A Difficulty for the Possible Worlds Analysis of Counterfactuals

A number of different accounts of counterfactual statements have been proposed in the literature. It has been thought that they should be understood in terms of the closeness of possible worlds, for example, with the counterfactual from A to C being true if all sufficiently close worlds in which A is true are worlds in which C is true or that they should be understood in terms of some notion of cotenability, with the counterfactual from A to B being true if A in conjunction with truths cotenable with A entails C. But a common presupposition of almost all of these accounts is that counterfactual claims should be intensional. If the sentences A and A′ or C and C′ are necessarily equivalent then the substitution of A′ for A or C′ for C in the antecedent or consequent of a counterfactual should preserve its truth-value. Thus, under the usual form of the possible worlds account, the truth-value of a counterfactual will simply turn on the possible worlds in which the antecedent and the consequent are true and so the account will be unable to distinguish between the truth-values of counterfactuals whose antecedents or consequents are true in the same possible worlds and hence are necessarily equivalent while, under the entailment-based accounts, the entailments will remain the same under the substitution of necessary equivalents and so the truth-values of the counterfactuals will also remain the same.[1]

It is the aim of this paper to show that no plausible account of counterfactuals should take them to be intensional and that if we are to describe the different kinds of counterfactual scenarios in the way we want and to reason about them in the way we would like, then the assumption of intensionality should be abandoned. Indeed, it is not merely the assumption of 'modal' intensionality that will fail but also the weaker assumption of'logical' or 'classical' intensionality. For the cases we shall consider are ones in which the substitution of A′ for A or C′ for C should not be permitted, even though they are *logical* and not merely *necessary* equivalents.

There are some familiar counter-examples to the substitution of necessary or logical equivalents. Suppose, for example, that the Continuum Hypothesis and Goldbach's Conjecture are both true. Then I might optimistically believe that if the Continuum Hypothesis were false then mathematicians would have shown it to be false. But this does not appear to constitute any reason for my believing that if Goldbach's Conjecture were false then mathematicians would have shown the Continuum Hypothesis to be false, notwithstanding my recognition that the Continuum Hypothesis and Goldbach's Conjecture are both necessarily true and hence necessarily equivalent. Or to take a counter-example that runs much closer to home, we might suppose that if I were to strike this match then it would light. But this does not seem to imply that if I were to strike the match when dry or to strike the match when not dry then it would light, notwithstanding the logical equivalence between my striking the match and my striking it when

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[1]The present paper expands on material in the first part of Fine [2011]. Much of the material was presented at the Whitehead Lectures at Harvard, 2009, the Townsend Lectures at Berkeley, 2010, the Nagel Lecture at Columbia, 2010, a conference on Propositions and Same-Saying at Sydney University, 2010, and at talks to the philosophy departments of University of Miami and Virginia Commonwealth University. I would like to thank the audiences at those meetings for many helpful comments.

dry or striking it when not dry.

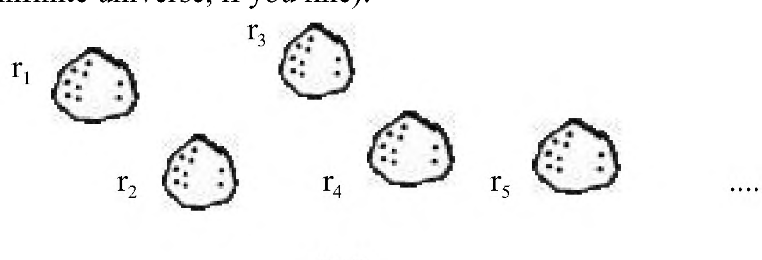
These counterexamples pose a serious challenge to the possible worlds theorist (and to the entailment theorist), but there are a number of responses that could and have, in fact, been made. In the second case, for example, it might be argued that the logical form of the second counterfactual (with its apparently disjunctive antecedent) is not what it appears to be; and, in the first case, it might be maintained that the account is only meant to apply to counterfactuals that do not involve counter-possible antecedents and that some different account should be given of the other cases.

But my own challenge to the possible worlds theorist is quite different and much more serious. For what I present is not simply a counter-example to Substitutivity but an argument to the effect that the principle is in conflict with well-established principles and well-grounded intuitions that he is (or should be) already inclined to accept. Thus he cannot simply hold on to Substitutivity in the face of the counter-example; he must also give up one of these other principles or intuitions and it is not at all clear which, if any, it should be. The possible worlds theorist is therefore under considerable internal pressure to accept the failure of Substitutivity in order that he might hold onto his other, more cherished, beliefs.

I begin by presenting the difficulty. This takes the form of some assumptions concerning the logic of counterfactuals and the truth of particular counterfactuals in a given hypothetical scenario (§§1-2). These assumptions are all very plausible and yet can be shown to lead to contradiction. I then consider the question of which of them should be given up and argue that the most plausible candidate is the principle of substitutivity for logically equivalent antecedents (§§3-5). I go on to consider a related puzzle concerning the substitutivity of logical equivalents in the *consequent* of a counterfactual (§6); and I conclude by attempting to dispose of some general arguments in favor of an intensional approach (§7). The present paper is entirely negative but, in other work (including Fine [2011]), I attempt to show how the puzzles might be avoided and a more satisfactory semantics for counterfactuals might be developed.

<u>§1 Rocks</u>

We lay out the assumptions that are required to derive a contradiction. To this end, we imagine an infinite landscape containing infinitely many rocks situated fairly close to one another (substitute stars in an infinite universe, if you like):



Use R_n for the sentence 'Rock n falls', R for the disjunction $R_1 \vee R_2 \vee R_3 \vee ...$, and '>' for the counterfactual operator (so that A > C may be read 'if were the case that A then it would be the case that C'). Let us suppose that none of the rocks actually falls but that if a given rock r_n were to fall it would always fall in the direction of rock r_{n+1}.

The following non-logical assumptions are then all plausibly taken to be true; and the ensuing logical assumptions (which take the form of rules of inference) are all plausibly taken to be valid.

Non-logical Assumptions

Positive Effect $R_n > R_{n+1}$

(if a given rock were to fall then the next rock would fall)

Negative Effect $R_{n+1} > \neg R_n$, $n = 1, 2, 3, \ldots$

(if a given rock were to fall then the previous rock would still stand)

Counterfactual Possibility $\neg(R > \neg R)$

(it is a counterfactual possibility that one of the rocks falls, i.e. a contradiction does not follow counterfactually from the supposition that a rock falls).

Logical Assumptions

Identity

$$\frac{}{A > A}$$

Substitution of Antecedents

$$\frac{A > C}{A' > C}$$ given that A and A′ are logically equivalent

Weakening

$$\frac{A > C}{A > C'}$$ given that C′ is a logical consequence of C

Transitivity

$$\frac{A > B \quad A \wedge B > C}{A > C}$$

Disjunction

$$\frac{A > C \quad B > C}{A \vee B > C}$$ given that A and B are logically exclusive

Finitary Conjunction

$$\frac{A > B \quad A > C}{A > B \wedge C}$$

Infinitary Conjunction
$$\frac{A > C_1 \quad A > C_2 \quad A > C_3 \ldots}{A > C_1 \wedge C_2 \wedge C_3 \wedge \ldots.}$$

Note that Identity and Weakening yield:

Entailment
$$\frac{}{C > C'} \quad \text{given that } C' \text{ is a logical consequence of } C$$

For C > C is derivable by Identity; and so C > C′ is derivable by Weakening.

The logical rules are largely familiar from the literature, though there are some differences. The Disjunctive Rule is usually stated without any restriction on the disjuncts A and B, while my rule is weaker since it is requires the disjuncts to be logically exclusive. I have my doubts about the unrestricted rule, for reasons that will later become clear, and so I have preferred to work with the restricted rule, even though it slightly complicates the derivation of the contradiction. The rule of Infinitary Conjunction is not a standard rule, if only because infinitary conjunctions are not normally allowed. But if they were allowed, then the rule would seem to have as much a right to be considered valid as its finitary counterpart.

From these assumptions we may, somewhat surprisingly, derive a contradiction. Details are left to the appendix but we may note here the broad outline of the proof. We show from the assumptions that if one of the rocks were to fall then it would not be the first and hence would be one that was second on. In the same way, we show that if one of the dominos from the second on were to fall, it would not be the second and hence would be one that was third on, from which it follows that if one of the dominos were to fall it would be one that was third on. Continuing in this way, we may show that if one of the dominos were to fall then it would not be any one of them, contrary to its being a counterfactual possibility that one of the dominos falls.

It should be noted that the assumptions upon which the reductio depend are stated entirely in the object language (and the derivation itself proceeds entirely within the object language). We say nothing about the connection between counterfactuals and the similarity of worlds or the existence of co-tenable sets of propositions or the like. It is therefore necessary to meet the argument on its own terms. We need to say which of *its* assumptions should be given up; and in this regard, it is perhaps advisable not to take for granted any particular view as to the general connection between counterfactuals and similarity or tenability or the like, since our intuitions about the truth of the particular counterfactual judgments and the validity of the particular counterfactual inferences are likely to be much more secure than any view we might have of these connections.

My own view is that, in the face of the contradiction, it is the Rule of Substitution that should be given up; we cannot in general substitute logical equivalents in the antecedent of a counterfactual salve veritate. But before considering this option - which would require giving up the standard forms of the possible worlds semantics and entailment-based accounts and the standard formulations of the logic of counterfactuals - we would do well to consider the possibility of rejecting some other assumption in the hope that some version of the standard views might thereby be retained. And this is what we shall do - beginning with the non-logical assumptions, turning next to the logical assumptions of Disjunction and Transitivity, and then to Infinitary Conjunction - before considering Substitution itself.

§2 Non-Logical Assumptions

There are three non-logical assumptions - Positive Effect, Negative Effect, and Counterfactual Possibility. Positive Effect and Counterfactual Possibility appear to be undeniable. But there are two qualms one might have about Negative Effect. The first is that it is not clear that the counterfactual 'if the second rock were to fall then the first would not fall' is true, for in entertaining the counterfactual supposition that the second rock falls, one should perhaps not keep fixed the fact that the first rock does not fall but allow that the second might fall by way of the first falling.

In responding to this objection, we should perhaps concede that there may be contexts in which, in entertaining the supposition that the second rock falls, we should consider the possibility of its falling by way of the first rock falling. Perhaps this is how the rocks normally fall - from 'first' to 'last'. But surely there is a reading of the counterfactual, and a much more natural reading at that, under which the possibility should not be taken into account. After all, there are many different things that might in principle cause the second rock to fall - a strong gust of wind, a falling meteorite, an earthquake etc. These possible causes of the second rock falling would not normally be taken into account. And so why single out for special treatment the possibility of the second rock falling by way of the first rock falling?

The second misgiving has to do with the lack of a connection between the antecedent and the consequent in Negative Effect. The second rock's falling, were it to happen, would not in any way be responsible for the first rock's not falling. But it might be thought that the truth of a counterfactual requires that the truth of the antecedent should somehow be responsible for - or connected to - the truth of the consequent.

Many philosophers would not be moved by this argument; and, indeed, it is sometimes taken to be an advantage of the 'similarity' analysis of counterfactual that it makes clear why a connection is not required, since similarity can be in points of fact that are unrelated to the antecedent (see §2 of Stalnaker [1968], for example).

Still, our argument should not be made to turn on such sensitive issues; and so let me give a more complicated version of the argument for which they do not arise (we might call it 'Goodman ad Infinitum' since it is based upon indefinitely duplicating Goodman's famous example of the match). Anyone not convinced by the original rock example should take it to be replaced by this more complicated example.

We imagine an infinity of matches m_1, m_2, m_3 ..., each in different, causally isolated, space-time regions of the universe. We suppose that each match is dry, that there is plenty of oxygen in the atmosphere surrounding the match and that, in general, the conditions for a struck match to light are as propitious as they could be. Use S_n for 'match n is struck', W_n for 'match n is wet', and L_n for 'match n lights'. Let S be $S_1 \wedge S_2 \wedge S_3 \wedge \ldots$ (each match is struck) and use:

M_1 for $S \wedge (W_1 \wedge \neg L_1) \wedge (W_2 \wedge \neg L_2) \wedge (W_3 \wedge \neg L_3) \wedge \ldots$
M_2 for $S \wedge (W_2 \wedge \neg L_2) \wedge (W_3 \wedge \neg L_3) \wedge (W_4 \wedge \neg L_4) \wedge \ldots$

$\vdots$

M_n for $S \wedge (W_n \wedge \neg L_n) \wedge (W_{n+1} \wedge \neg L_{n+1}) \wedge \ldots \wedge (W_{n+2} \wedge \neg L_{n+2}) \wedge \ldots$

$\vdots$

Thus M_1 say that all of the matches are struck but are wet and do not light, M_2 says that all of the matches are struck and that all from the second on are wet and do not light, and M_n, in general, says that all of the matches are struck and that all from the n-th on are wet and do not light.

Let us also suppose that no match is in fact struck. It can then be argued that the sentences $M_1, M_2, M_3, \ldots$ (in place of $R_1, R_2, R_3, \ldots$) will conform to the non-logical assumptions above:

Positive Effect M_{n+1} is a logical consequence of M_n (since it results from removing some of the conjuncts from M_n); and so $M_n > M_{n+1}$ by Entailment. Thus in this case there is no need for a special non-logical assumption; Positive Effect is guaranteed by the logic of counterfactuals alone.

Negative Effect It may surely be granted that $S_1 > L_1$ (if the first match were struck it would light). But $S_2, S_3, \ldots, W_2, \neg L_2, W_3, \neg L_3, \ldots$ are entirely irrelevant to L_1 being a counterfactual consequence of S_1, i.e. to whether the first match would light if struck, since they concern what happens in causally isolated regions of the universe; and so the counterfactual $S_1 \wedge S_2 \wedge S_3 \wedge \ldots \wedge (W_2 \wedge \neg L_2) \wedge (W_3 \wedge \neg L_3) \wedge \ldots > L_1$ (i.e., $M_2 > L_2$) should also be true. But $\neg M_1$ is a logical consequence of L_1 (since $\neg L_1$ is one of the conjuncts of M_1); and so $M_2 > \neg M_1$ should also be true. A similar argument establishes $M_{n+1} > \neg M_n$ for any n.

Counterfactual Possibility There appears to be nothing incoherent about the counterfactual supposition that all of the matches are struck but that all from some point on are wet and do not light.

One residual worry one might have is that what goes on in the other regions could be relevant in a non-causal way to what goes on in the given region. Thus, given that conditions for the match lighting were not propitious in all but finitely many of the specified regions, one might think that they would not then be propitious in the given region. But we may easily take care of this worry by making it part of the counterfactual supposition that there already are infinitely many regions which are propitious for the lighting of a match. More generally, there is no reason for there to be any uniformity in what goes on in the different regions; it could involve the striking of a match in one region, the falling of a rock in another, the turning on of a light in yet another, and so on. Thus it would not even be clear how what went on in the other regions could be relevant, either causally or non-causally, to what went on in the given region.[2]

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[2]In constructing the case I have relied on Goodman's example of a match. But there is a general recipe for constructing a case of the required sort. Suppose that $S > L$ is a counterfactual truth in some hypothetical situation which is (a) non-trivial in the sense that $S \wedge \neg L$ is a counterfactual possibility and (b) local in the sense that its truth only turns on how things are in a finite region of space-time. Now let $S_1 > L_1$, $S_2 > L_2$, ... be an infinite sequence of such counterfactuals $S > L$ that turn on how things are in different causally isolated regions of space-time. Then we may use $S_1, S_2, \ldots$ and $L_1, L_2 \ldots$ to construct the formulas $M_1, M_2, \ldots$ (for

The above example is infinite. But there are at least two different ways in which it might be 'finitized'. We might, in the first place, limit the non-logical assumptions of the argument to the first n statements R_1, R_2 ,..., R_n, with R_k counterfactually implying R_{k+1} and R_{k+1} counterfactually implying $\neg R_k$ for k = 1, 2, ..., n -1 and with the counterfactual possibility of $R_1 \vee R_2 \vee ... \vee R_n$ in place of the counterfactual possibility of $R_1 \vee R_2 \vee ....$ It then seems clear that the assumptions can be satisfied for each n = 1, 2, ...[3] But if for each n, then why not for all n? Surely, it is completely arbitrary to allow this sort of possibility for finitely many statements R_1, R_2 ,..., R_n but not for infinitely many statements R_1, R_2 , ...

In the second place, we may convert the infinite chain into a finite cycle.[4] Suppose that there are three rocks and that they are arranged in a circle so that if one were to fall then the next in line (going clockwise) would fall but that the previous one would not fall. This could be, for example, because a given rock falling as a result of the previous rock falling would not have the same impact as the given rock falling of its own accord and so would not result in the next rock falling. We then have the following finitary counterparts to the non-logical assumptions:

$R_1 > R_2$, $R_2 > R_3$, $R_3 > R_1$;
$R_2 > \neg R_1$, $R_3 > \neg R_2$, $R_1 > \neg R_3$;

$R_1 \vee R_2 \vee R_3 > \neg(R_1 \vee R_2 \vee R_3)$,

and, with the help of the logical assumptions (though Infinitary Conjunction is no longer required), a contradiction may be derived in the same way as before. Unfortunately, it does not appear possible to 'Goodmanize' this example and thereby disarm the objection that we should always allow for the possibility that a given rock might fall by way of the previous rock falling.

Finally, let me mention a somewhat more realistic version of the original case. Imagine a missile that has a mechanism for correcting its flight when it deviates from the intended path and suppose that the conditions for the flight of the missile are optimal and that the missile does not in fact deviate from its path. Let R_k be the sentence 'the missile deviates $(2^k -1)/2^k$ inches off course' (so R_1 is 'the missile deviates 1" off course', R_2 'the missile deviates ½" off course' and so on). Then the non-logical assumptions are all very plausible. If, for example, the missile were to deviate 1" off course then it would deviate ½" off course (as a result of having a continuous flight path) and if the missile were to deviate ½" off course, then it would not deviate 1" of course (since the best approximation to the actual conditions would be one in which the deviation was immediately corrected). And similarly for any other case in which we have a quantity that does not in fact change its value but that is capable of a continuous change in value under conditions in which the departure from its actual value will be minimized.

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simplicity, omitting reference to W_n). Thus all that is required to satisfy the non-logical assumptions is the existence of local and non-trivial counterfactual truths

[3]Indeed, for each n, the assumptions will be consistent in all of the standard counterfactual logics. By exploiting this fact, we can demonstrate the non-compactness of semantics for counterfactuals with the 'limit assumption'.

[4] I owe this suggestion to Rachel Briggs.

§3 Transitivity and Disjunction

We turn to the logical assumptions, dealing with Identity, Weakening, Transitivity and Disjunction in the present section and with Infinitary Conjunction and Substitution in the subsequent two sections.

A number of different axioms and rules for the logic of counterfactuals have been proposed and some have been found to be problematic. These include the rule A, C/ A > C that allows one to infer the truth of a counterfactual from the truth of its antecedent and consequent, the rule A > C, ¬(A > ¬B) / A ∧ B > C that allows one to add what might be true under a counterfactual supposition to the supposition, and the axiom of Conditional Excluded Middle, A > C ∨ A > ¬C, according to which any counterfactual supposition will have determinate consequences.

However, as far as I am aware, the logical assumptions presently under consideration have never been questioned. There appear to be no plausible counter-examples to them and their use in counterfactual reasoning is, on the face of it, completely unproblematic.[5] Moreover, even if we were to give up one of these assumptions, it would not save the standard version of the similarity semantics.

On what I am calling a *similarity* semantics (whether standard or not), it is assumed that there is a similarity ordering on worlds, according to which one world w can be said to be as close or similar (to a given 'base' world) as is the world v. It is also assumed that the relation of similarity (relative to a given base world) is reflexive and transitive. However, it is not assumed that any two worlds are always comparable with respect to closeness; and nor do we make the 'Limit Assumption' according to which any non-empty set of worlds V will always contain a closest world (one to which no world of V is closer).

Given a set of worlds V, let us say that V′ is a *close* subset of V if (i) for any world in V there is a world as close as that world (to the base world) in V′ and if (ii) any world in V as close (to the base world) as a world in V′ is also in V′. Then under what I am calling the *standard* version of the similarity semantics, the counterfactual A > C is taken to be true (at the base world) iff there is a close subset of the set of A-worlds in which C is true.[6] Thus the standard similarity semantics adopts a particular view as to how the truth of a counterfactual relates to the similarity of worlds.

Under this semantics, it is readily shown the logical rules presently under consideration - Identity, Weakening, Transitivity and Disjunction - are all valid; and so it only by giving up the semantics that it would be possible to question the validity of one of these rules.

One might well have thought that there was no plausible alternative to the *standard* version of the similarity semantics. But somewhat surprisingly, it turns out that this is not so and that there *is* an alternative which fails to validate all of the above rules and which is of some

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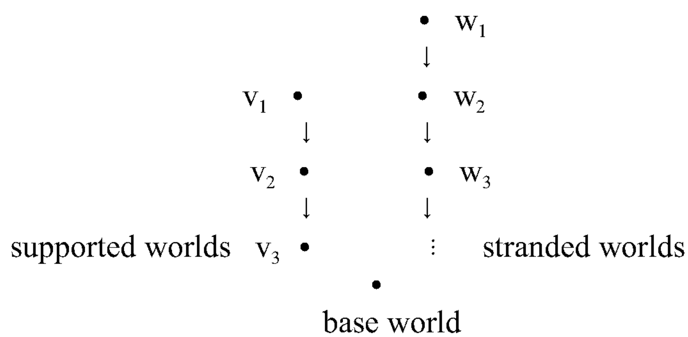
[5]We shall find reason in §6 to question Weakening, but the difficulties we will raise for it have no bearing on the present case.

[6]This is essentially equivalent to the account in Lewis [1981], 230. To simplify the exposition, I have assumed that all worlds are 'entertainable' from the given world.

interest, both in itself and through its bearing on the puzzle.[7]

Say that a member of a set of worlds is *supported* if there is a closest world of the set that is as least as close (to the given base world) as it is and otherwise say it is *stranded*. A supported world, when it looks 'down', can see a closest world to the base world but a stranded world can only see worlds that get closer and closer without end to the base world:



Say that a set of worlds is itself *supported* if each of its member worlds is supported and that otherwise it is *stranded*.

If the set of A-worlds is supported, then there would appear to be no reasonable alternative to the standard semantics: the counterfactual A > C will be true iff C is true in all of the closest A-worlds. But it is not so clear what we should say when the set of A-worlds is stranded. One *might* adopt the standard semantics. But an alternative to the standard semantics is to take A > C to be true if C is true in all of the closest and all of the stranded A-worlds. If we cannot get as close as possible to the base world, so to speak, then we do not even try.

We can bring out the difference between the two versions of the similarity semantics if we consider a chain of worlds w_1, w_2, w_3, ... that get closer and closer to the base world, as in the illustration above. Suppose that these worlds are exactly the A-worlds. Then on the standard version of the semantics, the counterfactual A > C will be true iff C is true in all but finitely many of the worlds. But on the alternative version, the counterfactual A > C will be true iff C is true in all of the worlds, since all of them are stranded.

It turns out that the alternative semantics will validate all of the logical rules (including Infinitary Conjunction) but with the single exception of Disjunction. For suppose that A_1 is true in the worlds w_1,w_2, that A_2 is true in the worlds w_3,w_4, ... and that C is true in all worlds but w_1:

w_1	•	A_1	
w_2	•	A_1	C
w_3	•	A_2	C
w_4	•	A_2	C
⋮	⋮	⋮	⋮
	•		

[7] I should perhaps add that it was consideration of the puzzle that led me to the present semantics.

Then $A_1 > C$ and $A_2 > C$ are both true while $A_1 \vee A_2 > C$ is not, since all of the worlds $w_1, w_2, \ldots$ in which $A_1 \vee A_2$ are true are stranded. (And when it comes to the argument itself, we might question the step in which we go from $[R_1 \vee (\neg R_1 \wedge R_2)] > \neg R_1$ (the first rock would not fall if the first or second were to fall) and $[\neg R_1 \wedge \neg R_2 \wedge (R_3 \vee R_4 \vee \ldots)] > \neg R_1$ (the first rock would not fall if one of the rocks from the third on were to fall) to $R_1 \vee (\neg R_1 \wedge R_2) \vee [\neg R_1 \wedge \neg R_2 \wedge (R_3 \vee R_4 \vee \ldots)] > \neg R_1$ (the first rock would not fall if one of the rocks were to fall).)

But this alternative also comes at great cost. For we will have to give up Disjunction, which we have seen no independent reason to question. And we will have to give up many of our intuitive judgements concerning the truth of particular counterfactuals.

Consider the rock case, for example. Then it may be shown, given Positive and Negative Effect, that for every R_1-world there will be a closer R_2-world, for every R_2-world a closer R_3-world, and so on.[8] It follows that each R-world is stranded. For any R world will be a R_k-world for some k and so, by Positive and Negative Effect, there will be a closer R_{k+1}-world and hence a closer R-world.

Given that each R-world is stranded, an arbitrary statement C will be a counterfactual consequence of R under the alternative semantics only if it is true in *all* of the R-worlds, i.e. only if it is entailed by R. More generally, let us say that the statement A is a *trivial* counterfactual supposition if $A > C$ is true only in the 'trivial' case in which A entails C. Then under the alternative semantics, A will be a trivial counterfactual supposition if (and, in fact, only if) each A-world is either a closest A-world or stranded.

But surely R is *not* a trivial counterfactual supposition. For a counterfactual consequence of $R = R_1 \vee R_2 \vee R_3 \vee \ldots$ is $R' = R_2 \vee R_3 \vee R_4 \ldots$; if one of the rocks from the first on were to fall then one of the rocks from the second on would fall. Yet R' is not entailed by R, since if only the first rock falls then R will be true and R' false. And similarly for many other cases of this sort.

§4 Infinitary Conjunction

As Pollock has observed, the infinitary rule of Conjunction appears to be 'just as obvious' as the finitary rule and should be valid 'for exactly the same reason'.[9] For what makes the finitary rule plausible is the more general principle that the logical consequences of the counterfactual consequences of a counterfactual supposition should also be counterfactual consequences of the supposition. But if this is the justification of the finitary rule, then it serves equally well to justify the infinitary rule.

But desperate times call for desperate remedies. Our only current hope, if we wish to cling

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[8] For take any R_k-world w. Then w will be at least as far as a stranded or closest R_k-world w'. By Positive Effect, w' will be a R_{k+1}-world; and so it will be at least as far as a stranded or closest R_{k+1}-world v. But w' cannot be just as far as v since otherwise Negative Effect would not hold.

[9] Pollock ([1976a], 471; [1976b], 20). He has somewhat different principles in mind but the point applies just the same. I should also note that it is not essential to the point that the notion of consequence in question be taken to be classical rather than intuitionistic, say, or relevant.

to Substitution, is to dismiss reject Infinitary Conjunction; and there is perhaps some comfort to be derived from the fact that the rule is not valid under Lewis' own semantics for the counterfactual. We should therefore do well to consider if there is any reasonable basis upon which it might be rejected.

There is perhaps a pragmatic consideration in favor of abandoning - or, at least, of not insisting upon - the rule. For whether valid or not, it is not a rule that we finite creatures could ever use; and so no great loss would come from our not having it.

It is not clear to me that the logic of counterfactuals should be conceived pragmatically in this way. But even if it is, the difficulties will still not disappear. For analogous problems arise with the use of quantificational expressions.

Corresponding to the infinitary conjunction rule is the following rule for the universal quantifier:

<u>Generalization</u>

$$\frac{\forall x(A > C(x))}{A > \forall x C(x)}$$

Now this rule is not generally valid. Consider the statement: everyone alive in the 20th Century would have been well fed if the food supply had been doubled. This is ambiguous between a narrow and wide scope reading of the quantifier 'everyone'. It could mean 'everyone alive in the 20th Century is such that he would have been well fed if the food supply had been doubled', which is of the form $\forall x(A > C(x))$; or it could mean 'if the food supply had been doubled, then everyone alive in the 20th Century (in those circumstances) would have been well fed', which is of the form $A > \forall xC(x)$. But one may well think that the first counterfactual is true but the second is false, since with the increase of the food supply would have come an increase in the population.

The reason the inference is not valid in this case is that there would have been more people than there actually are if the food supply had been doubled - the hypothetical domain of quantification would have contained objects not in the actual domain. But let us suppose that the domain does not expand in this way (so, where X is the class of objects that actually belong to the domain, it will be true that $A > \forall x(x \in X)$, i.e., under the counterfactual supposition A, every object would have been an actual object). The rule of Generalization will then be valid; given no hypothetical increase in the population, if *each* person would have been well fed had the food supply been doubled, then *everyone* would have been well fed had the food supply been doubled.

But it is now possible to reinstate the argument using finitary quantified statements in place of infinitary conjunctions and disjunctions. Let the quantifiers range over the rocks and take F(m) to mean that rock m falls. We can then avoid the infinitary apparatus. In place of of Possibility, for example, we would now have $\neg(\exists mF(m) > \neg\exists mF(m))$; and in the derivation, we would argue from $\forall n(\exists mF(m) > \neg F(n))$ to $(\exists mF(m) > \forall n\neg F(n))$, using Generalization in place of Infinitary Conjunction. This is a valid application of the rule since, within the given scenario, a rock's falling would have no effect on what rocks there are.[10]

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[10]One slight change in the argument is that we need to employ the substitution of (simple) arithmetical equivalents in place of the substitution of logical equivalents but I assume that, in

Of course, we must still allow that the rule should have application to cases in which the quantifiers range over an infinite domain. But there are many significant applications in the physical sciences where this will be so. For example, one often wishes to argue to a 'limit'. Thus one might know that within a certain time interval around a given instant t_0 a particle would under certain counterfactual circumstances have a certain velocity and that this velocity will approach a certain limit v_0 as the time intervals approach t_0. One then wishes to conclude that under those circumstances the particle would have v_0 as its velocity at t_0. But this line of reasoning involves Generalization. For from the fact that the particle would have such and such a velocity for *each* interval, one will wish to infer that the particle would have the velocity for *every* interval, from which its limit velocity v_0 is then inferred. Thus counterfactual reasoning within the physical sciences would be seriously compromised if the rule were given up.

But it seems to me that the consequences of giving up the infinitary rule (or its quantificational counterpart) would be devastating, not only for the way we reason with counterfactuals but also for some of the central uses that we wish to make of them. If we look at the derivation of a contradiction from our assumptions, we see that it does not require the full use of the infinitary rule but only the special case in which the counterfactual consequences are logically inconsistent with the counterfactual supposition.[11] Thus the rule can take the following special form:

Infinitary Consistency
$$\frac{A > C_1 \quad A > C_2 \quad A > C_3 \ldots}{A > C_1 \wedge C_2 \wedge C_3 \wedge \ldots.} \quad \text{where } C_1, C_2, C_3, \ldots \text{ are jointly inconsistent}$$

This means that if the relevant application of the rule is challenged, then it must be allowed that a statement A might be a counterfactual possibility (since $A > C_1 \wedge C_2 \wedge C_3 \wedge \ldots.$ is not true) even though the counterfactual consequences of A are jointly inconsistent. We might call a counterfactual supposition of this sort *paradoxical*.

But the trouble with paradoxical counterfactual suppositions is that they are of no use for two of the main purposes for which counterfactuals are required - decision making and theory testing. Suppose I am deciding between bringing it about that A or bringing it about that B. To this end, I may consider the counterfactual consequences of bringing about the one or the other and then make my decision on the basis of a comparison between the consequences. But what if one or both of the counterfactual suppositions that I bring about A and that I bring about B are paradoxical? Then how can I compare them, given that I can form no coherent conception of what one or both of them are? Perhaps one is a matter of giving you some pain and the other a matter of giving you some pleasure. One feels that one should be able to decide in favor of giving you some pleasure (other things being equal). But how is this possible on the present view when the alternatives involves giving you some pleasure though not any specific amount of pleasure -

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the present context, the one rule is as plausible as the other.

[11] Let R be the disjunction $R_1 \vee R_2 \vee \ldots.$ Then the argument only requires that we infer $R > R \wedge \neg R_1 \wedge \neg R_2 \wedge \ldots$ from $R > R$, $R > \neg R_1$, $R > \neg R_2, \ldots.$

not 1utile, ½ a utile, 1/4 utile etc., while the other involves giving you some pain though not any specific amount of pain - not 1 disutile, ½ a disutile, 1/4 a disutile etc. Even the most committed hedonist might feel challenged in these circumstances.

Or again, suppose I wish to test a theory. To this end, I take a particular counterfactual possibility A and consider what would follow, according to the theory, if A were to obtain. The theory is then disconfirmed if one or more of these consequences fail to hold when A is made the case and is otherwise confirmed. Perhaps the theory predicts what would happen if the temperature were to go up. One feels that one should be able to test the theory on the basis of this supposition. But what if the supposition is paradoxical according to the theory? The temperature goes up, but not by any particular amount - not by 4°C, not by 2°C, not by 1°C etc. Then one of its consequences must inevitably fail to hold (and, indeed, no actual testing of the theory would be required to see that this was so!). And yet we would not want to reject a theory simply because it tolerated paradoxical counterfactual suppositions, which could after all have been true.

Indeed, not only will these suppositions be of no use when they should be of use, they will also get in the way of making the counterfactual judgments that *can* properly be taken into account. For we do not want to find ourselves deciding between two alternatives, when one of them is paradoxical, or testing a theory on the basis of a paradoxical supposition. But this means that prior to making a decision or testing a theory, we will need to settle the question of whether the suppositions in question are paradoxical; and this is, in general, no easy task. Thus decision making and theory testing will become encumbered by the tricky preparatory exercise of determining whether the counterfactual suppositions can properly be made in the first place.

One further argument for rejecting Infinitary Conjunction (or Consistency) is implicit in Lewis [81]. Lewis ([73], 20) had previously argued that the so-called 'Limit Assumption' might fail to hold, with worlds getting closer and closer to the actual world without end. His example is of a line that is in fact under 1" in length. Consider now the worlds in which the line is longer than 1". Then for any such world there will be a closer world in which the line is closer to being 1" in length. Pollock ([1976a], 471; [1976b], 20) and Herzberger [1979] pointed out that such similarity judgments, in conjunction with the standard similarity semantics, will lead to the violation of such rules as Infinitary Conjunction and Consistency - for, within sufficiently close worlds, the line will be no more than 1½" in length, no more than 1¼" in length, ..., and so it will be true that if the line were longer than 1" then it would be no more than 1½" in length, no more than 1¼" in length, ...; and this led Pollock to question the similarity judgements on the basis of which the Limit Assumption had been thought to fail. His view was that worlds in which the line was 1¼" long, for example, should be regarded as being no closer, in the relevant sense, to the actual world than worlds in which the line was 1½" long.

In response to these criticisms, Lewis ([81], 229-30) attempted to argue that there are other cases in which the relevant similarity judgements are harder to deny. For suppose we are given infinitely many independent propositions, all true in the actual world. Then worlds in which infinitely many of them are false will get closer and closer to the actual world (keeping other things fixed); for whichever of them turn out to be false in a given world, there will always be another world in which fewer of them are false.

This argument is curious. For one might have thought that, in response to these criticisms, Lewis would have attempted to provide an intuitive counter-example to the validity of the rules, one in which the premisses seem to be true and the conclusion to be false. But he does not do this and attempts to show instead that certain similarity judgments, in conjunction with the similarity semantics, will imply that the rules are invalid. And nor does the case he presents even lead to an intuitive counter-example, for we have no inclination to say that if infinitely many of the independent propositions were false then each particular one of the propositions would be true.

Thus his response to the criticisms is made from a theoretical stance in which the correctness of the similarity semantics is simply taken for granted. He does indeed concede that there are some 'formal advantages' in adopting the rules but does not seem to think that they are sufficiently compelling to cast doubt on the semantics.

I myself would not wish to follow Lewis in his methodology but, even if we do, it is still not clear that we should accept his conclusions. For as we have already noted, the 'standard' similarity semantics in terms of sufficiently close worlds is not the only option available to us once the Limit Assumption is allowed to fail. There is also the possibility of requiring the consequent of the counterfactual to be true in *all* of the stranded worlds, and not merely the sufficiently close stranded worlds, in which the antecedent is true; and under this option, the infinitary rules can be retained (though Disjunction must then be rejected).

Moreover, this option is in some ways more palatable than Lewis'. For: (i) Disjunction is less intuitively compelling than Infinitary Conjunction and hence more plausibly rejected; and (ii) unwanted triviality is less bothersome than unwanted paradoxicality and less of an impediment to the use of counterfactuals in decision making and theory testing.[12]

Of course, it is also possible to question the similarity judgments that Lewis originally made about the case; and this is something that Lewis *is* willing to consider. He thinks that if we wish to retain the Limit Assumption then we should 'coarse-grain' and ignore 'most of the countless respects of difference that make possible worlds infinite in number' and that, although coarse-graining is a 'formal option', it is doubtful whether 'it can be built into an intuitively adequate analysis of counterfactuals'.

But the question immediately before us is whether the advocate of the Limit Assumption is able to deal with Lewis' particular case; and on this question, it might be thought that Lewis could have pressed his advantage. Recall that the independent propositions are true in the actual world while infinitely many of them are false in the worlds under consideration. Lewis wants to say that the fewer of them are false the closer we are to the actual world, thereby creating a counter-example to the Limit Assumption. But if we are to retain the Limit Assumption, then what are we to say instead? To fix ideas, let us suppose that there is a countably infinity of propositions p_1, p_2,p_3, ..., that all of them are actually true, and that infinitely many of them are false and infinitely many of them are true in the counterfactual worlds under consideration. It would then seem that the only plausible way to retain the Limit Assumption is to treat these worlds as all equally close or as all incomparable in closeness. But consider now the counterfactual: if infinitely many of p_2,

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[12]In all fairness, I should point out that paradoxicality under Lewis' proposal will be less widespread than triviality under our own; for the supposition A will be paradoxical for Lewis just in case every A-world is stranded while it will trivial for us just in case every A-world is a closest A-world or stranded. However, paradoxicality is *always* undesirable, triviality only *sometimes* so.

p_3, p_4,... were true and infinitely many false then p_1 would (still) be true. This counterfactual would appear to be true, since the truth-values of p_2, p_3, ... have no bearing on the truth-value of p_1. However, it would not be true under the proposed similarity relation since the closest worlds in which infinitely many of p_2, p_3, p_4,... are true and infinitely many are false will include $\neg p_1$ -worlds in addition to p_1-worlds. So it looks as if there is no reasonably way by which we might coarse-grain and thereby protect the Limit Assumption.

I do not think, however, that this is the correct diagnosis of what has gone wrong. For the argument here rests upon our having the intuition that if infinitely many of p_2, p_3, p_4,... were true and infinitely many false then p_1 would be true. But if we have this intuition then we should also have the intuition that if infinitely many of p_3,p_4, p_5,... were true and infinitely many false then p_2 would be true, that if infinitely many of p_4, p_5, p_6, ... were true and infinitely many false then p_3 would be true; and similarly for p_5, p_6, p_7, ..., p_6, p_7, p_8, ..., ... But each of these antecedents is necessarily equivalent (and, indeed, logically equivalent if expressed as an disjunction of conjunctions). So it follows that if infinitely many of p_1, p_2, p_3, ... were true and infinitely many false then each of p_1, p_2, p_3, ... would be true and hence that all of them would be true!

Thus we see that the intuition on which we have relied is already in conflict with Substitution and Infinitary Conjunction (given the counterfactual possibility that infinitely many of p_1, p_2, p_3,... are true and infinitely many false). Now one possible way to resolve the conflict is to reject Infinitary Conjunction (and hence to reject the Limit Assumption). But one could with far greater plausibility reject Substitution, for even though the proposition that infinitely many of p_2, p_3, p_4, ... are true and infinitely many false can plausibly be taken to counterfactually imply that p_1 is true, the proposition that infinitely many of p_1, p_2, p_3, ... are true and infinitely many false cannot plausibly be taken to counterfactually imply that p_1 is true. Thus rather than accept the counter-intuitive consequences of Substitution and refuse to apply Infinitary Conjunction to them, it would appear preferable not to accept the consequences in the first place.[13]

We are thereby free to accept Infinitary Conjunction. But it should not be thought that the Limit Assumption is thereby vindicated. For what we have shown is that the counterfactual judgments used in arguing against the Limit Assumption are incompatible with *any* similarity relation that might be imposed upon the worlds, whether in conformity with the Limit Assumption or not. Thus the Limit Assumption simply becomes irrelevant to the question of which counterfactuals should or should not be accepted.

If this is right, then a powerful argument against the possible worlds semantics was already implicit in the earlier discussion of the Limit Assumption. But so great, one assumes, was the commitment to some form of the similarity semantics, that neither side to the dispute was properly

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[13]In one way, the present argument against Substitution is an improvement on the earlier arguments, since it does not rest upon the assumptions of Disjunction or Transitivity. But in another way, the argument is less satisfactory, since it might be maintained that, with the change in the formulation of the antecedent, comes a change in the criterion of similarity, or what have you, by which the counterfactual is to be assessed, whereas this is not really on the cards with the earlier arguments.

able to appreciate its force.[14]

Let me note, finally, that the finitary cases of the puzzle do not even appeal to Infinitary Conjunction. It is plausible to suppose that the finitary and infinitary cases of the puzzle should have a common solution. But if that is so, then it cannot lie in the rejection of Infinitary Conjunction and some other form of solution should therefore be found.

§5 Substitution of Antecedents

The previous assumptions, in conjunction with Substitution, imply a contradiction. We have found no good reason from our earlier discussion to question the other assumptions; and this suggests that the fault lies with Substitution. And, indeed, when we examine the derivation of the contradiction, we see that this rule is the only one whose application in the reasoning of the puzzle is subject to intuitive doubt. For in order to derive the intermediate conclusion $[R_1 \vee (\neg R_1 \wedge R_2)] > \neg R_1$ (step (i) from the appendix), we have to make the inference from $R_2 > \neg R_1$ to $[(R_1 \wedge R_2) \vee (\neg R_1 \wedge R_2)] > \neg R_1$; and yet this inference does not appear to be valid. For granted that the first rock would not fall if the second rock were to fall, it hardly seems correct to say that the first rock would not fall if either the first and the second were to fall or the second but not the first were to fall. Thus it is not as if we are in the area of paradox with a number of assumptions and a number of steps, all equally compelling, leading to a contradiction. For one of the steps is far from compelling and, indeed, appears to be straightforwardly mistaken; and since nothing else about the argument appears to be problematic, the obvious way to escape the contradiction is to reject this one step and the more general principle of Substitution upon which it rests.

If asked why the critical step from $R_2 > \neg R_1$ to $[(R_1 \wedge R_2) \vee (\neg R_1 \wedge R_2)] > \neg R_1$ appears to be invalid, it is natural to point to a consequence of the latter statement not had by the former. For the latter statement appears to imply $R_1 \wedge R_2 > \neg R_1$ (that the first rock would not fall if the first and second rocks were to fall), which certainly does not follow from $R_2 > \neg R_1$ (that the first rock would not fall if the second rock were to fall). And, in general, it might be thought that we should accept a kind of converse to Disjunction:

Simplification $$\frac{A \vee B > C}{A\ (B) > C}$$

according to which the counterfactual from $A \vee B$ to C licenses the counterfactual from A to C and from B to C.

It turns out that there has been extensive debate over this rule in the literature, with one side insisting on its validity and the other side attempting to demonstrate its invalidity.[15] The rule is not of course valid under the standard similarity semantics (even assuming a single closest world). For if the closest $A \vee B$-world is an $A \wedge C$-world, then $A \vee B$ will counterfactually imply C

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[14] I myself was blind. I recall that, in writing a review of Lewis' 'Counterfactuals' (Fine [1975]), I removed a discussion of Infinitary Conjunction, thinking it to be of peripheral interest.

[15] A review of some of this work is to be found in §1.8 of Nute & Cross [2002]

and yet B will not counterfactually imply C given that the closest B-world is not a C-world. Moreover, given Substitution, Simplification will license the inference from $A > C$ to $A \wedge B > C$[16], which is commonly recognized to be invalid. But no matter how convincing these considerations may be for someone who already accepts the similarity semantics or the rule of Substitution, they have no force in the present context, where the semantics and the rule are themselves in question.

Some philosophers, however, have attempted to provide arguments against Simplification that are independent of the acceptance of Substitution or the similarity semantics; and the relationship between these arguments and the reasons we ourselves have offered for rejecting Substitution needs to be discussed. I previously said that it was *natural* to appeal to Simplification in explaining why it was independently plausible to reject the critical application of Substitution (in which one goes from $D_2 > \neg D_1$ to $[(D_1 \wedge D_2) \vee (\neg D_1 \wedge D_2)] > \neg D_1$); and if one does appeal to Simplification in this way, then any reason one has for rejecting Simplification will serve to undermine the plausibility of this response. But although it is natural to appeal to Simplification in this connection and although this is something that I myself would be happy to do, nothing so strong is actually required. It might be thought, for example, that $[(D_1 \wedge D_2) \vee (\neg D_1 \wedge D_2)] > \neg D_1$ should imply that $\neg D_1$ *might* be true if $D_1 \wedge D_2$ were true, not that it actually *would* be true; or it might be thought that, in this particular case, $[(D_1 \wedge D_2) \vee (\neg D_1 \wedge D_2)] > \neg D_1$ could not be true unless $(D_1 \wedge D_2) > \neg D_1$ were true, even though Simplification was not in general valid.

Indeed, subversion may well go in the opposite direction; for there is a way in which considerations arising from our argument serve to undermine the motivation that some philosophers have had for wanting to reject Simplification. For these philosophers have characteristically thought there may be some kind of mismatch between the ordinary language version of Simplification, expressed with vernacular ‘or’, and its formal version, expressed with symbolic ‘$\vee$’, and that even though the ordinary language version may be valid - or, at least, may appear to be valid for pragmatic reasons or the like - the formal version is not itself valid.[17] To such philosophers, we can say that our argument has been expressed entirely in symbolic terms. So use the logical symbols - $\vee$, $\wedge$ and $\neg$ - in just the way you think they should be used, without regard to ordinary language or pragmatic considerations, and then tell us where the argument goes wrong. Now the obvious place is in the critical application of Substitution. But our philosopher will have none of it. He tells us that the application only appears to be mistaken because of some kind of mismatch between the symbolism and ordinary language and that the mistake in the argument itself must lie elsewhere, even though it is not clear where it might be. One is left to wonder why our philosopher make things so difficult for himself. By denying the obvious connection between ordinary language and the symbolism, he deprives himself of the obvious - and, indeed, of the only obvious - explanation of where the reasoning goes wrong. But why not simply accept the obvious connection and the obvious explanation of the error that goes with it?

Even setting aside considerations of motivation, many of the arguments in favor of the apparent validity - or even of the actual validity - of ordinary language Simplification are

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[16]Since we can go from $A > C$ to $[(A \vee (A \wedge B)] > C$ by Substitution and then to $(A \wedge B) > C$ by Simplification.

[17]As suggested in Lewis [1977], for example, or McKay & van Inwagen [1977].

unconvincing. There is not the space to discuss these arguments in any detail, but one common defect is that they are not pitched at the right level of generality. The apparent (or actual) validity of ordinary language Simplification is an instance of a much broader phenomenon. Nute pointed out some time ago that 'not both' will work just as well as 'or'. His example (Nute [1980a], 33) is'if Nixon and Agnew had not both resigned, Ford would never have become President'. From this it appears to follow that if Agnew had not resigned, Ford would never have become president, in analogy to Simplification. But there are many other cases of this sort. The counterfactual 'if at least one of the women had been present then the conference would have been much better' is naturally taken to imply 'if Beatrice (Sharon) had been present then the conference would have been better', given that Beatrice and Sharon are among the women; the counterfactual 'if infinitely many of the propositions p_1, p_2, p_3, ... had been false, then things would have been very different' is naturally taken to imply 'if the propositions p_2, p_4, p_6, ... (p_1, p_3, p_5, ... etc.) had been false, then things would have been very different'; and the counterfactual 'if he were to earn more he would be happier' is naturally taken to imply 'if he were to earn a little more he would be a happier' as well as 'if he were to earn a lot more he would be happier'.[18] It seems to me that we should be suspicious of any explanation of the apparent or actual validity of ordinary language Simplification that does not extend to these other cases; and yet, as far as I am aware, none of them do - they all rely on features of 'or' that do not extend in any obvious way to the other logical particles (such as 'not both') or to the cases (as with 'earn more') in which no logical particles are employed.[19]

Another weakness in some of these arguments, from our point of view, is that they are essentially defensive. Someone like Loewer [76], for example, thinks that there is no mismatch in logical form between the formal and the ordinary language version of Simplification and he accepts the similarity semantics under which the formal argument would not be valid. He therefore faces the problem of explaining why the argument appears to be valid; and to this end, he provides a pragmatic explanation of how one might learn the truth of the conclusion from the assertion of its premise. Thus such a line of reasoning serves merely to defend his view against potential counter-example. There is nothing in it to suggest that someone else would be mistaken if he took the conclusion of ordinary language version of Simplification to be a semantic, rather than a pragmatic, consequence of its premises; and so there is no reason for someone who rejects Substitution on the basis of his acceptance of Simplification to be moved by this line of reasoning.

There is, however, an argument against Simplification that we do need to take seriously. For certain 'excluder' counterfactuals appear to be straightforward counter-examples to the rule. A case of McKay & van Inwagen [1977] is 'If Spain had fought with the Axis or the Allies, she would have fought with the Axis'. According to Simplification, this should imply 'If Spain had fought with the Allies in World War II she would have fought with the Axis'. Yet surely we do not want to accept this conclusion even if we are willing to accept the premise.

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[18] A similar example, proposed to a different end, is in Nute & Cross [2002], p. 23.

[19]This criticism applies, for example, to Loewer's [76] pragmatic explanation of the apparent validity of ordinary language Simplification and to Alonso-Ovalle's [2009] more recent attempt to defend the actual validity of ordinary language Simplification.

In the face of this apparent counter-example, I would like to suggest that Simplification *is* generally valid and that the counterfactual conclusion ‘if Spain had fought with the Allies she would have fought with the Axis’ does indeed follow from the counterfactual assumption ‘if Spain had fought with the Axis or the Allies then she would have fought with the Axis’. I therefore owe an explanation of why the inference appears to be invalid and why we are ready to accept the premise and yet not accept the conclusion.

The explanation has to do, I believe, with a pragmatic principle of ‘Suppositional Accommodation’, according to which we always attempt to interpret any counterfactual we employ in such a way that its antecedent A represents a genuine counterfactual possibility, i.e. one for which ‘if A were the case then ¬A would be the case’ is false (or at least not true). The intuitive basis for this principle is that a counterfactual should not hold ‘vacuously’, simply because we cannot make sense of what it would be for the antecedent to be true.

As a rule, a counterfactual will hold vacuously just in case its antecedent fails to represent a counterfactual possibility, i.e. ‘If A were the case then C would be the case’ will be vacuously true, and hence true for all C, just in case it is true for the case in which C is ¬A. But there may be exceptions. One might wish, for example, to assent to the counterfactual ‘if the Liar were true then it would not be true’ even though it does not hold vacuously but in virtue of a logico-semantic connection between the antecedent and the consequent; and similarly, one might wish to assent to ‘if everything were the case then not everything would be the case’. In these exceptional cases, it is non-vacuity in a more intuitive sense that is required.

Now it is plausible to suppose that the counterfactual possibilities will vary with context. In the context of testing a physical theory, for example, we would want to leave open what is physically possible and to exclude what is physically impossible while, in the context of wondering what to do, we would want to leave the future open, at least to the extent that it can be influenced by what we do, and to keep the past fixed. From this perspective, the principle might be seen as a constraint on the context, instructing us adjust the ‘space of possibilities’ so as to allow for the possible truth of any counterfactual supposition that we wish to entertain. Thus if we were to suppose that some law of nature or theorem of mathematics were false, then this must be allowed to be a genuine counterfactual possibility, even though in other, more normal contexts, we would not wish to countenance any violation of the laws of nature or the truths of mathematics.

If the Principle of Suppositional Accommodation is correct then we would never expect (the exceptional cases aside) to find ourselves wanting to assert a counterfactual of the form ‘if A were the case then ¬A is the case’ since, in using such a counterfactual, we would always adjust the context so as to ensure that the antecedent A was a counterfactual possibility. And this is how it is. Thus we never find ourselves wanting to say that if something were to travel faster than light then nothing would travel faster than light or that if Bizet were Verdi then Bizet would not be Verdi. We always attempt to make more discriminating judgements in such cases. But if no adjustment in the space of possibilities were required, then it is hard to see why we would not be willing to assert such counterfactuals simply on the grounds that their antecedents could not be true.

Let us now return to the Spanish predicament. I had wanted to say that the counterfactual ‘if Spain had fought with the Allies then she would have fought with the Axis’ does indeed follow from the excluder counterfactual ‘if Spain had fought with the Allies or the Axis then she would have fought with the Axis’, and I therefore had to explain why we might be willing to assert the latter and yet not willing to assert the former. The answer is that there is a shift in context. The initial context in which we assert the assumption is one in which it is a counterfactual possibility that Spain fought with the Axis but not a counterfactual possibility that she fought with the Allies. Indeed, the excluder counterfactual is a way of indicating that this is so; for the counterfactual ‘if Spain had fought with the Allies or the Axis then she would have fought with the Axis’ could not be true if it were a possibility that Spain fought with the Allies. When we move to the conclusion, the principle of Suppositional Accommodation would no longer be satisfied if we retained the original context in which it is not a counterfactual possibility that Spain fought with the Allies. The space of possibilities must therefore be extended to include this possibility; and once this is done, it is no longer true to say that if Spain had fought with the Allies she would have fought with the Axis.

Thus the argument appears invalid because the principle of Suppositional Accommodation requires a shift of context from premise to conclusion. Without a shift of context the argument would be valid, but it is impossible - or at least very difficult - to understand the counterfactual conclusion in violation of the principle. So we have a fallacy of equivocation, though of an odd sort, since, in the standard cases, it is through conflating a valid argument (without the equivocation) with an invalid argument (containing the equivocation) that an invalid argument is unwittingly taken to be valid whereas, in the present case, it is through such a conflation that a valid argument is unwittingly taken by our opponents to be invalid.[20]

§6 Substitution of Consequents

I wish to consider one further puzzle, which casts doubt on whether logical equivalents can be substituted salve veritate for the *consequent* of a counterfactual, and not merely for the antecedent. The current scenario is one in which we are considering whether a party to be held tomorrow in New York City will be a roaring success. We may suppose that it is correct to say:

(1) if Quentin were to come to the party it would be a roaring success ($Q > R$),

since Quentin is the heart and soul of any party, and that it is also correct to say:

(2) if Quentin were not to come to the party it would be a roaring success ($\neg Q > R$),

since if Quentin were not to come then Sally would come in his place and she is also the heart and soul of any party.

Consider now a possible situation, such as its raining in Peoria, which has absolutely no bearing on whether the party will be a roaring success. Then given (1) and (2), it would also appear to be reasonable to say:

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[20]Under a variant of the current view, the truth of the counterfactual $A > C$ might be taken to require the counterfactual possibility of A. The inference from $A \vee B > C$ to $A > C$ would not then be valid without the additional assumption that A was a counterfactual possibility. The truth of this additional assumption could legitimately be taken for granted in the applications of Simplification that are relevant to the failures of Substitution but could not be taken for granted in the case of excluder counterfactuals.

(1)′ if it were to rain in Peoria and Quentin were to come to the party then it would be a roaring success ($P \wedge Q > R$); and

(2)′ if it were to rain in Peoria and Quentin were not to come the party then it would be a roaring success ($P \wedge \neg Q > R$),

since its raining in Peoria makes no difference to the counterfactual impact of Quentin's either coming or not coming to the party. But in the envisaged circumstance we might also wish to deny or, at least, not wish to assert that:

(3) if it were to rain in Peoria then the party would be a roaring success ($P > R$).

But assumptions (1)′, (2)′ imply (3) under standard principles of the logic of counterfactuals. There are, in fact, two different ways to derive (3). Under the first, we go from $P \wedge Q > R$ and $P \wedge \neg Q > R$ to $((P \wedge Q) \vee (P \wedge \neg Q)) > R$ by Disjunction and then from $((P \wedge Q) \vee (P \wedge \neg Q)) > R$ to $P > R$ by Substitution of Antecedents.

The other derivation proceeds as before to the conclusion $((P \wedge Q) \vee (P \wedge \neg Q)) > R$. We now use a relatively unproblematic instance of Substitution of Antecedents to give us $P \wedge (Q \vee \neg Q) > R$. By Entailment (or by Substitution of Consequents applied to $P > (P \vee \neg P)$), we have $P > (Q \vee \neg Q)$; and so, an application of Transitivity to $P > (Q \vee \neg Q)$ and $P \wedge (Q \vee \neg Q) > R$ yields $P > R$.

The two derivations are set out below:

$$\dfrac{\dfrac{P \wedge Q > R \quad P \wedge \neg Q > R}{(P \wedge Q) \vee (P \wedge \neg Q) > R}}{P > R}$$

$$\dfrac{P > (Q \vee \neg Q) \qquad \dfrac{\dfrac{P \wedge Q > R \quad P \wedge \neg Q > R}{(P \wedge Q) \vee (P \wedge \neg Q) > R}}{P \wedge (Q \vee \neg Q) > R}}{P > R}$$

What are we to say? The third counterfactual judgement might, of course, be questioned. For the party would be a roaring success were Quentin to go or were he not to go; so the party will be a roaring success; and given that it will be a roaring success, it might then be argued that if it were to rain in Peoria then the party would be a roaring success, despite the lack of connection between antecedent and consequent. Indeed, this will follow from the standard similarity semantics under the most natural criterion of closeness. For given that the party will be a roaring success the closest worlds (or all sufficiently close worlds) in which it rains in Peoria will be ones which preserve the fact that the party will be a roaring success.

But even those who have taken this line have often thought that it should be possible to accommodate the connectionist point of view by adopting a looser criterion of closeness. For we may not care about preserving all particular matters of facts when these have no bearing on the change at hand. Thus worlds in which it rains in Peoria and the party is not a roaring success may be deemed just as close as worlds in which it rains in Peoria and the party is a roaring success, even though the party is in fact a roaring success. Given that this is so, then the counterfactual 'if it were to rain in Peoria the party would be a roaring success' would *not* be true under the similarity semantics.

However, what the example shows is that no change in the criterion of closeness will be capable of accommodating the kinds of judgement that the connectionist wishes to make. For take any counterfactual $P > R$ which he might wish to deny on the grounds that there does not exist the appropriate connection between antecedent and consequent. Then presumably the circumstances in which this is so will be compatible with there being another statement Q such that $Q > R$ and $\neg Q > R$ are both true (where Q will play the same bilateral role as the statement 'Quentin will be at the party' in our example). But this will then be incompatible, under the standard similarity semantics, with denying $P > R$. Thus it is far from clear that any reasonable connectionist point of view can be accommodated within the similarity semantics, no matter which criterion of closeness is adopted.

If one accepts the counterfactual judgements, as well one might, then Disjunction, Transitivity, Substitution or Entailment must be given up. There would appear to be no basis for rejecting the applications of Disjunction or Transitivity or the application of Substitution of Antecedents in the second argument; and this suggests that Entailment (or Substitution of Consequents) should be given up. And, indeed, when one examines the second argument this is the only step lacking in pre-theoretic plausibility. For one is disinclined to accept that if it were to rain in Peoria then the party would either be a roaring success or not be a roaring success, given that its raining in Peoria has no bearing on whether the party is a roaring success. We therefore seem to have good reason to reject Entailment (or Substitution of Consequents).

This response to the argument is not as drastic as it might appear. For one thing, it is to be expected that failures of substitution should extend from antecedent to consequent. For consequents appear as antecedents. In the application of Transitivity, for example, we argue from $A > B$ and $A \wedge B > C$ to $A > C$, with B appearing both as a consequent of the first premise and in the antecedent of the second premise. But this would suggest that the same aspects of the content of a statement will be relevant to either role, so that if the general substitution of $(A \wedge B) \vee (A \wedge \neg B)$ for A should fail in the antecedent, then it should fail in the consequent as well. In the second place, we have only shown that Entailment (or Weakening) fails for the classical notion of consequence. The failure in point (from P to $Q \vee \neg Q$) is a typical instance of a non-relevant implication; and so there is still a reasonable hope, within a connectionist framework, that Entailment might hold for an appropriately relevant notion of consequence.

§7 Some General Objections

I would like to conclude by considering some arguments of a more theoretical or methodological character. Although not directly responsive to the puzzle itself, they appear to provide strong support in favor of accepting Substitution or the similarity semantics and so they need to be cleared out of the way if the rule or the semantics are to be rejected. I shall focus on the earlier rock example, though similar considerations also apply to the party example.

One important consideration in favor of Substitution, I suspect, has been tacit in the minds of many philosophers though rarely articulated. The interpretation of the connectives - $\vee$, $\wedge$ and $\neg$ - employed in the symbolism of the logic of counterfactuals is up to us; and it is our intention that they should be understood classically, i.e. to be in conformity with the classical truth-conditions and the principles of classical logic. Perhaps the ordinary language counterparts of these connectives should be understood in the same way, perhaps not. But that is another question and in no way impinges on the intended interpretation of the connectives themselves.

Now there is some issue as to what it is for the connectives to be subject to the classical truth-conditions. But whether we explain these truth-conditions in terms of possible worlds or in some other way, it is plausible that logically equivalent statements of classical logic will have the same truth-conditions. It is also plausible that a counterfactual A > C will only be sensitive to the truth-conditions of A and C (once we fix the context). We are familiar with operators, such as those for belief or knowledge, that may be are sensitive to other features of a proposition, such as its structure or manner of justification. But the counterfactual operator does not appear to be of that sort. It belongs more with the modal operators of necessity and possibility than with the doxastic or epistemic operators and, as with the modal operators, it is hard to see how it need be sensitive to anything other than the truth-conditions of the statements upon which it operates. But if this is so then Substitution will be valid; for logical equivalents will have the same truth-conditions and hence will freely substitutable salve veritate within the antecedent of a counterfactual.

In order to respond to this objection, we must appeal to an important feature of counterfactuals, what one might call their 'wayward' character (where this is a good thing!). Consider a counterfactual from A to C. Then a necessary (and sufficient) condition for this counterfactual to be true is that, for any possible way in which A might be true, the counterfactual 'if A were the case in this way then C would be the case' should also be true. Thus any counterfactual is implicitly a number of counterfactuals, one for each of the possible way in which the antecedent might be true.

It is important to a proper understanding of waywardness that the ways be genuine counterfactual possibilities. Consider the counterfactual 'if I were to go to New York, it would take at least two hours'. Then in considering the ways in which I might go to New York I merely take into account the available means of transport (let us say) and not some futuristic spacecraft that could get me there in a couple of seconds. It is also important that the ways be proper or relevant. Striking a match when it is wet or when there is no oxygen in the atmosphere is not, properly speaking, a way of striking a match; and so there is no requirement on the truth of the counterfactual 'if I were to strike the match it would light' that the counterfactual 'if I were to strike the match when it is wet it would light' should also be true.

Consider now the possible ways in which a disjunction A $\vee$ B might be true. It is plausible that A, if possible, should constitute a possible way for A $\vee$ B to be true and that B, if possible, should also constitute a possible way for A $\vee$ B to be true. But the general validity of Simplification then follows. For suppose that the counterfactual A $\vee$ B > C is true. Then for any possible way W in which A $\vee$ B is true, the counterfactual W > C should also be true. Now consider A (the argument is similar for B). Either it is possible or it is not. If it is not possible, then the counterfactual A > C is vacuously true. If it is possible, then it is a possible way W for A $\vee$ B to be true; and so, again, the counterfactual A > C will be true. Thus the validity of Simplification (and the consequent invalidity of Substitution) follows from the general wayward character of the counterfactual operator and the more specific wayward character of disjunction.

Moreover, these explanations are in perfect conformity with the classical conception of the connectives and with the requirement that the truth-value of a counterfactual should only be sensitive to the truth-conditions of its component sentences. For we might take the different possible ways in which a sentence is true to be constitutive of its truth-conditions and, in particular, it will be constitutive - or partly constitutive - of the truth-conditions for a disjunction $A \vee B$ that each of A and B is one of the ways in which $A \vee B$ might be true. Thus there is no need for us to give up the classical conception of the connectives or the requirement of 'sensitivity'. Where the usual line of thinking goes amiss is in having an unduly narrow conception of what the truth-conditions are. It insists that the truth-conditions for logically equivalent statements such as A and $(A \wedge B) \vee (A \wedge \neg B)$ should be the same, whereas on a natural conception of truth-conditions - and the one most relevant to the evaluation of counterfactuals - the truth-conditions will not in general be the same, since $(A \wedge B)$ will constitute a way in which $(A \wedge B) \vee (A \wedge \neg B)$ is true though not in general a way in which A is true.[21]

It is worth noting that the wayward character of counterfactuals also promises to provide an explanation of the general phenomenon of 'multiple realizability' of which the rule of Simplification was an instance. For consider one of the other examples, 'if he were to earn more he would be happier'. One of the possible ways for him to earn more is for him to earn a little more and another is for him to earn a lot more (we may suppose). But then it should follow from the wayward character of the counterfactual that if he were to earn a little more (a lot more) then he would be happier. Similarly for the other examples (and perhaps also for other constructions, such as those for permission or obligation, which appear to behave in an analogous way).

The second general objection appeals to the stipulative character, not of the classical connectives, but of the counterfactual operator itself. Let us concede that the counterfactual operator found in ordinary language is not in general subject to Substitution. Still, surely we can come to a new understanding of the counterfactual operator that *is* subject to Substitution and that may, for this or other reasons, be preferable to our ordinary understanding of the operator. After all, something similar is already common in the case of belief or knowledge. We do not normally suppose that our beliefs or that knowledge are closed under logical equivalence. But still it is useful for general theoretical purposes to posit an 'ideal' cognizer, not subject to the usual cognitive limitations of human beings, for whom closure under logical equivalence (and perhaps under logical consequence as well) is assured.

I have no objection in principle to the task of providing idealized versions of ordinary notions. But idealization can go too far and result in something that we no longer recognize as a

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[21]I attempt to defend this alternative conception of truth-conditions in §5 of Fine [2011] and I argue that $(A \wedge B)$, if possible, should also constitute a possible way in which $A \vee B$ is true. In this case, the Disjunction Rule (without the restriction to exclusive disjuncts) will be invalid, since the inference to $A \vee B > C$ will require the premise $(A \wedge B) > C$, in addition to $A > C$ and $B > C$. It is not clear to me that ordinary language 'or' behaves in this way but cognate expressions of ordinary language do. For example, 'if Nixon and Agnew had not both resigned, Ford would never have become President' would appear to imply 'if Nixon and Agnew had both not resigned, Ford would never have become President' and 'if at least one of the women had been present then the conference would have been better' would appear to imply 'if at least two of the women had been present then the conference would have been better'. If this is correct, then there is a further difficulty for those who wish to provide an explanation of the apparent or actual validity of ordinary language rules of simplification of explaining why, in such cases, this additional inference would also appear to hold.

satisfactory version of the original notion. We already know - no matter how the counterfactual is interpreted - that if we accept the non-logical assumptions of the reductio, which we will surely want to do, then one of the logical assumptions must be given up. So idealizing the notion in order to save Substitution, will already require us to give up Disjunction or Transitivity or Infinite Conjunction. But things look even worse when we consider in concrete detail how the idealized interpretation might go.

In the case of belief, there is a natural way to understand the idealized version in terms of the ordinary notion, for we might take someone to believe that A in an idealized sense if he believes that A′ in the ordinary sense for some A′ logically equivalent to A. [***perhaps emphasize that we now get more rules of inference to be valid, in contrast to the counterfactual case] Substitution is then guaranteed. In the same way, we might suppose that $A >_I C$ (A counterfactually implies C in the idealized sense) if $A' > C$ (A′ counterfactually implies C in the ordinary sense) for some A′ logically equivalent to A. Again, Substitution is guaranteed. But we thereby lose Transitivity and Finite Conjunction. To see why Transitivity fails, note that $R_1 \vee R_2 > R_2$ and $R_2 > \neg R_1$ in the original rock example. Now R_2 is logically equivalent to $(R_1 \vee R_2) \wedge R_2$; and so $R_1 \vee R_2 >_I R_2$ and $(R_1 \vee R_2) \wedge R_2 >_I \neg R_1$. But not $R_1 \vee R_2 >_I \neg R_1$ (no logical equivalent of $R_1 \vee R_2$ counterfactually implies $\neg R_1$). To see why Finite Conjunction fails, consider a variant of our original rock scenario in which an odd numbered rock being the first to fall will also somehow result in the first rock falling while an even numbered rock being the first to fall will only result in the subsequent rocks falling. Then $R = (R_1 \wedge R_2 \wedge R_3 \wedge ...) \vee (R_3 \wedge R_4 \wedge R_5 ...) \vee (R_5 \wedge R_6 \wedge R_7 \wedge ...) \vee ...$ (an odd domino onwards falls) will counterfactually imply R_1 and $R' = (R_2 \wedge R_3 \wedge R_4 \wedge ...) \vee (R_4 \wedge R_5 \wedge R_6 ...) \vee (R_6 \wedge R_7 \wedge R_8 \wedge ...) \vee ...$ (an even domino onwards will fall) will counterfactually imply $\neg R_1$ in the ordinary sense '>' of the counterfactual operator. But R and R′ are logically equivalent; and so $R >_I R_1$ and $R >_I \neg R_1$. But not $R >_I R_1 \wedge \neg R_1$ (no logical equivalent of R is a counterfactual impossibility).[22]

Thus the consequences for the logic are calamitous; and nor is it clear how a more palatable version of the notion might be obtained. So even though the infringements of the substitutivity of logical equivalents are far less common in the case of counterfactuals than in the case of knowledge or belief, they are also far less tractable and seem to leave no room for a conception of counterfactual reasoning in which they do not arise. It is somehow integral to our understanding of the counterfactual that the failures in substitutivity should be allowed to stand.

There is one final objection I would like to consider. It is that if we give up Substitution or the similarity semantics, then it is not clear if there is anything better to put in their place. Nute ([1980b], p. 161) has voiced such a concern. He writes, 'If we reject SE [Substitution], then we must replace it with a complicated set of substitution principles which will certainly seem as ad hoc as the translation lore we are trying to avoid'; and for us, of course, the problem is more severe since 'translation lore' is not going to tell us what is wrong with the reductio. In any case, we face the challenge of providing an alternative semantics and an alternative logic; and if nothing

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[22]We *do* retain Disjunction. For suppose $A >_I C$ and $B >_I C$ for A and B logically exclusive. Then $A' > B$ and $B' > C$ for some A′ logically equivalent to A and some B′ logically equivalent to C. But A′ and B′ are then logically exclusive and so $A' \vee B' > C$ by Disjunction for >; and so $A \vee B >_I C$, given that $A \vee B$ is logically equivalent to $A' \vee B'$.

better is at hand, then there may be no choice but to work as best we can with the logic and semantics that we already have.

In response to this objection, all I can do is deliver a promissory note. It seems to me that it is possible to work up our previous informal remarks about the wayward character of counterfactuals into a formal semantics, based upon the idea of a possible state rather than of a possible world, which is just as elegant and natural in its own way as the similarity semantics and which, in addition, provides a much more realistic model of the basis upon which we make the counterfactual judgments that we do. Thus it should not be thought that there is anything essentially unsystematic or anomalous about the phenomena under consideration. They should be seen, not as wrinkles on an otherwise smooth account, but as the inevitable outcome of our conceiving of the truth-conditions for counterfactual statements in terms of possible states rather than possible worlds.

Appendix

We give the derivation of the contradiction from the assumptions. To this end, it will be helpful to make use of the following derived rule:

<u>Transitivity′</u>

$$\frac{A > B \quad B > C}{A > C} \qquad \text{(as long as A is a logical consequence of B)}$$

For suppose that A is a logical consequence of B. Then B and $A \wedge B$ are logically equivalent; and so by Substitution, $A \wedge B > C$ is derivable from $B > C$. By Transitivity, $A > C$ is derivable from $A > B$ and $A \wedge B > C$; and so $A > C$ is derivable from $A > B$ and $B > C$.

(i) In order to derive a contradiction, we first show that $[D_1 \vee (\neg D_1 \wedge D_2)] > \neg D_1$ is derivable:

(1) $D_2 > \neg D_1$	Negative Effect
(2) $[(D_1 \wedge D_2) \vee (\neg D_1 \wedge D_2)] > \neg D_1$	from (1) by Substitution
(3) $D_1 > D_2$	Positive Effect
(4) $D_1 > (D_1 \wedge D_2)$	from (3) by Entailment & Finite Conj.
(5) $D_1 > [(D_1 \wedge D_2) \vee (\neg D_1 \wedge D_2)]$	from (4) by Weakening
(6) $\neg D_1 \wedge D_2 > [(D_1 \wedge D_2) \vee (\neg D_1 \wedge D_2)]$	by Entailment
(7) $[D_1 \vee (\neg D_1 \wedge D_2)] > [(D_1 \wedge D_2) \vee (\neg D_1 \wedge D_2)]$	from (5) and (6) by Disjunction
(8) $[D_1 \vee (\neg D_1 \wedge D_2)] > \neg D_1$	from (7) and (2) by Transitivity′

(ii) We next show that:

(1/¬1) $D_1 \vee D_2 \vee D_3 \vee ... > \neg D_1$

is derivable. $[D_1 \vee (\neg D_1 \wedge D_2)] > \neg D_1$ is derivable by (i) above and $[\neg D_1 \wedge \neg D_2 \wedge (D_3 \vee D_4 \vee ...)] > \neg D_1$ is derivable by Entailment. So $D_1 \vee (\neg D_1 \wedge D_2) \vee [\neg D_1 \wedge \neg D_2 \wedge (D_3 \vee D_4 \vee ...)] > \neg D_1$ is derivable by Disjunction. But $D_1 \vee (\neg D_1 \wedge D_2) \vee [\neg D_1 \wedge \neg D_2 \wedge (D_3 \vee D_4 \vee ...)]$ is logically equivalent to $D_1 \vee D_2 \vee D_3 \vee ....$ So $D_1 \vee D_2 \vee D_3 \vee ... > \neg D_1$ is derivable by Substitution.

(iii) We are now in a position to derive a contradiction. From the derivability of (1/¬1) above, it follows by Entailment and Finite Conjunction that $D_1 \vee D_2 \vee D_3 \vee .... > \neg D_1 \wedge (D_1 \vee D_2 \vee D_3 \vee ...)$ is derivable. But $D_2 \vee D_3 \vee ...$ is a logical consequence of $\neg D_1 \wedge (D_1 \vee D_2 \vee D_3 \vee ...)$; and so by Weakening:

(1/2) $D_1 \vee D_2 \vee D_3 \vee ... > D_2 \vee D_3 \vee ...$

is derivable.

In exactly the same manner in which we derived (1/¬1), we can also derive:

(2/¬2) $D_2 \vee D_3 \vee D_4 \vee ... > \neg D_2$.

But $D_1 \vee D_2 \vee D_3 \vee ...$ is a logical consequence of $D_2 \vee D_3 \vee D_4 \vee ...$; and so by Transitivity′ applied to (1/2) and (2/¬2):

(1/ ¬2) $D_1 \vee D_2 \vee D_3 \vee ... > \neg D_2$

is derivable.

Proceeding in this manner, we establish the derivability of:

(1/ ¬n) $D_1 \vee D_2 \vee D_3 \vee ... > \neg D_n$, for n = 1, 2, ...

So by Infinitary Conjunction, $D_1 \vee D_2 \vee D_3 \vee ... > \neg D_1 \wedge \neg D_2 \wedge \neg D_3 \wedge ...$ is derivable; and hence $D_1 \vee D_2 \vee D_3 \vee ... > \neg(D_1 \vee D_2 \vee D_3 \vee ...)$ is derivable by Weakening, contrary to Possibility.

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