [2010]. I should like to thank Fabrice Correia, Jose Martinez, and Graham Priest and audiences at the 2009 Boulder Conference on dependence, the 2009 workshop in Fortaleza, the 2009 ASL Conference in Notre Dame, the 2009 Northwestern Conference in Bellingham, the 2009 Australasian Association of Philosophy Conference and the 2009 meeting of the Portugese Analytic Philosophy Association for helpful discussion.

[2] I especially have in mind the kind of solution to the semantic paradoxes to be found in Kripke [1975].

partly in virtue of p_0 being true. But p_0 is true partly in virtue of every proposition being true or not true, since p_0 is the proposition that every proposition is true or not true; and so every proposition is true or not true partly in virtue of every proposition being true or not true. Which is again an absurdity.

There is an exactly analogous version of the puzzle with sentences in place of propositions. The starting point is now that every sentence is true or not true and the trouble-maker is the sentence 'every sentence is true or not true'. The reasoning is then as before but with reference to propositions replaced by reference to sentences.

There are also versions of the puzzle which use an existential generalization in place of the universal generalization. In the case of facts, we start with the claim that *some*thing exists. Let f_1 be the fact that something exists. Then something exists partly in virtue of f_1's existing. But f_1 exists partly in virtue of something existing; and so something exists partly in virtue of something existing. And similarly for the other cases.

There are no doubt other versions of the puzzle, appealing to somewhat different entities or to somewhat different features of the entities. But the general nature of the difficulty will be the same; and I hope that nothing will be lost by restricting our attention to the present cases.[3]

§2 Notions

In preparation for a more formal presentation of the puzzle, let us lay out the notions and assumptions that we shall need to use.

We shall need to appeal, in the first place, to certain logical notions. These are: negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$), absurdity ($\perp$), universal quantification ($\forall xA$), existential quantification ($\exists xA$), identity ($=$), and existence ($E(x)$). For present purposes, we can take $E(x)$ to be stipulatively defined as $\exists y(x = y)$ (there is something to which x is identical).

We shall also need to appeal to the extra-logical notions of *fact*, *proposition*, and *sentence*. Where A is a sentence, we use [A] for 'the fact that A', |A| for 'the proposition that A', and $\ulcorner A \urcorner$ as a name for the sentence 'A' itself[4]. We use T(x) ambiguously to indicate that x is a true sentence or proposition (depending upon the context).

Finally, we shall appeal to the idea of one thing holding *in virtue of* – or being *grounded in* – another. We take ground to be *partial* ground. Thus in saying that p is grounded in q, we are allowing that p may hold *partly* in virtue of q. We also take ground to be sentential operator, applying to two sentences to form a sentence. Thus a canonical statement of ground will take the form:

it is the case that B (partly) in virtue of its being the case that A,

where A and B are sentences. For symbolic purposes, we write this as:

$A \prec B$ (read: A helps ground B).[5]

Other ways to formulate statements of ground are possible. Thus we may treat ground as a relation between propositions and propositions or between facts and facts or even between facts and propositions - with the fact grounding (the truth of) the proposition. I suspect that the puzzle will straightforwardly apply to these other formulations via a suitable translation from the

---

[3] Dennis Whitcomb has pointed out that related argument might be used against God's omiscience. For suppose God is omniscient (i.e. that He knows everything that is the case). Then His ominiscience is partly grounded in his knowing that He is omniscient (this being one of the things he knows); and his knowing that He is ominiscient is partly grounded in His being omniscient.

[4] For the sake of definiteness, we might take $\ulcorner A \urcorner$ to be a structural-descriptive name for 'A'.

[5] For further discussion of the relevant concept of ground, see Correia [2010], Fine[2009] or Rosen[2010].

sentential mode. Thus instead of saying ‘A $\succ$ B’, we may say ‘|A| grounds |B|’, using a predicate on propositions in place of the sentential operator; and the presentation of the puzzle in the sentential mode will thereby translate into a presentation in the propositional mode (and similarly for the other cases).

§3 Assumptions

Let me list all of the assumptions that will be used in the various formulations of the puzzle. The assumptions divide into three groups, corresponding to the three types of notion. Within a given group, there may also be subdivisions according as to whether the argument concerns facts or propositions or sentences. Since the argument for sentences is so similar to that for propositions, I shall not usually give it separate treatment.

I. Logical Assumptions

These can be stated in purely logical terms and are commonly taken to be logical truths.

Universal Middle: $\forall x(A(x) \vee \neg A(x))$
(Everything is an A or not an A)

Particular Middle $\exists x(A(x) \vee \neg A(x))$
(Something is an A or not an A)

Universal Existence: $\forall x E(x)$
(Everything exists)

Particular Existence $\exists x E(x)$
(Something exists)

II. Extra-Logical Assumptions

These assumptions concern the notions of *fact*, *proposition* and *sentence*. We use ‘A’ and ‘B’ and the like for arbitrary sentences; and we use $A_1, A_2, \ldots, A_n$ / B to indicate that B can be inferred from $A_1, A_2, \ldots, A_n$. We shall *not* assume the validity of conditional proof, according to which $A \supset B$ can be inferred from $A_1, A_2, \ldots, A_n$ if B can be inferred from $A_1, A_2, \ldots, A_n$, and A. In this way, we can see that the reasoning of the puzzle will hold in various logics, sometimes proposed in connection with the semantic paradoxes, for which the rule of conditional proof and various other indirect rules are not valid. Of course, if one were willing to accept the conditional statement $A_1 \supset (A_2 \supset \ldots \supset (A_n \supset B) \ldots)$ corresponding to the inference rule $A_1, A_2, \ldots, A_n$ / B, then the conditional statement could be substituted for the rule and application of the rule could be replaced with application of modus ponens.

Factual Existence $A / \exists f(f = [A])$
(Given A, then there is the fact that A)

Propositional Existence $\exists p(p = |A|)$

(There is the proposition that A)

Sentential Existence $\exists s(s = \ulcorner A \urcorner)$
(There is the sentence 'A')

Truth Introduction $p = |A|, A \,/\, T(p)$
$s = \ulcorner A \urcorner, A \,/\, T(s)$
(Given A, then the proposition that A or the sentence 'A' is true)

For our purposes, we could state Proposition and Sentential Existence in the weaker forms:

$$A \,/\, \exists p(p = |A|)$$
$$A \,/\, \exists s(s = \ulcorner A \urcorner)$$

Thus the existence of the proposition or sentence could be made to depend, as in the case of facts, on its content being the case. This might not matter much for sentences but it could be of significance for propositions if it was thought that any proposition should be true or false, for the premise A could then be seen to ensure that the proposition that A was true.

III. Ground-theoretic Assumptions

We shall make use of some general and some special principles concerning ground (again stated in rule form).

General Ground-theoretic Assumptions

Transitivity $A \prec B, B \prec C \,/\, A \prec C$
(Given that A helps ground B and B helps ground C, then A helps ground C)

Irreflexivity $A \prec A \,/\, \perp$
(It cannot be supposed that A helps ground itself)

Factivity $A \succ B \,/\, A$
$A \succ B \,/\, B$
(Given that A helps ground B then A and B are the case)

All of the central arguments will go through without the benefit of Factivity. Note that we do not assume that the relation of grounding is well-founded (in the sense that infinite chains of the form $A_2 \prec A_1, A_3 \prec A_2, A_4 \prec A_3, \ldots$ are forbidden) or that the grounds of any truth that is grounded will 'bottom out' in truths that are ungrounded.

Special Ground-theoretic Assumptions

There are two sets of special assumptions about ground which we shall need to make – one concerned with the grounds for logically complex statements and the other concerned with the grounds for factual existence and propositional (or sentential) truth.

Disjunctive Grounding $A \,/\, A \prec (A \vee B)$

$B / B \prec (A \vee B)$

(Given the truth of any disjunct, it will help ground a disjunction.)

Universal Grounding $\forall x A(x), E(y) / A(y) \prec \forall x A(x)$

(Given that everything is an A and that y exists, then y's being an A helps ground that everything is an A.)

Existential Grounding $A(y), E(y) / A(y) \prec \exists x A(x)$

(Given that A(y) and that y exists, then y's being an A helps ground that something is an A.)

Factual Grounding $E(f), f = [A] / A \prec E(f)$

(Given that f exists and is the fact that A, then A helps ground that f exists.)

Propositional Grounding $T(p), p = |A| / A \prec T(p)$

(Given that p is true and is the proposition that A, then A helps ground that p is true.)

Sentential Grounding $T(s), s = \ulcorner A \urcorner / A \prec T(s)$

(Given that s is true and is the sentence 'A', then A helps ground that s is true.)

Some philosophers would be willing to accept stronger versions of Universal and Existential Grounding in which the existential supposition E(y) is absent:

$$\forall x A(x) / A(y) \prec \forall x A(x)$$
$$A(y) / A(y) \prec \exists x A(x)$$

They may be right. But since there is an issue as to whether the free variable is capable of ranging over objects not in the range of the quantifier $\forall x$ or $\exists x$, I have preferred to adopt the more cautious formulation.

It is arguable that Disjunctive Grounding should be seen to rest on two more basic principles (and similarly for Existential Grounding). These are:

Core Disjunctive Grounding $A, (A \vee B) / A \prec (A \vee B)$
$B, (A \vee B) / B \prec (A \vee B)$

(Given a true disjunct and disjunction, the disjunct will help ground the disjunction),

Disjunction Introduction $A / (A \vee B)$
$B / (A \vee B)$

For it might be thought that in claiming that A grounds $(A \vee B)$ given A, we are presupposing that $(A \vee B)$ will be the case given A. Thus given A, we first get $A \vee B$ by Disjunctive Introduction and then, given A and $A \vee B$, we get that A grounds $A \vee B$ by Core Grounding. However, the distinction is a subtle one and it is only towards the end of the paper that we will appeal to it.

§4 Formal Argument

I now present more formal versions of the arguments: first for facts from the assumption that everything exists; then for propositions from the assumption that everything is true or not true; and, finally for facts again, but from the assumption that something exists (and, by analogy, for propositions from the assumption that some proposition is true or not true).

Since we have stated the assumptions in rule form, we shall only require one logical rule of inference (and no logical axioms). This is the rule of Existential Elimination ($\exists$E), according to which C may be inferred from $\exists xB(x)$ and the auxiliary assumptions $A_1, A_2, \ldots, A_n$ if C can be inferred from A(x), E(x) and $A_1, A_2, \ldots, A_n$. Some philosophers may prefer to state the rule in the somewhat weaker form under which C can be inferred from $\exists xB(x)$ and the auxiliary assumptions as long as it can be inferred from B(x) and the auxiliary assumptions (without the additional support of E(x)). But the difference between the two formulations will not matter given that we can make the eminently reasonable inference from $\exists xB(x)$ to $\exists x(E(x) \wedge B(x))$.

<u>The Universal Argument for Facts</u>

(1) $\forall xE(x)$ (which we abbreviate to F_0) — by Universal Existence
(Everything exists)

(2) $\exists f(f = [F_0])$ — from (1) by Factual Existence
(There is the fact that everything exists)

(3)(a) $E(f_0)$
(b) $f_0 = [F_0]$ — Assumptions (in preparation for $\exists$E)
(Let f_0 be the fact that everything exists)

(4) $E(f_0) \prec F_0$ — from (1) & (3)(a) by Universal Grounding
(f_0 existing helps ground that everything exists)

(5) $F_0 \prec E(f_0)$ — from (3) by Factual Grounding
(Everything existing helps ground that f_0 exists)

(6) $F_0 \prec F_0$ — from (4) and (5) by Transitivity
(Everything existing helps ground that everything exists)

(7) $\perp$ — from (6) by Irreflexivity

The derivation of the contradiction at line (7) rests on the following assumptions: Universal Existence, Factual Existence, Universal Grounding, Factual Grounding, Transitivity and Irreflexivity.[6] Whether or not we apply the rule of reductio to line (7), the non-dialethists among us will assume that there is something unsatisfactory about deriving a contradiction from the given assumptions and that one of them should therefore be rejected.

The argument could be simplified if we were to allow that the existence of the fact that everything exists helps ground that everything exists ($E([\forall xE(x)]) \prec \forall xE(x)$). But I have preferred to stick with the more cautious formulation of Universal Grounding (and similarly for the other arguments below).

[6] As Annand Pillay has pointed out to me, in this and in the other argument for facts, we can get by with the weaker assumption of Asymmetry in place of Transitivity and Irreflexivity.

<u>Universal Argument for Propositions</u>

(1) $\forall x(T(x) \vee \neg T(x))$ (which we abbreviate to P_0) by Universal Middle
(Everything is true or not true)

(2) $\exists p(p = |P_0|)$ by Propositional Existence
(There is the proposition that everything is true or not true)

(3)(a) $E(p_0)$
(b) $p_0 = |P_0|$ Assumptions (in preparation for $\exists$E)
(Let p_0 be the proposition that everything is true or not true)

(4) $T(p_0) \vee \neg T(p_0) \prec P_0$ from (1) & (3)(a) by Universal Grounding
(p_0 being true or not true helps ground that every proposition is true or not true)

(5) $T(p_0)$ from (1) & (3)(b) by Truth Introduction
(The proposition that everything is true or not true is true)

(6) $T(p_0) \prec T(p_0) \vee \neg T(p_0)$ from (5) by Disjunctive Grounding
(p_0 being true helps ground that p_0 is true or not true)

(7) $P_0 \prec T(p_0)$ from (3)(b) & (5) by Propositional Grounding

(8) $P_0 \prec P_0$ from (4), (6) & (7) by Transitivity
(Everything being true or not true helps ground that everything is true or not true)

(9) $\perp$ from (8) by Irreflexivity

The assumptions of the argument are: Universal Middle, Propositional Existence, Universal Grounding, Truth Introduction, Disjunctive Grounding, Propositional Grounding, Transitivity and Irreflexivity. There is an exactly analogous version of the argument for sentences.

<u>Particular Argument for Facts</u>

(1) $\exists x E(x)$ (which we abbreviate to F_1) Particular Existence
(Something exists)

(2) $\exists f(f = [F_1])$ from (1) by Factual Existence
(There is the fact that something exists)

(3)(a) $E(f_1)$
(b) $f_1 = [F_1]$ Assumptions (in preparation for $\exists$E)
(Let f_1 be the existing fact that something exists)

(4) $E(f_1) \prec \exists x E(x)$ from (1) & (3)(a) by Existential Grounding

(The existence of f_1 helps ground that something exists)

(5) $\exists xE(x) \prec E(f_1)$ from (3)(a)(b) by Factual Grounding
(Something existing helps ground that f_1 exists)

(6) $\exists xE(x) \prec \exists xE(x)$ from (4) and (5) byTransitivity
(Something existing helps ground that something exists)

(7) $\perp$ from (6) by Irreflexivity

The assumptions of the argument are Particular Existence, Factual Existence, Existential Grounding, Factual Grounding, Transitivity and Irreflexivity.

There are similar versions of the particular argument for the case of propositions. In this case, all we need is the assumption $\exists x(T(x) \vee \neg T(x))$ (something is true or not true) in place of the assumption $\forall x(T(x) \vee \neg T(x))$. And similarly for the case of sentences.

Particular Existence (at line (1) above) is evident. But it can also be justified on the basis of Factual Existence. For start with any statement A that one is willing to assert and then proceed as follows:

(1) A
(2) $\exists f(f = [A])$ from (1) by Factual Existence
(3) (a) $E(f_2)$
(b) $f_2 = [A]$ Assumptions (in preparation for $\exists$E)
(4) $\exists xE(x)$ from (3)(a) by Existential Generalization

§5 The Conflict

How should we respond to the various arguments? A number of plausible assumptions seem to lead by impeccable reasoning to a contradiction. Which of the assumptions or which of the steps in the reasoning should be rejected? We are in the vicinity of the paradoxes and so it is hard to come to a definitive view on the matter. But let me first consider the various assumptions and the basis upon which they might be questioned or defended and then turn, in the next section, to the question of the options that are plausibly left open to us.

As I previously noted, the only *logical* rule of inference that we require is Existential Elimination. This should be acceptable under many different approaches, including those of classical and intuitionistic logic and the logics of Kleene's 'weak' and 'strong' truth-tables.[7] It is true that this rule might be questioned under some supervaluational approaches. For it might be thought that it could be determinately true that something is an A ($D\exists xA(x)$) even though there is nothing which is determinately an A ($\exists xDA(x)$), which is what would in general be required for a legitimate application of the rule. However, such doubts would appear to have no application in the present context. For the relevant instances of the existential premise are of the form $\exists f(f = [A])$ or $\exists p(p = |A|)$; and, in such a case, there would appear to be no indeterminacy as to which fact or proposition is in question.

[7]Let me remind the reader that, under the strong tables, a disjunction will be true if one of the disjuncts is true, regardless of whether or not the other disjunct has a truth-value, while, under the weak tables, it is required that the other disjunct should also have a truth-value; and similarly for the other cases.

The assumptions of Factual and Propositional Existence might also be questioned. They amount, in effect, to the supposition that one can have an impredicative domain – one whose members include objects 'defined' by reference to the whole domain. In the case of Propositional Existence, for example, the proposition p which witnesses $\exists p(p = |A|)$ will itself be in the range of the quantifiers that occur in A. Russell wished to ban impredicative domains. But this restriction is very severe and it is of interest to see if we can do without it.

The rules of Truth Introduction have been challenged under certain approaches to the semantic paradoxes. But it is worth noting that if we are prepared to accept Factivity for ground and the rule of Disjunctive Elimination, then it is possible to get by with the weaker assumption that a statement A cannot be grounded by the *non*-truth of the proposition that A or, more formally:

(*) $p = |A|,\ \neg T(p) \prec A\ /\ \perp$

For at line (4) in the Universal Argument for Propositions, we have:

(4) $T(p_0) \vee \neg T(p_0) \prec P_0$

By Factivity, this gives: $T(p_0) \vee \neg T(p_0)$. We now separately suppose $T(p_0)$ and $\neg T(p_0)$ (in preparation for an application of Disjunctive Elimination). We already know that the supposition of $T(p_0)$ will lead to contradiction. But the supposition of $\neg T(p_0)$ leads to the conclusion $\neg T(p_0) \prec T(p_0) \vee \neg T(p_0)$ by Disjunctive Ground and so, by Transitivity, we get $\neg T(p_0) \prec P_0$ with the help of (4). But we then have $p_0 = |P_0|$ and $\neg T(p_0) \prec P_0$, from which a contradiction follows by (*).

It is hard to see how the general principles of ground (Transitivity, Irreflexivity or Factivity) might reasonably be rejected for the relevant notion of ground. One might, of course, understand ground to be *immediate ground*. Transitivity will then fail. But the relevant sense of ground would then be one which was the ancestral of *immediate ground* and transitivity would then automatically hold and Non-Circularity would also hold as long as there were no cycles A $=A_0, A_1, ..., A_{n-1}, A_n = A$ in which each of $A_0, A_1, ..., A_{n-1}$ was an immediate ground of its successor.

A more serious objection to Transitivity and Irreflexivity would permit cycles as long as there were an informative account of how they held. Thus as a general matter A would not be a ground, either immediate or mediate, for A but there might be special cases in which A in one capacity, so to speak, was a ground for A in another capacity. The truth-teller might in this sense be a ground for itself; and perhaps the cycles involved in our various arguments might also be of this sort.

I do not wish to deny that there may be such a notion of ground. But there is still a plausible demand on ground or explanation that we are unable to evade.. For given a truth that stands in need of explanation, one naturally supposes that it should have a 'completely satisfactory' explanation, one that does not involve cycles and terminates in truths that do not stand in need of explanation. Let us now understand ground in following manner: the truth A will ground the truth C iff C stands in need of explanation and either (i) A does not stand in need of explanation and there is a completely satisfactory explanation of C in terms of A (and perhaps other truths) or (ii) A does stand in need of explanation and any completely satisfactory explanation of A can be extended to a completely satisfactory explanation of C. The principles we have laid down will then be plausible for this notion of ground, even if they do not hold for some other notion of ground.

We are therefore left with a conflict between the relevant logical assumption and the special principles of ground: between Universal Existence and Universal and Factual Ground in the case of the Universal Argument from Facts; between Universal Middle and Universal, Propositional and Disjunctive Ground in the case of the Universal Argument from Propositions; between Particular Existence and Existential and Factual Ground in the case of the Particular Argument from Facts; and between Particular Middle and Existential, Propositional and Disjunctive Ground in the case of the Particular Argument from Propositions. We have in each case a conflict between a deeply entrenched logical view, on the one side, and extremely plausible metaphysical views, on the other side.

Part of what makes the metaphysical views so plausible is the thought that there should be a classical ground for any logically complex truth (with the possible exception of the negations of atomic statements). Surely there should be a ground, or 'simpler' truth, in virtue of which any conjunctive or disjunctive truth or any universal or existential truth is the case? Combine this thought now with the view that the classical truth-conditions should provide us with a guide to ground and the present principles of ground for disjunctive and quantificational statements would appear to be forced upon us. If, for example, the truth of a disjunction is a 'matter' of one of the disjuncts being true, then how can the truth of a disjunct fail to be a ground for the disjunction? The first of these assumptions (that every logically complex truth should have a ground) might be called 'Complex Ground' and the second (that the ground, if it exists, should be in conformity with the classical truth-conditions) might be called 'Classicality'. Thus Complex Ground and Classicality, together, would appear to require the present principles.

It has often been observed that the classical truth-conditions are essentially without content under a deflationary reading of 'true'. For if ''A' is true' generally has the same content as 'A', then the classical truth-conditions for conjunction, say:

(*) $T(p \wedge q) \equiv (Tp \wedge Tq)$

will have the same content as:

(**) $(p \wedge q) \equiv (p \wedge q)$.

And similarly for when the truth-conditions are stated in rule form.

But things are quite different when we give the truth-conditions a 'direction' and read them as a guide to ground. For we can now say that p helps grounds $p \wedge q$ or that Fa helps ground $\forall xFx$, which goes well beyond the kind of triviality stated by (**). It is the ground-theoretic reading which gives real substance to the truth-conditions and, indeed, I suspect that it is this understanding of the truth-conditions that philosophers have often had in mind rather than the straight bi-conditional reading. It is, after all, the truth of the conjuncts that is seen as the condition for the truth of the conjunctions and not the truth of the conjunction that is seen as a condition for the truth of the conjuncts. But, of course, what is then giving substance to the truth-conditions, as so conceived, is the idea of a 'condition' rather than the idea of 'truth'. What matters is not so much that the *truth* of p should help ground the *truth* of $p \wedge q$ but that p should help *ground* $p \wedge q$.[8]

We have not yet discussed the principles of Factual and Propositional Grounding, but it seems to me that they might be justified in a somewhat similar way. Of course, if existence and truth are simple notions, then we will not have exactly the same reasons of logical complexity for thinking that the existence of a fact or the truth of a proposition should have a ground.[9] But it might plausibly be maintained that when f is the fact that A, then the grounds for the existence of

[8] Related ideas are pursued in Schneider [2008] as a means of characterizing truth-functionality.

[9] The existence of x (E(x)) is often defined, as it is here, in terms of there being something identical to x ($\exists y(x=y)$). But it might be argued that this definition is unacceptable from a ground-theoretic point of view, since $x = x$ helps ground $\exists y(x = y)$ but does not help ground E(x).

f (i.e. E(f)) should be the same as the grounds for the existence of the fact that A (E([A]) and, similarly that, when p is the proposition that A, then the grounds for the truth of p (i.e. T(p)) should be the same as the grounds for the truth of the proposition that A (T(|A|)). But these latter statements (E([A]) and T(|A|) do enjoy a kind of logical complexity; and so it is only natural to suppose that this complexity should disappear under a suitable statement of their ground.

In the case of propositions or sentences, it is hard to see how A might not ground or might not help to ground the truth of the proposition that A or of the sentence 'A'. But the case of facts is not so clear. On a thin conception of facts – under which they might be propositions, say, which merely happened to be true – A's being the case would not in general be relevant to the existence of the fact (or proposition) that A. Or again, on a 'thin' conception of existence – under which an object's existing was to be distinguished from its being 'substantial' or 'concrete' – A's being the case would also not be relevant to the existence of the fact that A. I myself do not find these objections to Factual Grounding to be at all plausible, but there are alternative formulations of the puzzle that are able to avoid them.

For even under a thin conception of facts or of existence, there will be a notion of 'obtains' under which the fact that A will obtain just in case A is so. If facts are propositions, for example, then a fact (i.e. a proposition) will obtain just in case it is true. We may now restate the particular version of the Argument for Facts but using the assumption 'something obtains' in place of the assumption 'something exists'. The reasoning will then go through much as before and the grounding assumption for facts (viz. that A helps ground that the fact that A *obtains*) will be unobjectionable. When facts are taken to be propositions and obtaining is taken to be truth, we are thereby able to generate a version of the paradox from the assumption that something is true.

A related move might be made in the case of the Universal version of the Argument for Facts. We can no longer replace the assumption that everything exists with the assumption that everything obtains, since it is presumably not true that everything obtains. But this may plausibly be taken to be true if the quantifiers are restricted (either implicitly or explicitly) to the facts that obtain. Thus in case the facts that obtain are taken to be true propositions, we may obtain a generate a version of the puzzle from the assumption that every true proposition is true.

A rather different objection to Factual Grounding accepts the thick conception of facts and existence but reverses the connection between the existence of a fact and its ground. Rather than A being a ground for the existence of the fact that A, the existence of the fact that A is taken to be a ground for A. Such a view might be taken to be implicit in the truth-maker approach, with its emphasis on facts as the ground of truth.

However, the reversal of Factual Grounding has no plausibility as a completely general principle. For then any truth A whatever would lead to an infinite regress of grounds – with the existence of the fact f_1 that A being a ground for A, the existence of the fact f_2 that the fact that f_1 exists being a ground for the existence of f_1, and so on ad infinitum; and I take it that, even if we are open in principle to the possibility of an infinite regress of grounds, the present regress, which is automatically generated by repeated application of the fact-forming operator, is one that we would wish to reject. Now there may be some basic truths A whose truth is grounded in the existence of the fact that A, where the existence of the fact that A is itself ungrounded. But not all truths can be of this sort, on pain of an unacceptable regress. And so even if this exception to Factual Grounding is allowed, it is not altogether implausible that that for a non-basic truth such as that something exists ($\exists xE(x)$), the existence of the fact that something exists ($E([\exists xE(x)])$) should be grounded in something's existing ($\exists xE(x)$).

It has been suggested to me that the existence of facts might be subject to a parallel treatment to things being the case. Thus just as either P or Q might be taken to be a ground for $P \vee Q$, the existence of the fact that P or the existence of the fact that Q might be taken to be a ground for the existence of the fact that $P \vee Q$. But such an approach is in danger of leading directly to paradox. For it will take the existence of the fact that $\exists xF(x)$ to be grounded in the existence of the fact that F(y), given that F(y) is indeed the case; and so by this principle, the existence of the fact that $\exists xE(x)$ will be grounded in the existence of the fact that $\exists xE(x)$.

We might also note that there are some versions of the factual argument for which the analog of Factual Grounding is less easily rejected. We might plausibly maintain, for example, that there is not just the fact that something exists but also the *knowledge* that something exists. And if the puzzle is stated appealing to knowledge that A in place of the fact that A, then it would be bizarre in the extreme to suppose that it was not A that helped ground the existence of the knowledge that A but the existence of the knowledge that A that helped ground A.

This leaves us with the logical assumptions. Universal Middle has often been questioned. It is not accepted under intuitionism, for example, or under the weak or strong Kleene tables. Even Particular Middle (that some thing - or proposition - is true or not true) might be questioned under the weak Kleene tables, since the truth that something is true or not true requires that each instance of something's being true or not true should have a truth-value and should therefore be true (given that it cannot be false). But Universal and Particular Existence have never been questioned to my knowledge and it is hard to see how any of the standard logical approaches might fail to validate them or on what basis they might plausibly be rejected. These cases therefore represent especially stubborn versions of the puzzle for, even if we are inclined to reject one of the logical assumptions in the other cases, it is hard to see how we might do so here.

§6 Some Options

It is hard to see how one might give up the general assumptions concerning ground without giving up the favored concept of ground; and it is also hard, though perhaps not so hard, to see how we might give up the special assumptions of Factual Grounding, Propositional Grounding or Truth Introduction. So let us keep these assumptions fixed for the rest of the discussion. This still leaves the standard predicative approach of Russell, according to which Factual or Propositional Existence are taken to be in violation of the vicious circle principle. But since our interest is in avoiding the restrictions imposed by the predicative approach, let us provisionally keep these other assumptions fixed as well.

We are then left with the logical principles of Universal and Particular Middle and of Universal and Particular Existence and with the ground-theoretic principles of Universal, Existential and Disjunctive Grounding. What is interesting about these remaining principles is that they may all be regarded as constitutive of the classical approach to logic. There are two aspects of the classical approach to which they relate. The first concerns the classical approach to logical truth; and the second concerns the classical approach to truth-conditions, interpreted here as conditions of ground. The logical principles fall under the first head, they are all instances of classical logical truths; and the ground-theoretic principles fall under the second head, they all belong to the classical account of truth-conditions.

Normally, these two aspects of the classical approach are seen as complementary and even as mutually reinforcing, each being plausible in the light of the other. But we see that, under seemingly evident assumptions, these two aspects of the classical approach, far from being complementary, are in conflict with one another. Classical logic, seen as a unitary approach to logical truth and truth-conditions, is in tension with itself; and one of the two aspects of the approach must be given up if the other is to be retained.

In the face of this conflict, it is natural to suppose that one should either go with the classical conception of logical truth or with the classical conception of truth-conditions. But a more nuanced stance is possible. For consider again the principle of Disjunctive Grounding, which states that $A \vee B$ will be grounded by A should A be the case and by B should B be the case. This principle is unnecessarily strong if our intention is merely that any disjunctive truth should have a ground for, in the case in which a disjunction has two true disjuncts, the principle will require that both should be a ground whereas all that is required by satisfaction of the intention is that one of them should be a ground. And similarly for the principle of Existential Grounding, which requires that any true instance of an existential truth should be a ground, whereas all that is strictly required for the existential truth to be grounded is that one of its instances should be a ground.

This suggests that in place of Disjunctive and Existential Grounding, we should adopt the weaker principles:

<u>Weak Disjunctive Grounding</u> $(A \vee B) / [A \prec (A \vee B)] \vee [B \prec (A \vee B)]$
(Any disjunctive truth is grounded by a disjunct)

<u>Weak Existential Grounding</u> $\exists xA(x) / \exists y[A(y) \prec \exists xA(x)]$
(Any existential truth is grounded by an instance)

The Particular Argument for Facts will not then go through, since we will have no guarantee that the existence of the fact that something exists will be a ground for something existing, even though it is an instance of something existing; and similarly, the Particular Argument for Propositions will not go through. We are therefore free to hold onto Particular Existence and Particular Middle while still allowing that each disjunctive and existential truth will have a ground.

Unfortunately, we cannot save ourselves from the Universal Argument for Propositions in this even way, even though it appeals to the principle of Disjunctive Grounding. For the disjunction to which the principle is applied (to the effect that a given proposition is true or not true) is one in which only one of the disjuncts is the case and so there is no relevant difference between the principle and its weakened form. Nor can we save ourselves from the Universal Argument for Propositions or for Facts by appealing to a weakened form of Universal Grounding, since we cannot properly take a universal truth to be grounded by some of its instances to the exclusion of the others.

It might be thought that Universal Grounding should be rejected on the basis that (i) an instance Fa of a universal truth $\forall xFx$ will only be a partial ground for the universal truth if all of the instances Fa_1, Fa_2, ... are a full ground for the universal truth and (ii) all of the instances Fa_1, Fa_2, ... cannot be a full ground for the universal truth since the objects a_1, a_2, ... might not be the only objects that there are.[10] This may be. But there would appear to be a relative notion of full ground, of a full ground relative to a certain presupposition. Thus given A, B will be a full ground for A & B and B, C will be a full ground for A & (B & C). Presupposing that a_1, a_2, ... are all the objects that there are, all of the instances Fa_1, Fa_2, ... will be a full ground for the

[10] I owe this objection to Fabrice Correia.

universal truth ∀xFx and each of the instances will be a corresponding partial ground for the universal truth. Of course, this is not to say what the absolute full ground for a universal truth should be, but that problem need not be solved in order to pose the puzzle.

We appear to be left with four main options, given the aim of retaining as much of the classical approach as we can. One is to give up the existence assumptions and adopt some form of predicativism. This enables us to embrace classical logic in its entirety, without any impairment to its conception of logical truth or truth-conditions. None of the other options enable us to do this and so, from this point of view, it is predicativism, with its severe constraint on what can properly be taken to exist, that is most within the spirit of the classical approach.

But suppose we wish to accept impredicative domains. One option then is to adopt a compromise position. We accept neither the relevant classical logical truths nor the relevant classical truth-conditions in their entirety. But by accepting a weakened form of Disjunctive and Existential Grounding, we hold on to some form of the classical truth-conditions and also hold on to Particular Existence and Middle, though not to Universal Existence or Middle.

The alternative is to adopt an extreme position, under which one either accepts the classical truth-conditions in their entirety while abandoning even a compromised form of the logical truths or one accepts the classical logical truths in their entirety while abandoning even a compromised form of the truth conditions. In the first case, the principles of Universal and Particular Existence and of Universal and Particular Middle will be given up and, in the second case, the principles of Disjunctive or Existential Grounding (in even their weakened form) will be given up, as will the principle of Universal Grounding.

Given that one abandons the standard principles of grounding for disjunctive or existential or universal truths, then what should be put in their place? One possibility is to retain the view that these logically complex truths are grounded but deny that they are grounded in anything like the classical way. Thus one might suppose that it is the status of A as a logical truth that grounds the truth of A rather than some simpler truth although, of course, this then leads to the question of what grounds the status of A or of what grounds the logically complex truths that are not themselves logical truths. The other possibility is to deny that logical truths have a ground. It is odd to think of these truths as basic and so this version of the position is perhaps best construed as a form of instrumentalism or non-factualism about logic. The logically complex truths - or, at least, the logical truths - are not grounded since there is nothing in the world to which they can properly taken to correspond.[11]

We map out the options in the chart below (with LT indicating logical truths and TC indicating truth conditions):

/ \
/ \
Predicativist Impredicativist
LT TC / \
/ \
Compromise Extremist
~~L~~T ~~T~~C / \
/ \
~~LT~~ TC LT ~~TC~~

[11] See Fine [2001] for a general account of nonfactualism and its connection with ground.

<u>§7 Grounding and Fixed Points</u>

It will be of interest to relate the previous discussion of grounding to Kripke's work on the semantic paradoxes. It turns out that there is a natural correspondence between the various approaches to the grounding puzzles that we have considered and the various approaches to the semantic paradoxes that Kripke has considered. Each can be used to illuminate the other. For the various versions of Kripke's fixed point construction provide a concrete model for the various approaches to the grounding puzzles, one that shows that they do indeed correspond to a coherent point of view; and the different approaches to the grounding principles can be seen to provide an underlying motivation for the different versions of the construction.

Kripke starts with an interpreted first-order language L (such as that of arithmetic). To this is added an (as yet) uninterpreted truth-predicate T, thereby giving us the language L^+. Let us assume, if only for simplicity, that there is a name in L for each object in the domain of the intended interpretation of L, that each sentence of L receives a truth-value – True or False – in conformity with the classical truth-conditions, and that the the sentences of L^+, or at least their surrogates, are among the objects of the domain of L^+ so that there will be a name $\ulcorner A \urcorner$ in L for each sentence A of L^+.

Kripke's aim is to extend the interpretation of the original language L to the language L^+. He states the rules for extending the interpretation in semantic fashion. But it will be helpful for our purposes to state them in proof-theoretic fashion, though the two approaches are essentially the same.

To this end, we suppose that A is an axiom for each truth A of L and that not-A is an axiom for each falsehood A of L. We are then allowed to infer the truth of $T(\ulcorner A \urcorner)$ from A and the falsehood of $T(\ulcorner A \urcorner)$ from $\neg A$ for any sentence A of L^+:

$$\frac{A}{T(\ulcorner A \urcorner)} \qquad\qquad \frac{\neg A}{\neg T(\ulcorner A \urcorner)}$$

It remains to give rules for determining the truth-values of logically complex sentences – be they negations (of the form $\neg A$), conjunctions ($A \wedge B$), disjunctions ($A \vee B$), universal quantifications ($\forall x A(x)$) or existential quantifications ($\exists x A(x)$).

Kripke suggests three different ways in which this might be done: strong Kleene truth-tables; weak Kleene truth-tables; and supervaluationism. According to the strong tables, a disjunction is true iff one of the disjuncts is true, regardless of whether the other disjunct has a truth-value and a disjunction is false iff both of its disjuncts are false; and similarly, an existential sentence $\exists x A(x)$ is true iff some instance $A(c)$ is true, regardless of whether the other instances have a truth-value, and an existential sentence is false iff all of its instances are false. And analogously for negation, conjunction and universal quantification.

The truth-table for disjunction under the strong tables corresponds to the following rules of inference:

$$\frac{A}{A \vee B} \qquad \frac{B}{A \vee B} \qquad \frac{\neg A \;\; \neg B}{\neg(A \vee B)}$$

and, similarly, the semantic clauses for the existential quantifier corresponds to the following rules:

$$\frac{A(c)}{\exists x A(x)} \qquad\qquad \frac{\neg A(c_1)\ \ \neg A(c_2)\ \ldots}{\neg\exists x A(x)}$$

where c_1, c_2, … are names for all of the objects c_1, c_2, … of the domain. (And analogously for negation, conjunction and universal quantification.)

According to the weak tables, a disjunction is true iff either of the disjuncts is true and if, in addition, the other disjunct has a truth-value, True or False, and a disjunction is false iff both disjuncts are false; and similarly, the existential sentence $\exists x A(x)$is true iff some instance A(c) is true and if, in addition, the other instances of $\exists x A(x)$ have a truth-value, and an existential sentence is false iff all of its instances are false.

This corresponds to adopting the following rules for disjunction:

$$\frac{A\ \ \pm B}{A \vee B} \qquad\qquad \frac{B\ \ \pm A}{A \vee B} \qquad\qquad \frac{\neg A\ \ \neg B}{\neg(A \vee B)}$$

and the following rules for existential quantification:

$$\frac{A(c)\ \ \pm A(c_1)\ \pm A(c_2)\ \ldots}{\exists x A(x)} \qquad\qquad \frac{\neg A(c_1)\ \neg A(c_2)\ \ldots}{\neg\exists x A(x)}$$

where $\pm A$ is used ambiguously for the sentence A or $\neg A$. (And analogously when it comes to the rules for negation, conjunction and universal quantification.)

According to supervaluationism, a sentence is true (relative to some partial information concerning the application of the truth-predicate) if the sentence is true under any way in which the information might be completed (with each truth-predication being either true or false). Under this approach, it is also possible to impose certain constraints on how the information is to be completed. It might be required, for example, that the assignment of truth under the completions should be in conformity with the classical truth-conditions.

To state the supervaluational approach in proof-theoretic form, we should suppose that we are given some truth-predications $T(\ulcorner A \urcorner)$ or their denials $\neg T(\ulcorner A \urcorner)$. Suppose they take the form $\pm T(\ulcorner A_1 \urcorner)$, $\pm T(\ulcorner A_2 \urcorner)$ … (where A_1, A_2, … need not be a list of all the sentences and might even be empty). The corresponding supervaluational rule of inference then takes the form:

$$\frac{\pm T(\ulcorner A_1 \urcorner),\ \ \pm T(\ulcorner A_2 \urcorner)\ \ldots}{B}$$

where B should be a consequence of the premises in the sense that it will be true in any classical model in which the premises are true and the intended interpretation of L remains the same (if the completions are subject to constraints then these must also be built into the understanding of consequence).

Under each of these approaches, the ‘fixed points’ of Kripke will correspond to those sets of sentences that are closed ‘two-way’ under the rules. Thus in the case of truth-predications, $T(\ulcorner A \urcorner)$ will be in the set just in case A is in the set and, in the case of disjunctions under the strong truth-tables, $A \vee B$ will be in the set just in case either A or B is in the set. It is easy to show that there will be a least fixed point, one contained in any other fixed point. This will correspond to repeatedly applying the rules to the axioms until they yield nothing new.

If we are to relate this framework to the previous discussion of ground, then there are two things that must be done. The first is to read off a relation of ground from the derivation of truths.[12] The natural suggestion is that the truth A should be taken to be a (partial) ground for the truth B if it may be used to generate B, i.e. if A appears (and is actually used) in a derivation of B.

This proposal works well in the case of the weak truth-tables. Indeed, there is essentially only one way to derive a given truth under the weak truth-tables and so, in regard to grounding, there is no basis for distinguishing one derivation of a given truth from another. But the proposal leads to circularities in the case of the other two approaches. For consider the sentence ‘$\exists xT(x)$’ (some sentence is true). This may be derived from $\forall x(x = x)$. For from $\forall x(x = x)$, we obtain $T(\ulcorner \forall x(x = x) \urcorner)$; and from $T(\ulcorner \forall x(x = x) \urcorner)$, we obtain $\exists xT(x)$ under the strong truth-tables and supervaluationism (though not under the weak tables). So far so good. But from $\exists xT(x)$, we may obtain $T(\ulcorner \exists xT(x) \urcorner)$; and from $T(\ulcorner \exists xT(x) \urcorner)$, we may obtain $\exists xT(x)$ – thereby grounding $\exists xT(x)$ in itself!

Some restriction on the derivations which may give rise to a relation of grounding is therefore required. Suppose we construct derivations in conformity with the following precept: always derive a sentence if you can unless it has already been derived (do not delay and do not repeat!). The previous difficulty will not then arise. For at the start of constructing derivations, we will have $\forall x(x = x)$ as an axiom; and we may then successively derive $T(\ulcorner \forall x(x = x) \urcorner)$, $\exists xT(x)$, and $T(\ulcorner \exists xT(x) \urcorner)$. But we cannot go on to derive $\exists xT(x)$ from $T(\ulcorner \exists xT(x) \urcorner)$ thereby generating a circularity, since $\exists xT(x)$ has already been derived.

Derivations constructed in conformity with the precept might be called *efficient.* We might then take A to be a ground for B if A appears (and is used) in an efficient derivation of B. Partial ground, as so defined, will then be a irreflexive and transitive relation and it is not implausible, at least in the case of the weak and strong truth-tables, that any case of ground in this sense should be an intuitive case of ground.[13]

---

[12] Some related ideas concerning the role of ground or dependence in an account of truth are pursued in Yablo[1982] and Leitgeb[2005].

[13] Each sentence appearing in a derivation may be assigned an ordinal rank in a natural manner. Efficient derivations will then be ones in which each sentence is assigned a minimal rank (one that is less than or equal to the rank of the sentence in any other derivation). It might be thought that partial ground, as so defined, is too restrictive. For consider the sentence ‘$\forall x(x = x) \vee T(\ulcorner \forall x(x = x) \urcorner)$. Then it can be directly derived from the axiom $\forall x(x = x)$ and so cannot be efficiently derived from ‘$T(\ulcorner \forall x(x = x) \urcorner)$. And yet it appears to be perfectly acceptable to take $T(\ulcorner \forall x(x = x) \urcorner)$ to be a ground for $\forall x(x = x) \vee T(\ulcorner \forall x(x = x) \urcorner)$. To take care of this difficulty, we might broaden the notion of ground as follows. Say that a derivation is *non-circular* if it does not contain two occurrences of the same sentence; and say that the truth A is *essentially non-circular* if any derivation of A is non-circular. We may then take A to be a partial ground of B if either it is a partial ground in the previous sense or if A is essentially non-circular and there is a derivation of B from A. $T(\ulcorner \forall x(x = x) \urcorner)$ will then be a ground for $\forall x(x = x) \vee T(\ulcorner \forall x(x = x) \urcorner)$ since $T(\ulcorner \forall x(x = x) \urcorner)$ is essentially non-circular. The question of how the relation of ground might be defined within the Kripkean

The second modification to the framework concerns its extension to facts. Under the original framework, the intended domain is given in advance and the construction of a fixed point provides us with a method for determining the extension of the truth-predicate over this domain. But we now wish to use the construction to provide us with an ontology of facts, rather than with an ideology of truth - with the truth of A assuring us of the existence of the fact that A and the truth of ¬A assuring us of the existence of the fact that ¬A.

How is this to be done? The natural suggestion is that we should be start the construction with a putative domain of facts, one that includes a putative fact that A for each sentence A of the given language. At the beginning of the construction, we are given no information about which of the putative facts are genuinely in the domain or genuinely outside the domain. But given A, we may infer that the fact that A is in the domain and, given ¬A, we may infer that the fact that A is not in the domain. In this way, we obtain more and more information about what is in or outside the domain.

Under this approach, what clauses should we give for the quantifiers, given that at no stage of the construction do we have full information about what the domain will eventually be? To fix our ideas, let us first attempt to answer this question under the adoption of the strong tables. So when should an existential sentence ∃xA(x) or a universal sentence ∀xA(x) be taken to be true?

Under this scenario, the truth of an instance A(c) of ∃xA(x) cannot in general be taken to be sufficient for its truth. For we want the truth-values assigned to sentences to remain stable under an increase of information. But c might denote a putative object *c* that is eventually shown to be outside the domain; and the assignment of Truth to ∃xA(x) on the basis of A(c) would not then be justified. What we therefore require for the truth of ∃xA(x) is the truth of both A(c) and E(c); and the possibility that the object *c* might later be shown to be in the domain is thereby excluded.

Similarly, the truth of all of the instances A(c) for which the object *c* denoted by c has been shown to belong to the domain will not, in general, be sufficient for the truth of ∀xA(x). For some putative object *c* might subsequently be shown to belong to the domain; and the truth of A(c) would then also be required. What we therefore require for the truth of ∀xA(x) is the truth of A(c) for all those putative objects *c* which have not yet been shown to be outside the domain; and the possibility of there later being a new object in the domain to consider is thereby excluded.[14]

If we translate these semantical remarks into our inferential framework, we obtain the following rules for existence, non-existence and the quantifiers:

$$\frac{A}{E([A])} \qquad\qquad \frac{\neg A}{\neg E([A])}$$

$$\frac{E(c)\ A(c)}{\exists x A(x)} \qquad\qquad \frac{A(c_1)\ A(c_2)\ \ldots\ \neg E(d_1)\ \neg E(d_2)\ \ldots}{\forall x A(x)}$$

framework is of great general interest; and I have only touched upon some of the issues that arise.

[14] There are some potential complications over the identity of putative objects on this account which I have not gone into.

where all of the putative objects of the domain are among $c_1, c_2, ..., d_1, d_2, ...$ Thus the existential quantifier ranges, in effect, over the putative objects that have been shown to be in the domain while the universal quantifier ranges in effect over the putative objects that have not been shown not to be in the domain. There are similar clauses for the negations of existential and universal sentences; and analogous clauses can also be given under the other approaches.

Once the Kripkean framework is extended in these two ways, we may note some remarkable parallels. We should observe that for present purposes (in which the identity conditions for propositions are not in question) it does not much matter whether we think of $\ulcorner A \urcorner$ as denoting a sentence or a proposition. We then see that the strong Kleene approach will correspond to the compromise approach to classical logic. We will have Particular Middle and Existence. For from $\forall x(x = x)$, say, we can derive $T(\ulcorner \forall x(x = x) \urcorner)$; from $T(\ulcorner \forall x(x = x) \urcorner)$ we can derive $T(\ulcorner \forall x(x = x) \urcorner) \vee \neg T(\ulcorner \forall x(x = x) \urcorner)$, from which we can derive $\exists x(T(x) \vee \neg T(x))$. Similarly, from $\forall x(x = x)$ we can derive $E([\forall x(x = x)])$, from which we can derive $\exists x E(x)$.

However, we will not have Universal Middle or Universal Existence. Universal Middle will fail for familiar reasons (when instanced with the Liar, for example). But Universal Existence ($\forall x \exists y(x = y)$) will also fail in general. For what it requires, in effect, is that every putative fact [A] that, at a given stage, has not been shown not to be in the domain will have been shown to be in the domain. But this then requires that the truth-value of every sentence (including the Liar) should have been settled. So although $\forall x \exists y(x = y)$ might appear to be a relatively innocuous logical truth, it will fail, in the present setting, for much the same reasons as the Law of Excluded Middle.

Turning to ground, we see that the principles of Universal Grounding and of Weak Disjunctive and Existential Grounding will hold under strong Kleene. Any disjunctive truth $A \vee B$, for example, will have an efficient derivation in which $A \vee B$ is obtained from either A or B; and so either A or B will be a ground for $A \vee B$. However, the stronger forms of Disjunctive or Existential Grounding will not hold. For there is no efficient derivation of $\exists x T(x)$, say, from $T(\ulcorner \exists x T(x) \urcorner)$, even though it is true, and, similarly, there is no efficient derivation of $\exists x E(x)$ from $E([\exists x T(x)])$.

The failure of the original unqualified forms of Disjunctive or Existential Grounding might appear odd, for the truth of a disjunct is sufficient for the truth of a disjunction and the truth of an instance is sufficient for the truth of an existential quantification under the strong tables. And so how come they are not grounds? But there is a general reason for thinking that the unqualified principles might fail. For given a disjunction with true disjuncts, then one of those disjuncts may ground the disjunction and the disjunction in its turn may ground the other disjunct, thereby preventing the other disjunct from grounding the disjunction.[15]

The weak Kleene approach, by contrast, will correspond to the extreme position in which we sacrifice logical truths for grounding principles. Thus all of the principles of Universal or Particular Middle and Universal or Particular Existence will fail. The first two will fail for familiar reasons. But even Particular Existence ($\exists x E(x)$) will fail in general, since its truth will require that each instance $E([A])$ should have a truth-value and this means, in effect, that the truth-value of each sentence A should be settled.

Under our account of grounding for weak Kleene, Universal Grounding and the core principles of Disjunctive and Existential Grounding will hold. So, for example, given the truth

[15] An especially clear case in which the stronger form of Disjunctive Grounding fails is with the sentence $(\forall x(x = x) \vee T(c))$, where c denotes the very sentence '$(\forall x(x = x) \vee T(c))$'. $\forall x(x = x)$ will ground $(\forall x(x = x) \vee T(c))$ but $T(c)$ will not, even though it is true.

of A and $A \vee B$, $A \vee B$ will be derivable with the help of A and so must have A as a ground; and similarly, given the truth of $\exists xA(x)$ and $A(c)$, $\exists xA(x)$ must be derivable with the help of $A(c)$ and so must have $A(c)$ as a ground.

The original principles of Disjunctive and Existential Grounding will not hold in general. For given A, we cannot infer that A grounds $A \vee B$ since B may lack a truth-value; and given $A(c)$, we cannot infer $\exists xA(x)$ since other instances $A(c')$ of $\exists xA(x)$ may lack a truth-value. But properly viewed, what we have here is a further sacrifice of logical principles for principles of ground. For it is the failure of Disjunction Introduction under weak Kleene that prevents us from inferring Disjunctive Grounding from core Disjunctive Grounding (and an analogous failure of Existential Introduction that prevents us from inferring Existential Grounding from core Existential Grounding).

Although Disjunctive and Existential Grounding will fail, we may still assert a 'compositional' form of grounding under this approach, with an upward flow of truth from less to more complex truths. For the truth A, together with either the truth B or the truth $\neg B$, will be a full ground for $A \vee B$; and the truth $A(c)$, together with the truths $\pm A(c_1)$, $\pm A(c_2)$, ..., where c_1, c_2, ... are all the objects of the domain, will be a full ground for $\exists xA(x)$. Thus the classical truth-conditions, when understood as statements of full ground under a complete specification of the grounding conditions, can still be seen to hold.

Finally, the supervaluational approach corresponds to the extreme position in which we sacrifice grounding principles for logical truths. All of the classical logical truths will hold, since they will each hold in any given 'completion' of some partial information (or be derivable by the supervaluational rule from any premises). However, none of the classical grounding principles will hold, even in weakened or modified form. For grounds are always of the form $T(\ulcorner A \urcorner)$ or $\neg T(\ulcorner A \urcorner)$ and so there is no assurance in the case of a true disjunction $A \vee B$, for example, that A or B will be of the required form. Indeed, we cannot even say that A or B will be a ground $A \vee B$ when they *are* of the required form. The disjunction $T(\ulcorner A \urcorner) \vee \neg T(\ulcorner A \urcorner)$, for example, will be true but it will, like the axioms, be derivable at the very first stage from zero premises and so the subsequent derivation of $T(\ulcorner A \urcorner) \vee \neg T(\ulcorner A \urcorner)$ from $T(\ulcorner A \urcorner)$ or $\neg T(\ulcorner A \urcorner)$ will not serve to show that either $T(\ulcorner A \urcorner)$ or $\neg T(\ulcorner A \urcorner)$ is a ground.

A further oddity of our account of ground under the supervaluational approach is that logical truths will have no (partial) grounds. For each logical truth B will be derivable at the first stage from zero premises and so no subsequently derived truths of the form $T(\ulcorner A \urcorner)$ or $\neg T(\ulcorner A \urcorner)$ can be used in an efficient derivation to obtain B. If one wants logical truths to have partial grounds under the supervaluational approach, then some other account of ground must be given. As I have mentioned, perhaps there is some account of logical truth under which its being a logical truth that A can be regarded as a ground for A and perhaps this account of ground can somehow be related to the supervaluational approach, since the supervaluational truth of A under zero information about the truth-predicate will effectively amount to the truth of A in all models. But failing that, one might wish to stick to the present account of ground and yet wed it to something like an instrumentalist or nonfactualist view of logical truth, under which grounds for logical truths would not be required.

We may sum up the correspondence between the various solutions to the ground-theoretic puzzles and the different kinds of fixed point in the following extension of the earlier chart:

/ \

/ \
Predicativist Impredicativist
LT TC / \
/ \
Compromise Extremist
~~LT~~ ~~TC~~ / \
Strong Kleene / \
~~LT~~ TC LT ~~TC~~
Weak Kleene Supervaluational

References

Audi Paul [2010?] 'Grounding', to appear **

Correia F., [2005] 'Existential Dependence and Cognate Notions', Philosophica Verlag.

Fine K., [2001] 'The Question of Realism', Imprint, vol. 1, no. 1, 2001; reprinted in 'Individuals, Essence and Identity: Themes of Analytic Philosophy' (ed. A. Bottani, M Carrara, P. Giaretta), Dordrecht: Kluwer 2002, 3- 41.

Leitgeb H., [2005] 'What Truth Depends On', Journal of Philosophy 34, 155-192.

Rosen G., [2010] 'Metaphysical Dependence: Grounding and Reduction', to appear in a collection of papers with OUP.

Schaffer J., [2009?] 'Grounding as the Primitive Concept of Metaphysical Structure',

Schnieder B. [2008] 'Truth-Functionality', Review of Symbolic Logic 1, 64-72.

Schneider B., [2010?] 'A Logic of 'Because', work in progress.

Yablo S., [1982] 'Grounding, Dependence and Paradox', Journal of Philosophy 11, 117-137.