Guide to Ground

A number of philosophers have recently become receptive to the idea that, in addition to scientific or causal explanation, there may be a distinctive kind of metaphysical explanation, in which explanans and explanandum are connected, not through some sort of causal mechanism, but through some constitutive form of determination. I myself have long been sympathetic to this idea of constitutive determination or 'ontological ground'; and it is the aim of the present paper to help put the idea on a firmer footing - to explain how it is to be understood, how it relates to other ideas, and how it might be of use in philosophy.[1]

§1 The Notion of Ground

There is an intuitive notion of one thing holding in virtue of another. Here are some examples:

(1)The fact that the ball is red and round obtains in virtue of the fact that it is red and the fact that it is round;

(2) The fact that the particle is accelerating obtains in virtue of the fact that it is being acted upon by some net positive force;

(3) The fact that his action is wrong obtains in virtue of the fact that it was done with the sole intention of causing harm.

There are some alternative - more or less equivalent - ways of saying the same thing. Thus instead of (2), we might say that the particle was accelerating *because* it was acted upon by a positive force or that

[1] A number of other philosophers (they include Audi [2010], Batchelor [2009], Schaffer [2009],Correia ([2005], [2010]), M. Raven [2009], Rosen [2010], Schnieder [2010]) have done related work in defense of the notion; and I have not attempted to make a detailed comparison between their ideas and my own. I am grateful to the participants at the Boulder conference on dependence and to Neil Tennant for many helpful comments on an earlier draft of the paper.

the particle's being acted upon by a positive force *made it true* that the particle was accelerating; and similarly for (1) and (3).

In each of the above cases, there would appear to be some sort of modal connection between explanandum and explanans. Thus from (1) - (3), it would appear to follow that:

(1) Necessarily, if the ball is red and it is round then it is red and round;

(2) Necessarily, if the particle is acted upon by some positive force then it is accelerating;

(3) Necessarily, if the action was done with the intention of causing harm then it is wrong.

However, it is arguable that the 'force' or 'strength' of the modal operator is different in each case. The first conditional ('if the ball is red ...') holds of *metaphysical* necessity, the second of *natural* necessity, and the third of *normative* necessity.[2]

Whether or not this is so, there would appear to be something more than a modal connection in each case. For the modal connection can hold without the connection signified by 'in virtue of' or 'because'. It is necessary, for example, that if it is snowing then 2 + 2 = 4 (simply because it is necessary that 2 + 2 = 4), but the fact that 2 + 2 = 4 does not obtain in virtue of the fact that it is snowing; and it is necessary that if the ball is red and round then it is red but the fact that the ball is red does not obtain in virtue of its being red and round. In addition to the modal connection, there would also appear to be an *explanatory* or *determinative* connection - a movement, so to speak, from antecedent to consequent; and what is most distinctive about the in-virtue-of claims is this element of movement or determination.

We may call an in-virtue claim a statement of *ontological* or *metaphysical ground* when the conditional holds of metaphysical necessity and I shall talk, in such cases, of the antecedent fact or facts *grounding* or being a *ground for* the consequent fact. Thus we may say, in the first of the cases above,

---

[2] I have attempted to argue in Fine ([2005], chapter 7) that these are the basic forms of necessity, with no one of them reducible to the others.

that the fact that the ball is red and round is grounded in the fact that it is red and the fact that it is round. Just as metaphysical necessity is the strictest form of necessity (at least as compared to natural and normative necessity), so it is natural to suppose that statements of metaphysical ground are the strictest form of in-virtue-of claim. In the other cases, we may sensibly ask for a stricter or fuller account of that in virtue of which a given fact holds. So in the case of the particle, for example, we may agree that the particle is accelerating in virtue of being acted upon by a positive force but think that there is some kind of gap between the explanans and explanandum which could - at least in principle - be filled by a stricter account of that in virtue of which the explanandum holds. But if we were to claim that the particle is accelerating in virtue of increasing its velocity over time (which is presumably a statement of metaphysical ground), then we have the sense that there is - and could be - no stricter account of that in virtue of which the explanandum holds. We have as strict an account of the explanandum as we might hope to have.

It is for this reason that it is natural in such cases to say that the explanans or explanantia are *constitutive of* the explanandum, or that the the explanandum's holding *consists in nothing more* than the obtaining of the explanans or explanantia. But these phrases have to be properly understood. It is not implied that the explanandum just *is* the explanans (indeed, in the case that there are a number of explanantia, it is clear that this requirement cannot be met). Nor need it be implied that the explanandum is unreal and must somehow give way to the explanantia. In certain cases, one might wish to draw these further conclusions. But all that is properly implied by the statement of (metaphysical) ground itself is that there is no stricter or fuller account of that in virtue of which the explandandum holds. If there is a gap between the grounds and what is grounded, then it is not an explanatory gap.[3]

---

[3] My remarks on this point in Fine ([2001], p. 16) have been over-interpreted by a number of authors.

I have remarked that to each modality - be it metaphysical, natural or normative - there corresponds a distinct relation of one thing holding in virtue of another. It is plausible to suppose that the natural in-virtue-of relation will be of special interest to science, the normative relation of special interest to ethics, and the metaphysical relation of special interest to metaphysics. Each of these disciplines will be involved in its own explanatory task, that will be distinguished, not merely by the kinds of things that explain or are explained, but also by the explanatory relationship that is taken to hold between them.

It is an interesting question whether each of these explanatory relations should be defined in terms of a single generic relation. Thus it might be thought that 'metaphysical ground' should be defined by:

> the fact that A grounds the fact that B iff the fact that B obtains in virtue of the fact that A (in the generic sense) and it is a metaphysical necessity that if A then B;

and similarly for the other cases, but with another modality in place of metaphysical necessity. It might, on the contrary, be thought that each basic modality should be associated with its 'own' explanatory relation and that, rather than understanding the special explanatory relations in terms of the generic relation, we should understand the generic relation as some kind of 'disjunction' of the special relations. If there is a generic notion here, it is that which *connects* the modality to the corresponding explanatory relationship and that has no status as an explanatory notion in its own right.

I myself am inclined to favor the latter view. For consider the fact that a given act was right or not right ($R \vee \neg R$). This is grounded, we may suppose, in the fact that it is right (R). The fact that it is right, we may suppose, is (normatively) explained by the fact that it maximizes happiness (R'). So the fact that the given act is right or not right is explained in the generic sense by the fact that it maximizes happiness. But it is a metaphysical necessity that if the act maximizes happiness then the act is right or not right ($\Box(R' \supset R \vee \neg R)$), since it is a metaphysical necessity that the act is right or not right ($\Box(R \vee$

$\neg$R)). However, the fact that the act maximizes happiness does not metaphysically ground the fact that it is right or not right, contrary to the proposed definition. Nor is it altogether clear how the definition might be modified so as to avoid counter-examples of this sort.

§2 The Importance of Ground

Once the notion of ground is acknowledged, then I believe that it will be seen to be of general application throughout the whole of philosophy. For philosophy is often interested in questions of explanation - of what accounts for what - and it is largely through the employment of the notion of ontological ground that such questions are to be pursued. Ground, if you like, stands to philosophy as cause stands to science.

But the principal importance of the notion is to the question of reality. We may distinguish, in a broad way, between two main branches of metaphysics. The first, which I call *realist* or *critical*, is concerned with the question of what is real. Is tense real? Is there genuinely tense in the world and not merely in language? Are values real? Are there genuinely values out there in the world and not merely in our minds? Are numbers real? Are numbers out there in the world waiting to be discovered or merely something that we have invented? The second branch of metaphysics, which I call *naive* or *pre-critical*, is concerned with the nature of things without regard to whether they are real. We might ask, for example, whether material things exist in time in the same way as they exist in space (with the four-dimensionalists thinking they do and the three-dimensionalists thinking they do not) or we might ask whether fictional characters are genuinely created by their authors; and these are questions that we can properly consider even if we decide at the end of the day to adopt a position in which the reality of the external world is rejected in favor of the reality of a purely phenomenal world or in which fictional characters are dismissed in favor of the literary works or acts by which they are introduced.

Questions of ground are not without interest to naive metaphysics, but they are *central* to realist metaphysics. Indeed, if considerations of ground were abolished, then very little of the subject would remain. For the anti-realist faces an explanatory challenge. If he wishes to deny the reality of the mental, for example, then he must explain or explain away the appearance of the mental. It is likewise incumbent upon the realist, if he wishes to argue against his opponent, to show that this explanatory challenge cannot be met.

The question now is: how is this explanatory challenge to be construed? What is it to explain the appearance of a world with minds in terms of a mindless world or the appearance of a world with value in terms of a purely naturalistic world? My own view is that what is required is that we somehow *ground* all of the facts which appear to presuppose the reality of the mental or of value in terms of facts which do not presuppose their reality.[4] Nothing less and nothing else will do.

It will not do, for example, to say that the physical is *causally* determinative of the mental, since that leaves open the possibility that the mental has a distinct reality over and above that of the physical. Nor will it do to require that there should be an analytic definition of the mental in terms of the physical, since that imposes far too great a burden on the anti-realist. Nor is it enough to require that the mental should modally supervene on the physical, since that still leaves open the possibility that the physical is itself ultimately to be understood in terms of the mental.

The history of analytic philosophy is littered with attempts to explain the special way in which one might attempt to 'reduce' the reality of one thing to another. But I believe that it is only by embracing the concept of a ground as a metaphysical form of explanation in its own right that one can adequately explain how such a reduction should be understood. For we need a connection as strong as that of metaphysical necessity to exclude the possibility of a 'gap' between the one thing and the other;

[4] The above account of *ground* and of its role in realist metaphysics is further discussed in Fine [2001]. I do not presuppose that the one set of facts must ground-theoretically supervene on the other set of facts.

and we need to impose a form of determination upon the modal connection if we are to have any general assurance that the reduction should go in one direction rather than another.

The explanatory challenge constitutes the core of realist metaphysics. An anti-realist position stands or falls according as to whether or not it can be met. And so given that the challenge is to be construed in terms of ground, the subject of realist metaphysics will be largely constituted by considerations of ground. We must attempt to determine what grounds what; and it will be largely on this basis that we will be in a position to determine the viability of a realist or anti-realist stand on any given issue.

In addition to this grand role, the notion of ground has a humbler role to play in clarifying the concepts and claims of interest to other branches of philosophy. Let me give one of my favorite examples. How are we to distinguish between a three- and four-dimensionalist view of the nature of material things? The distinction is often put in terms of the existence of temporal parts, with the three-dimensionalist denying that material things have temporal parts (or a suitable range of temporal parts) and the four-dimensionalist insisting that they have such parts. But even the three-dimensionalist might be willing to admit that material things have temporal parts. For given any persisting object, he might suppose that 'in thought', so to speak, we could mark out its temporal segments or parts. But his difference from the four-dimensionalist will then be over a question of ground. For he will take the existence of a temporal part at a given time to be grounded in the existence of the persisting object at that time, while his opponent will take the existence of the persisting object at the time to be grounded in the existence of the temporal part. Thus it is only by introducing the notion of ground that this account of the difference between the two positions can be made at all plausible.[5]

---

[5] Cf. Hawthorne ([2006], p. 100). Rosen ([2010], fn. 1) has another example concerning the analysis of *intrinsic property* and Correia ([2005], chapters 4) provides various accounts of dependence in terms of ground.

§3 Ground and Truth-Making

The notion of ground is a close cousin of the notion of truth-making. Both are bound up with the general phenomenon of what accounts for what, but there are some significant differences in how they structure the phenomenon.[6]

The relation of truth-making relates an entity in the world, such as a fact or state of affairs, to something, such as a statement or proposition, that represents how the world is; and the intended understanding of the relation is that the *existence* of the worldly entity should guarantee the *truth* of the representing entity. Ground, on the other hand, is perhaps best regarded as an operation (signified by an operator on sentences) rather than as a relation (signified by a predicate). But in so far as it is regarded as a relation, it should be seen to hold between entities of the same type and, in so far as a choice needs to be made, these entities should probably be taken to be worldly entities, such as facts, rather than representational entities, such as propositions. Thus it is that the ball is red and that the ball is round that makes it the case that the ball is red and round, and not the existence of the *facts* that the ball is red and that the ball is round that make the *proposition* that it is red and round true.

From the perspective of the theory of ground, truth-maker theory has an unduly restricted conception of what is grounded. One might of course be interested in the ground for the truth or correctness of our representations of the world (in which case, it is presumably not merely the truth of one class of representations that will be of interest, but the class of all representations - be they linguistic, mental or abstract). But these are by no means the only questions of ground or 'making' that will arise. For whenever we consider the question of what makes the representation that P true, there will also arise the question of what, if anything, makes it the case that P. Indeed, it might well be

---

[6] Truth-making has been widely discussed (see, for example, Mulligan et al [1984], Fox [1987] and Armstrong ([1997], [2004])) and a number of authors (most notably Schnieder [2006] and Horwich [2008]) have made some related criticisms concerning the connection between truth-making and ground. Cameron [2011] provides a recent survey of the literature.

thought that the question concerning the representation will always divide into two parts, one concerning the ground for what it is for the representation that P to represent P and the other having nothing to do with representations as such, but concerning the ground for P.

From the perspective of the theory of ground, truth-maker theory is also unduly restrictive in its conception of what grounds. For it insists that grounds should take the form of existential attributions; it is always the *existence* of something that properly accounts for the truth of the representation. But there is no reason in principle why the ultimate source of what is true should always lie in what exists. Perhaps it can lie in something relational, *a* standing in the relation R to *b*, or the negation of something relational, *a* not standing in the relation R to *b*, or in something of some other form.

Indeed, the existential view of ground is somewhat suspect in itself. For it is much more natural to suppose that it is because P (e.g. it is raining) that the fact that P exists, rather than the other way round. One can only conjecture as to why truth-maker theorists might have built such an implausible view into their conception of truth-making. One possible reason is that they wanted something that would clearly indicate that the grounds were in the world and, just as truth indicates that what is grounded lies on the side of representation, so existence indicates that what grounds lies on the side of the world. Be that as it may, it is clearly preferable if our conception of what accounts for what should remain neutral over the form of the relata.

The lack of uniformity between what grounds and what is grounded gives rise to another limitation in truth-maker theory. For the attempt to determine what grounds what naturally proceeds in stages - one first determines the relatively immediate grounds for the truths in question, then the relatively immediate grounds of those grounds, and so on until one reaches the ultimate grounds. So one might first ground the normative in the natural, for example, then the natural in the physical, and then the physical in the micro-physical, thereby establishing that the normative was grounded in the micro-physical.

But the existence/truth dichotomy that is built into the notion of truth-making makes it ill-suited to this step by step procedure. For what is grounded is a truth and what grounds is the existence of something, which is not of the right form for itself to be grounded. Thus the truth of the normative will be grounded in the existence of the natural, which is not of the right truth-theoretic form to be grounded in the existence of the physical.

It might of course be suggested that whenever:

(i) the existence of the fact that q makes true the proposition that r; and

(ii) the existence of the fact that p makes true the proposition that q,

then:

(iii) the existence of the fact that p will make true the proposition that r.

The legitimacy of 'chaining' is thereby preserved. But why think this chaining principle holds, given the shift in the middle term from the existence of the fact that q in (i) to the truth of the proposition that q in (ii)? Presumably, this can only be because the truth of q is some kind of ground for the existence of the fact that q. But this goes against the whole tenor of truth-making theory, which takes the existence of the fact to ground the truth of the proposition, rather than the other way round.

Perhaps there is some other, more ingenious, way to establish the legitimacy of chaining on the truth-maker approach. But why go through these contortions when there is a simple and natural alternative under which the grounds are already suited to have grounds? The truth-maker theorist is like someone who, faced with the problem of fitting a round peg into a round hole, first makes the round peg square and then attempts to solve the problem of fitting a square peg into a round hole.

The difficulties over the relation of truth-making do not merely concern the relata; they also concern the relation itself. For the relation is usually explicated in modal terms: f will be a truth-maker for p if the existence of f necessitates the truth of p ($\Box E(f) \supset T(p)$). But as has often been pointed out, this lets in far too much. Any necessary truth, for example, will be grounded by anything and, not only will the fact that Socrates exists be a truth-maker for the proposition that singleton Socrates exists, the fact that singleton Socrates exists will be a truth-maker for the proposition that Socrates exists. Thus whereas the form of the relata makes truth-making too restrictive, the nature of the relation makes it too liberal.

It is conceivable that the restrictions on the relata were a way for compensating for the deficiencies in the relation. For if P were taken to be a truth-maker for Q whenever P necessitated Q, then every truth would trivially be a truth-maker for itself. By insisting that the grounds should take the form of something that exists and that what is grounded should take the form of something that is true, we avoid trivializations of this sort; and we can even ensure that the relation be irreflexive and anti-symmetric, since the objects to the right and left of the relation will be of different type.

But we have here a mere chimera of substantiality. Indeed, on certain quite plausible metaphysical views, there will still exist wholesale trivializations of the truth-making project. One might well think, for example, that for any truth p, the fact that p will exist and will require the truth of p for its existence. The fact that p will then be a truth-maker for any true proposition p. Or one might think that the world exists and could not exist without being the way it is. The world would then be a truth-maker for any true proposition. But such innocuous metaphysical views cannot legitimately be regarded as enabling us to find a truth-maker for every truth.

The notion of truth-making is thoroughly ill-suited to the task for which it was intended: it arbitrarily restricts the relata between which the relation should be capable of holding; it does not allow truth-making connections to be chained; and it trivializes the project of finding truth-makers. Perhaps the best that can be said in its favor is that it provides a necessary condition for the intended relation: if P genuinely grounds Q then the fact that P will be a truth-maker for the proposition that Q. It is therefore possible that, by looking for truth-makers while guided by a sense of what is really in question, we will alight on genuine grounds. But it should not be pretended that the relation of truth-making is anything but a pale and distorted shadow of the notion of genuine interest to us.

§4 The Grammar of Ground

How should we formulate statements of ground? My preferred view is that the notion of ground should be expressed by means of a sentential operator, connecting the sentences that state the ground to the sentence that states what is grounded. If we use ‘<’ for this connective, then (1) above might be formulated as:

The ball is red, The ball is round < The ball is red and round.[7]

Perhaps the closest we come to an ordinary language formulation is with ‘because’. Thus we might say ‘the ball is red and round because the ball is red and the ball is round’. But, of course, ‘because’ does not convey the distinctive sense of *ground* and is not able to distinguish between a single conjunctive antecedent and a plurality of non-conjunctive antecedents.

Corresponding to the notion of ground as a sentential operator is a notion of ground as a sentential predicate. If we use ‘$\lhd$’ for the predicate, then ‘A, B < C’, say, will be true just in case ‘A’, ‘B’ $\lhd$ ‘C’. Likewise, corresponding to the notion of ground as a sentential operator is a notion of ground as a propositional predicate (or as a predicate of facts). If we use $\blacktriangleleft$ for this predicate, then ‘A, B < C’ will be true just in case that-A, that-B $\blacktriangleleft$ that-C.

But is important here that the notion of proposition be properly understood. For the truth of ‘A, B < C’ might be taken to depend not merely upon the propositions expressed by ‘A’, ‘B’ and ‘C’ but

---

[7] Different authors have used different symbols for this notion. Audi [2010] uses ‘ $_g$’, Correia [2005] uses ‘$\triangleright$’, and Rosen [2010] uses ‘$\leftarrow$’. My own notation derives partly from the need to distinguish between a strict (<) and weak ($\leq$) notion of ground and partly from the metaphor in which the ground is ‘lower’ than what it grounds.

also upon *how* these propositions are expressed; and, in this case, the clauses 'that-A', 'that-B' and 'that-C' should also be taken to indicate the manner in which the proposition is expressed in addition to the proposition itself. One might think, for example, that this is water in virtue of its being H_2O. But if the proposition that this is water is the same as the proposition that it is H_2O then we would have here an unacceptable case of a proposition being a ground for itself. So if we are to use the propositional mode to express this statement of ground, we must adopt a richer conception of what propositions are in question. One of the advantages of the operator approach is that it enables us to remain neutral on such questions.

The grounding operator '<' is variably polyadic; although it must take exactly one argument to its 'right', it may take any number of arguments to its 'left' – be they of zero, finite or infinite number. This means that there is both a conjunctive and a disjunctive sense in which a given statement C may be multiply grounded. It will be *conjunctively* grounded in A_1, A_2, A_3,…in so far as A_1, A_2, A_3,… collectively ground C; and it will be *disjunctively* grounded in A_1, A_2, A_3, … (or, more generally, in A_{11}, A_{12}, … and A_{21}, A_{22}, … and A_{31}, A_{32}, …, …) in so far as it is grounded in A_1 and in A_2 and in A_3, … (or in A_{11}, A_{12}, … and in A_{21}, A_{22}, … and in A_{31}, A_{32}, …, …).

The case in which a given statement is *zero*-grounded, i.e. grounded in zero antecedents, must be sharply distinguished from the case in which it is *un*grounded, i.e. in which there is no number of statements - not even a zero number - by which it is grounded. We may bring out the difference by means of an analogy with sets. Any non-empty set {a, b, ...} is generated (via the 'set-builder') from its members a, b, ... The empty set {} is also generated from its members, though in this case there is a zero number of members from which it is generated.

An urelement such as Socrates, on the other hand, is ungenerated; there is no number of objects - not even a zero number - from which it may be generated. Thus 'generated from nothing' is ambiguous between being generated from a zero number of objects and there being nothing - not even a zero plurality of objects - from which it is generated; and the empty set will be generated from nothing in the one sense and an urelement from nothing in the other sense.

We might imagine that we have a machine that manufactures sets. One feeds some objects into one end of the machine and turns it on; and the set of those objects then emerges from the other end of the machine. The empty set is the object that emerges from the machine when no objects are fed into it, while the urelements are those objects that never emerge from the machine.

There is a similar distinction to be drawn between being zero-grounded and ungrounded. In the one case, the truth in question simply disappears from the world, so to speak. What generates it, just as what generates the empty set, is its zero-ground. But in the case of an ungrounded truth, just as in the case of an urelement, the truth is not even generated and simply stays in place.

Of course, in any putative case of a zero-grounded statement, we should provide some explanation as to how it might be zero-grounded. But this may not be impossible. Suppose we thought that there was a operator of conjunction '$\bigwedge$' that could apply to any number of sentences A, B, ... It might then be maintained, as a general principle, that the conjunction $\bigwedge$(A, B, ...) was grounded in its conjuncts A, B, ... So in the special case in which the operator $\bigwedge$ was applied to zero statements, the resulting conjunction $\top = \bigwedge ()$ would be grounded in its zero conjuncts.

Indeed, the case of zero-grounding may be more than an exotic possibility. For suppose that one held the view that any necessary truth was ultimately to be grounded in contingent truths. Now, in the case of some necessary truths, it may be clear how they are to be grounded in contingent truths. It might be thought, for example, that the statement $A \vee \neg A$ was always to be grounded in either A or $\neg$A. But in other cases - as with Socrates being identical to Socrates or with Socrates belonging to singleton

Socrates - it is not so clear what the contingent truths might be; and a plausible alternative is to suppose that they are somehow grounded in nothing at all.[8]

§5 Conceptions of Ground

The general notion of ground comes in different 'strengths' - normative, natural and metaphysical. But it also comes in different 'flavors'. There will be different ways of conceiving of the notion, even when its strength has been fixed. I begin with some familiar distinctions in how the notion is to be conceived, and then turn to some less familiar distinctions.

Factive/Non-factive

There is a familiar distinction between the factive and a non-factive conception of ground. On the factive conception, we can only correctly talk of something factive - such as a true statement or a fact - being grounded; and what grounds must likewise be factive. But on the non-factive conception, we can also correctly talk of something non-factive - such as a false statement or a merely possible fact - being grounded; and what grounds may likewise be non-factive. Thus on the factive conception, $A \wedge B$ can only be grounded in A and B if $A \wedge B$ (and hence A and B) are indeed the case while, on the non-factive conception, $A \wedge B$ can be grounded in A and B even if A or B (and hence $A \wedge B$) is not the case.

We may define the factive notion ($<$) in terms of the non-factive notion ($<_{\circ}$) as follows:

(F-N) $\Delta < A$ iff $\Delta <_{\circ} A$ and Δ (i.e. each statement of Δ is the case).

Some statements Δ factively ground another statement A another iff they non-factively ground the other statement and are the case. This equivalence (and even its necessary truth) would appear to be relatively unproblematic.

---

[8] A similar strategy may be on the cards when what is to be explained is not a necessary truth but the necessity of a truth, as with the dilemma posed by Blackburn [1987].

We might also appear to be able to define the nonfactive notion in terms of the factive notion:

(N-F) $\Delta <_{\circ} A$ iff $\Diamond(\Delta < A)$.

Some statements non-factively ground another if they possibly factively ground the other. In this way, we may extend the 'field' of the relation from the actual facts, so to speak, to the possible facts.

However, this definition is subject to a difficulty. For presumably a statement of the form A can non-factively ground $A \vee B$ (i.e. it is possible that A factively grounds $A \vee B$, viz. when A is the case) and $A \vee B$ and $\neg A$ together can nonfactively ground $(A \vee B) \wedge \neg A$ (i.e. it is possible that $A \vee B$ and $\neg A$ factively ground $(A \vee B) \wedge \neg A$, viz. when $\neg A$ and B are the case). It should then follow (upon replacing $(A \vee B)$ in the previous statement of ground with A) that A and $\neg A$ can non-factively ground $(A \vee B) \wedge \neg A$. But this is not possible according to the definition given that it is impossible that A and $\neg A$ should both be the case.

The difficulty arises from the fact that the antecedents Δ must be jointly possible if they are nonfactively to ground A and this is too restrictive. But it does not help to allow the antecedents Δ to non-factively ground A whenever they are jointly impossible. For then A would non-factively ground A whenever A is impossible and, presumably, we do not want any statement to ground itself.

The difficulties in coming up with a straightforward account of non-factive ground, in addition to the more intuitive considerations, strongly suggest to me that the factive notion is the more fundamental notion; the difficulties of the nonfactive notion are a product, so to speak, of its artificiality. But that is not to say that we should give up on the non-factive notion. For we can think of it as being obtained from the factive notion by a process of 'rounding out', in which the possible cases of factive grounding are extended with cases of grounding from impossible antecedents in such a way that the basic principles governing the behavior of ground are preserved (it is in much the same way that we extend the number system). However, this is not the place to consider how this might be done.

Full/ Partial Ground

Another familiar distinction is between full and partial ground. Ground in the previous sense is *full* ground. A is a *partial* (or what we shall later call a *partial strict*) ground for C if A, on its own or with some other truths, is a ground of C (i.e. A, $\Gamma < B$, where Γ is a possibly empty set of 'other truths'). Thus given that A, B is a full ground for $A \wedge B$, each of A and B will be a partial ground for $A \wedge B$. Each will be relevant to the grounding of $A \wedge B$, even though neither may be sufficient on its own.

Partial ground has been defined in terms of full ground, but it would not appear to be possible to define full ground in terms of partial ground. For the partial grounds of $A \vee B$ and $A \wedge B$ are the same, viz. A and B when A and B are the case. But each of A and B is a full ground of $A \vee B$ though not, in general, of $A \wedge B$. And so how are we to distinguish between the full grounds of $A \vee B$ and $A \wedge B$ if appeal is only made to their partial grounds? It is for this reason that pride of place should be given to the full notion in developing an account of ground.[9]

Mediate/Immediate Ground

The third distinction, between mediate and immediate ground, is not so familiar. Ground in the previous sense is *mediate*. An *immediate* ground, by contrast, is one that need not be seen to be mediated. The statement that $A \wedge (B \wedge C)$ is mediately grounded in the statements that A, B, C, since the grounding must be seen to be mediated through B, C grounding $(B \wedge C)$ and A, $(B \wedge C)$ grounding $A \wedge (B \wedge C)$. The statements B, C, by contrast, immediately ground $B \wedge C$, since the grounding in this case is not mediated through other relationships of ground.[10]

I say that an immediate ground is one that 'need not be seen to be mediated' rather than 'is not mediated' because there are cases of immediate ground that are in fact mediated even though they need

[9] In contrast to the approach of Schneider [2010].

[10] I have proposed drawing a similar distinction in the case of essence (Fine [2003], 61-2).

not be seen to be mediated. The truth that A, for example, is a ground for A $(A \vee A)$. It is also an immediate ground for A $(A \vee A)$ since A in its capacity as a left disjunct, so to speak, is not a mediated ground for A $(A \vee A)$. However, A is an immediate ground for $(A \vee A)$ and $(A \vee A)$ is an immediate ground for A $(A \vee A)$; and so A also stands in a mediated relationship of ground to A (A A).

Mediate ground can be defined in terms of immediate grounds. For all relationships of mediate ground can be obtained by appropriately chaining relationships of immediate ground. Thus each relationship of immediate ground can be taken to be a degenerate case of a mediate ground; and given that Δ is an immediate ground for A and Γ_1, A, Γ_2 a mediate ground for B, we can take $\Gamma_1, \Delta, \Gamma_2$ to be a mediate ground for A. But a definition in the other direction would not appear to be possible. For, in the example above, the truth that A is an immediate ground for $A \vee (A \vee A)$. But, as we have seen, the ground is also mediated, with A being an immediate ground for $(A \vee A)$ and $(A \vee A)$ for $A \vee (A \vee A)$. We wanted to say that the ground A for $A \vee (A \vee A)$ need not be *seen* to be mediated, but it is hard to see how to convert this idea into a definition from mediate ground.

The notion of immediate ground would appear to be give us something genuinely new; and I find it remarkable how strong our intuitions are about when it does and does not hold. It surely is the case, for example, that $A \wedge B$ is *immediately* grounded in A and B and that statements about cities are, at best, *mediately* grounded in statements about atoms. It is the notion of immediate ground that provides us with our sense of a ground-theoretic hierarchy. For given any truth, we can take its immediate grounds to be at the next lower level. Thus as long as mediate grounds are always mediated through immediate grounds, any partial ground for the truth will always be at some finite level below the level of the truth.

<u>Weak/Strict Ground</u>

The fourth distinction, between weak and strict ground, is less familiar still. Ground in the previous sense is *strict* and does not allow a truth to ground itself, while ground in the weak sense allows - and, indeed, requires - that a truth should ground itself.

We might perhaps express weak ground by means of the locution 'for - and for - and ... is for -', where the last 'for' specifies the statement to be grounded and the first 'for''s specify its grounds. Thus for John to marry Mary is for John to marry Mary, for John to marry Mary is for Mary to marry John, and for John to marry Mary and for Mary to marry John is for John to marry Mary. Or to take a somewhat different example, for Hesperus to be identical to Phosphorus and for Phosphorus to be a planet is for Hesperus to be a planet (in this case, it might be argued that, in contrast to the others, the grounded truth does not weakly ground any of its grounds).

What is characteristic of these cases is that any explanatory role that can be played by the given truth can also be played by their grounds. Thus if John's marrying Mary accounts for the existence of the married couple John and Mary, then Mary's marrying John also accounts for the existence of the married couple. Or if John's marrying Mary accounts for John's marrying Mary or Bill's marrying Sue then Mary's marrying John will also account for John's marrying Mary or Bill's marrying Sue.

We might think of strict ground as moving us down in the explanatory hierarchy. It always takes us to a lower level of explanation and, for this reason, a truth can never be strict ground for itself. Weak ground, on the other hand, may also move us sideways in the explanatory hierarchy. It may take us to a truth at the same level as what is grounded and, for this reason, we may always allow a given truth to be a weak ground for itself.[11]

Given the notion of weak ground, it would appear to be possible to define strict ground. For we may say that Δ strictly grounds C if (i) Δ weakly grounds C and (ii) C, on its own or with some other

[11] We might compare truths which are weak grounds for one another with 'equations' in systems of term rewriting (Terese[2003]). Term rewriting, in general, has deep analogies with the calculus of ground.

statements, does not weakly ground any member B of Δ - i.e. for no Γ does Γ, C weakly ground B. Thus a strict ground is a weak ground which cannot be 'reversed', with the explanandum C helping to explain one of the explanantia.

It would also appear to be possible to define the weak notion of ground in terms of the strict notion. For we may say that Δ weakly grounds C if Δ subsumes the explanatory role of C, i.e. if Δ, Γ strictly grounds B whenever C, Γ strictly grounds B.

There is an interesting question as to which of the notions, if either, is more fundamental than the other. We naturally gravitate towards the strict notion and think of the weak notion as an artificial offshoot. But my own inclination is to think of the weak notion as more fundamental (just as one might see the notion of proper-or-improper part as more fundamental than the notion of proper part). As we shall see, it has a simpler semantics; and it seems to provide a simpler and more natural starting point in developing a general theory of ground.

Varieties of Strict/Partial Ground

Once we introduce the notion of weak ground, it is possible to make further distinctions in the notion of strict/partial ground. There is first of all the partial notion that is the natural counterpart of strict full ground and that we naturally think of as *the* notion of partial ground. P will be a partial ground of Q in this sense if P, on its own or with some other truths, is a strict full ground for Q. We have called this notion *partial strict* ground and might symbolize it by $\prec^*$.

There is then the notion of strict/partial ground that is the natural counterpart to the notion of weak partial ground. P is weak partial ground for Q if P, possibly with some other truths, is a weak full ground for Q; and P will be a corresponding strict/partial ground for Q if P is a weak partial ground for Q but Q is not a weak partial ground for P. We call this notion *strict partial* ground and symbolize it by $\preceq$.

Finally, there is the result of chaining partial strict ground with weak partial ground. P will be a partial/strict ground for Q in this sense if, for some truth R, P is a partial strict ground for R and R is a weak partial ground for Q. We call this notion *part strict* ground and symbolize it by $\prec'$. There is also a corresponding notion that results from chaining weak partial ground with partial strict ground. But it gives nothing new - the result is coincident with partial strict ground.

It is readily shown, under plausible assumptions, that if P is partially strict ground for Q then P is a part strict ground for Q and that if P is a part strict ground for Q then P is a strictly partial ground for Q. However, there is no obvious way to establish the reverse implications and it may plausibly be argued - either by reference to the semantics or by appeal to counter-examples - that they fail to hold.

The notion of strict partial ground provides us with a natural partial notion of ground for which a partial ground need not always be part of a full ground. One might wish to say, for example, that the truth that P is a partial ground for knowledge that P, even though there is nothing one might add to P to obtain a strict full ground for knowledge that P (as in the view of Williamson [2000]). But let it be granted that P and the knowledge that P is a weak full ground for knowledge that P. Then knowledge that P is presumably no part of a weak full ground for P and so P will be a strict partial ground, in the intended sense, for knowledge that P.[12]

Distributive/non-distributive Ground

Finally, let us note that although we have taken ground to be a many-one connection, allowing any number of antecedents on the left but requiring a single consequent on the right, there are natural ways in which we may interpret it as a many-many connection, i.e. as a connection between any number of statements on either the left or the right. The obvious way to do this is to

[12] See Fine [2011a] for further discussion.

understand $\Delta < \Gamma$ to mean that Δ *simultaneously* grounds each statement of Γ, i.e. that $\Delta < C$ for each C in Γ. However, for a number of purposes, it is more natural to understand $\Delta < \Gamma$ to mean that Δ *distributively* grounds Γ, i.e. there is a decomposition of Δ into subsets $\Delta_1, \Delta_2, \ldots$ (with $\Delta = \Delta_1 \cup \Delta_2 \cup \ldots$) and a corresponding decomposition of Γ into members $C_1, C_2, \ldots$ (with $\Gamma = \{C_1, C_2, \ldots\}$) such that $\Delta_1 < C_1, \Delta_2 < C_2, \ldots$ (and similarly when weak full ground is in question).

This understanding of ground is naturally involved in statements of supervenience. It might be thought, for example, that the psychological supervenes (in the sense of *ground*) upon the physical. Let Γ be the set of all psychological truths $C_1, C_2, \ldots$. Then for an appropriate set Δ of physical truths (one containing a physical ground Δ_i for each psychological truth C_i) Δ will be a distributive ground for Γ. We shall see that the notion of distributive ground is also naturally involved in formulating ground-theoretic principles for the logical connectives.

§6 The Pure Logic of Ground

We might divide the logic of ground into two parts: 'pure' or structural; and 'impure' or applied. The pure logic of ground is simply concerned with what follows from statements of ground without regard to the internal structure of the truths that ground or are grounded. Thus the pure logic of ground might state that if A grounds B and B grounds C then A grounds C. The impure logic of ground, on the other hand, also takes into account the internal logical structure of the truths. Thus the impure logic of ground might state that A is a ground for A   B (given that A is the case).

In developing the pure logic of ground, there are a number of choices of the ground-theoretic primitives that one might make, but it turns out to be convenient to adopt the following four operators as primitive[13]:

---

[13] The pure logic of ground is developed in much great detail in Fine [2011a].

$\leq$ weak full

$\preceq$ weak partial

$<$ strict full

$\prec$ strict partial

We may set these out in a chart as follows:

	Strict	Weak
Full	$<$	$\leq$
Partial	$\prec$	$\preceq$

There are four corresponding statements of ground:

$\Delta \leq C$ weak full

$A \preceq C$ weak partial

$\Delta < C$ strict full

$A \prec C$ strict partial,

where Δ in the statements of full ground is used to indicate an arbitrary number of antecedent truths $A_1, A_2, \ldots.$

Let us place the premisses of a valid rule of inference above a horizontal line and the conclusion below the line. The pure theory of ground, with these primitives, may then be taken to be constituted by the following rules:

Subsumption

$$(<\,/\preceq):\quad \frac{\Delta < C}{\Delta \preceq C} \qquad\qquad (\prec\,/\preceq):\quad \frac{A \prec C}{A \preceq C}$$

$$(<\,/\prec):\quad \frac{\Delta, A < C}{A \prec C} \qquad\qquad (\leq\,/\preceq):\quad \frac{\Delta, A \leq C}{A \preceq C}$$

Cut$(\leq/\leq)$:

$$\frac{\Delta_1 \leq A_1 \quad \Delta_2 \leq A_2 \ \ldots \quad A_1, A_2, \ldots \leq C}{\Delta_1, \Delta_2, \ldots \leq C}$$

Transitivity

$$(\preceq/\preceq):\ \frac{A \preceq B \quad B \preceq C}{A \preceq C} \qquad (\preceq/\prec):\ \frac{A \preceq B \quad B \prec C}{A \prec C} \qquad (\prec/\preceq):\ \frac{A \prec B \quad B \preceq C}{A \prec C}$$

Identity$(\preceq)$: $\overline{\qquad}$

Non-Circularity$(\prec)$: $A \prec A$

$$A \leq A \qquad\qquad \frac{\qquad}{\perp}$$

Reverse Subsumption($\leq/\prec$):

$$\frac{A_1, A_2, \ldots \leq C \quad A_1 \prec C \quad A_2 \prec C \ldots}{A_1, A_2, \ldots < C}$$

The Subsumption Rules tell us how to weaken a statement of ground. They enable us to go from strict ground (either full or partial) to weak and from full ground (either strict or weak) to partial. The Cut Rule allows us to chain statements of ground; the antecedents in a weak statement of ground may be replaced by their grounds. The Transitivity Rules allow us to chain two partial statements of ground, and the resulting statement will be strict as long as the one of the given statements is strict. According to Identity, we may infer (from zero premises) that any truth is a weak ground for itself; and according to Non-Circularity, no truth is a strict partial ground for another. Reverse Subsumption permits us to go from a weak statement $A_1, A_2, \ldots \leq C$ of ground to the corresponding strict statement $A_1, A_2, \ldots < C$ as long as all of the antecedents $A_1, A_2, \ldots$are strict partial grounds for the consequent C.

A couple of comments are in order:

(1) The most striking difference from the structural rules for classical consequence is the absence of Weakening. Even though Δ is a strict (or weak) ground for C, we cannot infer that Δ together with an arbitrary truth A is a strict (or weak) ground for C. This is because all of the grounds must be relevant to conclusion. Indeed, if Weakening held, then Non-circularity could no longer be maintained. For surely at least one strict statement of ground $\Delta < C$ is true. But given

Weakening, $\Delta, C < C$ should then hold and so C would be a strict partial ground for C ($C \prec C$) - contrary to Non-Circularity.

(2) From the above rules, we can derive the following Amalgamation Rule for strict ground:

$$\frac{\Delta_1 < C \quad \Delta_2 < C \quad \ldots}{\Delta_1, \Delta_2 \ldots < C}$$

In other words, the strict grounds for a given truth can be amalgamated, or combined, into a single ground. It is not usual to include this rule (or anything from which it might be derived) among the rules for ground. But the plausibility of the rules from which it can be derived provide a strong argument for its adoption; and I doubt that there is simple and natural account of the logic of ground that can do without it.

We may also argue more directly in favor of the rule without appeal to the weak notion of ground. For consider the case in which $\Delta < C$ and $\Gamma < C$ (the case in which there are more than two premises to the rule is similar). Now C, C is a ground for $C \wedge C$ (this is an instance of the general truth that A, B is a ground for $A \wedge B$). So by the application of Cut for $<$, $\Delta, \Gamma < C \wedge C$, i.e. Δ, Γ is a strict ground for $C \wedge C$. But then how can Δ, Γ fail to be a strict ground for C? What difference in the relationship of ground could be marked by such a distinction?

It follows from Amalgamation that there will always be a maximum ground for a grounded truth. In other words, if $\Delta < A$ then there will be a Δ^+ such that (i) $\Delta^+ < A$ and (ii) $\Gamma \subseteq \Delta^+$ whenever $\Gamma < A$. For we may simply let Δ^+ be the union of all the Γ for which $\Gamma < A$. On the other hand, there may be no minimum ground for a grounded truth A, i.e. a Δ^- for which (i) $\Delta^- < A$ and (ii) $\Delta^- \subseteq \Gamma$ whenever $\Gamma < A$. For suppose A is a truth of the form $(p_1 \wedge p_2 \wedge p_3 \wedge \ldots) \vee (p_2 \wedge p_3 \wedge p_4 \wedge \ldots) \vee (p_3$

$\wedge\ p_4 \wedge\ p_5 \wedge\ \ldots) \vee \ldots$.Then for each k = 1, 2, …, p_k , p_{k+1}, p_{k+2}, … < A and so, if there were to be a minimum ground for A it would have to be empty.

This example also puts paid to the idea that we might get at the idea of relevance through minimality. In other words, starting with a notion of ground $<^+$ that is subject to Weakening one might hope to define a 'relevant' notion of ground < that is not subject to Weakening via the definition:

$\Delta < C$ if $\Delta <^+ C$ and for no proper subset Δ' of Δ does $\Delta' <^+ C$.

But this would then prevent there from being any relevant ground for $A = (p_1 \wedge p_2 \wedge\ p_3 \wedge\ \ldots) \vee (p_2 \wedge\ p_3 \wedge\ p_4 \wedge\ \ldots) \vee (p_3 \wedge p_4 \wedge\ p_5 \wedge\ \ldots) \vee \ldots$ in the example above.

§7 The Ground of Logic (Introduction Rules)

We turn to the 'impure' logic of ground. The central question concerns the ground for truth-functional and quantificational truths; and, for this reason, we may think of this part of the logic as ground as constituting an account of the ground of logic.

There are two kinds of ground-theoretic rules that we might provide for logically complex truths, loosely corresponding to the introduction and elimination rules of classical logic. The first provides sufficient conditions for something to ground a logically complex truth of a specified form; the second provides necessary conditions. We consider the first kind of rule here and the second in the next section.[14]

What are the grounds for a logically complex truth - be it a conjunctive truth $A \wedge\ B$, a disjunctive truth $A \vee B$, a universal truth $\forall xFx$, an existential truth $\exists xFx$, or a negative truth $\neg A$? Let us begin with conjunction and disjunction, then turn to the quantifiers, and finally deal with the case

[14] I hope to give a more detailed account of the impure logic elsewhere.

of negation. I shall only concern myself with the question of strict ground, although there is a parallel question of weak ground that might also be considered.

It has usually been supposed that conjunction and disjunction should be subject to the following 'introduction' rules:

$\wedge$I $\quad$ $A , B < A \wedge B$

$\vee$I-L $\quad$ $A < A \vee B$ $\qquad$ $\vee$I-R $\quad$ $B < A \vee B$

Thus A and B, together, are a ground for $A \wedge B$, while A or B, separately, is a ground for $A \vee B$.

These rules are fine as far as they go, but there is a way in which the rule for disjunction may be inadequate. For, as we have already argued, ground should be subject to Amalgamation. This means that if A and B are separate grounds for $A \vee B$ then A, B should be a collective ground for $A \vee B$. Thus in addition to $\vee$I.L and $\vee$I.R, we should also have:

$\vee I^+$ $\quad$ $A, B < A \vee B$

(under the assumption, of course, that A and B are both true).

We may argue for the plausibility of this further rule in the same way as before. For suppose that A and B is each a ground for $A \vee B$, according to the original rules. Then A, B is a ground for $(A \vee B) \wedge (A \vee B)$ by the rule for conjunction. But do we then want to deny that A, B is a ground for $(A \vee B)$, thereby creating what appears to be an invidious distinction in the grounds for $(A \vee B)$ and $(A \vee B) \wedge (A \vee B)$?

Of course, if the original rules are stated within a context in which the Amalgamation Rule can be derived, then there is no need for the additional rule. But the contexts within which these rules are stated are not usually taken to be of this sort; and the additional rule is then required.[15]

We turn to the quantifiers. The obvious rules, in analogy to rules for $\wedge$ and $\vee$, are:

$\forall$I $\quad B(a_1), B(a_2), \ldots < \forall xB(x)$

$\exists$I. $\quad B(a) < \exists xB(x)$

where $a_1, a_2, a_3, \ldots$ are names for all of the objects of the domain and a is the name for one such object.[16] As with the rule for disjunction, we should also allow (through either stipulation or derivation) for the application of Amalgamation $\exists$I:

$\exists I^+$. $\quad B(a), B(b), B(c), \ldots < \exists xB(x)$

But there is another difficulty with $\exists$I. For take the case in which B(x) is x = x (a similar difficulty also arises when B(x) is of the form $\neg(Fx \wedge \neg Fx)$ for some predicate F). Then Socrates being identical to Socrates (a = a) will ground the truth that something exists (in the sense $\exists x(x = x)$). But then necessarily, if Socrates is identical to Socrates, Socrates being identical to Socrates will ground the truth that something exists and hence will imply that something exists. But necessarily,

[15] There are some extraordinary logical circumstances, related to the paradoxes, that may lead one to question some of these rules and some of the associated rules for the quantifiers. They are discussed in Fine [2010b] but will not be considered here.

[16] I assume for simplicity that the quantifiers are unrestricted although an analogous account could be given for restricted quantification.

Socrates is identical to Socrates. And so necessarily, something exists – which is not so (at least on most views of the matter).

The standard solution to this difficulty is to take the ground for $\exists xB(x)$ to be not simply B(a) but B(a) along with the truth that a exists (Ea). Thus the rule now takes the form:

$$B(a), Ea < \exists xB(x)$$

However, this proposal is often coupled with the suggestion that the relevant existential claim Ea should be understood as the existential $\exists x(x = a)$. But let B(x) be x = a. According to the revised version of $\exists$I, a = a, $\exists x(x = a)$ will then ground $\exists x(x = a)$ - contrary to the non-circularity of ground.

One could try to circumvent this difficulty by placing restrictions on the B for which B(a), Ea grounds $\exists xB(x)$, but I doubt that there is any reasonable way in which this might be done and it would be desirable, in any case, to have uniform rules of ground for the logical constants. The proposal also has other counter-intuitive consequences. For if allowed, $\exists x(x = a)$ would in general be a partial ground for $\exists xB(x)$ and, since a = a partially grounds $\exists x(x = a)$, a = a would in general be a partial ground for $\exists xB(x)$. But surely the truth that a = a is in general irrelevant to the truth of $\exists xB(x)$ - Socrates being identical to Socrates, for example, is irrelevant to someone being a philosopher.

Rather than give up the idea that the existence claim is irrelevant to the ground of the existential truth, I should like to suggest that it is relevant but should not itself be understood in terms of existential quantification. This is not to deny that there is a necessary equivalence between Socrates existing and there being something that is Socrates. But once we are sensitive to ground we will be sensitive to differences between necessarily equivalent statements that turn on differences in their ground. $A \vee \neg A$ and $B \vee \neg B$, for example, are necessarily equivalent (since each is necessarily true) but will generally differ with regard to their ground, with the grounds for A, should it be true, being a ground for $A \vee \neg A$ though not in general a ground for $B \vee \neg B$. And similarly, or so it might be thought, in the present case.

We can get at the relevant notion of existence by asking what grounds $\exists x(x = a)$. Intuitively, it is not just $a = a$ (which is the only relevant instance) but also something about a, what we might call its existence. But then the existence of a in this sense, if it is to serve as a partial ground for $\exists x(x = a)$, cannot itself be understood as $\exists x(x = a)$. Indeed, it would not normally be supposed that the identity of an object with itself is a ground – or even a partial ground – for the existence of the object. What grounds Socrates' existence might turn on whether he was born of such and such parents, for example, but it is hard to see how it could turn, even indirectly, on the identity of Socrates to himself.

There is a dual difficulty for the universal quantifier. Suppose that $a_1, a_2, a_3, \ldots$ are (names for) all the individuals that there are. Let B(x) be the condition that x is identical to one of the individuals that there are, something we may write as:

$$x = a_1 \vee x = a_2 \vee x = a_3 \vee \ldots.$$

Then given the rule $\forall$I above, the truths:

$$a_1 = a_1 \vee a_1 = a_2 \vee a_1 = a_3 \vee \ldots,$$

$$a_2 = a_1 \vee a_2 = a_2 \vee a_2 = a_3 \vee \ldots,$$

$$a_3 = a_1 \vee a_3 = a_2 \vee a_3 = a_3 \vee \ldots,$$

...

will ground $\forall x(x = a_1 \vee x = a_2 \vee x = a_3 \vee \ldots)$. And since:

$a_1 = a_1$ grounds $a_1 = a_1 \vee a_1 = a_2 \vee a_1 = a_3 \vee \ldots$,

$a_2 = a_2$ grounds $a_2 = a_1 \vee a_2 = a_2 \vee a_2 = a_3 \vee \ldots, \ldots$,

$a_3 = a_3$ grounds $a_3 = a_1 \vee a_3 = a_2 \vee a_3 = a_3 \vee \ldots$,

…

it follows that $a_1 = a_1$, $a_2 = a_2$, $a_3 = a_3$, … grounds $\forall x(x = a_1 \vee x = a_2 \vee x = a_3 \vee \ldots)$. But each of $a_1 = a_1$, $a_2 = a_2$, $a_3 = a_3$, … is necessary; and so it should be necessary that $\forall x(x = a_1 \vee x = a_2 \vee x = a_3 \vee \ldots)$, i.e. necessary that $a_1, a_2, \ldots$ are all of the objects that there are - which is not so (at least on most views of the matter).

This difficulty is usually solved by appeal to the 'totality' claim that $a_1, a_2, \ldots$ are all of the individuals that there are. Let us signify this truth by $T(a_1, a_2, \ldots)$. Then the ground for $\forall xB(x)$ should be taken to be not simply $B(a_1)$, $B(a_2)$, $B(a_3)$, … but $B(a_1)$, $B(a_2)$, $B(a_3)$, … along with $T(a_1, a_2, \ldots)$.

However, this proposal is usually coupled with the suggestion that the totality claim $T(a_1, a_2, \ldots)$ should itself be understood as the universal claim: $\forall x(x = a_1 \vee x = a_2 \vee x = a_3 \vee \ldots)$. But let $B(x)$ be the condition $(x = a_1 \vee x = a_2 \vee x = a_3 \vee \ldots)$. Then the ground for $\forall xB(x)$, i.e. for $\forall x(x = a_1 \vee x = a_2 \vee x = a_3 \vee \ldots)$, will be the truths:

$a_1 = a_1 \vee a_1 = a_2 \vee a_1 = a_3 \vee \ldots$,

$a_2 = a_1 \vee a_2 = a_2 \vee a_2 = a_3 \vee \ldots$,

$a_3 = a_1 \vee a_3 = a_2 \vee a_3 = a_3 \vee \ldots$,

...

along with $\forall x(x = a_1 \vee x = a_2 \vee x = a_3 \vee \ldots)$ - contrary to the non-circularity of ground.

One could try to circumvent this difficulty by placing restrictions on the $B(x)$ for which $B(a_1)$, $B(a_2)$, $B(a_3)$, … in conjunction with $T(a_1, a_2, \ldots)$ grounds $\forall xB(x)$ but, again, I doubt that there is any reasonable way in which this might be done and it would be desirable, in any case, to have uniform rules of ground for the logical constants. The proposal also has other counter-intuitive consequences. For the totality claim $T(a_1, a_2, \ldots)$ will in general be a partial ground for $\forall xB(x)$ and each of $a_1 = a_1$, $a_2 = a_2$, $a_3 = a_3$, …will partially ground $T(a_1, a_2, \ldots)$ under the proposed account of $T(a_1, a_2, \ldots)$; and so the identities $a_1 = a_1$, $a_2 = a_2$, $a_3 = a_3$, … will in general be a partial ground for the universal truth

$\forall xB(x)$. But surely the truth of $a = a$ is in general irrelevant to the truth of $\forall xB(x)$ - Socrates being identical to Socrates, for example, is irrelevant to the truth that everything is either not a man or is mortal.

What I would like to suggest, in the same way as before, is that $T(a_1, a_2, \dots)$ should indeed be taken to be part of the ground of any universal truth $\forall xB(x)$ but that it should not itself be understood as a universal truth. Thus even though $\forall x(x = a_1 \vee x = a_2 \vee x = a_3 \vee \dots)$ and $T(a_1, a_2, \dots)$ are necessarily equivalent, they differ with respect to their grounds. The identities $a_1 = a_1$, $a_2 = a_2$, $a_3 = a_3$, … are directly relevant to the grounds of $\forall x(x = a_1 \vee x = a_2 \vee x = a_3 \vee \dots)$ but are neither directly nor indirectly relevant to the grounds of $T(a_1, a_2, \dots)$.

There is a variant on the above position which I am inclined to prefer on general theoretical grounds. We have taken the totality claim $T(a_1, a_2, \dots)$ in the weak sense that $a_1, a_2, \dots$ are *at most* the objects that there are, but we might also take it in the strong sense that $a_1, a_2, \dots$ are *just* the objects that there are (something which is equivalent to $\forall x(x = a_1 \quad x = a_1 \quad \dots) \quad Ea_1 \quad Ea_2 \quad \dots)$. We may then take the ground for an existential truth $\exists xB(x)$ to consist of some true instances and the appropriate totality claim. Thus it will be part of the grounds for a quantificational truth, whether it be universal or existential, that the objects of the domain are what they are; and a separate category of existence facts will not be required.

The issue of the ground for universal truths has caused a great deal of puzzlement in the philosophical literature, going back to Russell [1918] and continuing to this day (Armstrong [2004]). But if I am right, there is a purely logical aspect to the problem which is readily solved once one draws a distinction between the totality claim and the corresponding universal claim. Of course, this still opens the question of the grounds, if any, for the totality claim. But this is a question that lies on the side of metaphysics, so to speak, rather than of logic; and it should not be supposed that there is anything in our general understanding of the quantifiers or of the concept of ground that might indicate how it should be answered.

We turn finally to negation. In this case, it is hard to see how one might state the grounds for $\neg A$ in terms of A, since if $\neg A$ is a truth then A is a falsehood. What we might do instead is to take the case in which A is logically complex and then state grounds for $\neg A$ in terms of the components of A. There are five cases in all:

$\neg\wedge$I. $\quad \neg A < \neg(A \wedge B) \qquad \neg B < \neg(A \wedge B)$

$\neg\vee$I. $\quad \neg A, \neg B < \neg(A \vee B)$

$\neg\neg$I. $\quad A < \neg\neg A$

$\neg\forall$I. $\quad \neg Fa_1 \;\; Ea < \neg\forall xFx$

$\neg\exists$I. $\quad \neg Fa_1 \;\; \neg Fa_2 \;\; \ldots \;\; T(a_1, a_2, \ldots) < \neg\exists xFx$

Given these rules, the grounds for negations can be driven inwards until we reach atomic truths and their negations.

§8 The Ground of Logic (Elimination Rules)

We have provided sufficient conditions for a logically complex truth to be grounded by simpler truths. But we would also like to know something about the necessary conditions under which a logically complex truth will be grounded. We know, for example, that the truths A and B will ground $A \wedge B$. But when in general will an arbitrary set of truths Δ ground $A \wedge B$? For all that we have said, any set of truths could ground $A \wedge B$; and so clearly, something more should be said on the question if such unpalatable possibilities are to be excluded.

This further question, to my knowledge, has been almost completely ignored; and the little that has been said has not been accurate. It turns out that, in order to provide an adequate formulation of the necessary conditions, we need to appeal to the weak notion of ground ($\leq$), even when it is only the strict grounds for a given truth that are in question. This is therefore another case in which appeal to the weak notion is critical in developing an adequate theory of ground.

For consider again the question of when a set of truths Δ is a (strict full) ground for $A \wedge B$. We naturally want to say that any grounds for $A \wedge B$ should be mediated through A and B; the conjuncts are the conduit, so to speak, through which truth to the conjunction should flow. But we cannot express this as the thought that Δ must divide into two parts Δ_1 and Δ_2 (with $\Delta = \Delta_1 \cup \Delta_2$) which are respectively strict grounds for A and B. For A and B ground $A \wedge B$ and so, when $\Delta = \{A, B\}$, the required division of Δ into strict grounds for A and B will not exist. Nor can we say that either Δ must divide into two such parts Δ_1 and Δ_2 or should be identical to $\{A, B\}$, since the same difficulty will arise when Δ consists of ground-theoretic equivalents A' and B' of A and B.

What we should say instead is:

if Δ is a strict full ground for $A \wedge B$ ($\Delta < A \wedge B$) then, for some division of Δ into the parts Δ_1 and Δ_2, Δ_1 and Δ_2 are weak full grounds for A and B respectively ($\Delta_1 \leq A$ and $\Delta_2 \leq B$),

using the notion of weak ground on the right in place of the notion of strict ground. An alternative way to express the consequent is that Δ should be a (weak) distributive ground for $\{A, B\}$, given our previous notion of distributive ground. Let us use '$\Delta \leq \Gamma$', with set-symbols to the left and right, to indicate that Δ is a weak distributive ground for Γ. The 'elimination' rule for $\wedge$ will then take the form[17]:

$$\wedge E. \qquad \frac{\Delta < A \wedge B}{\Delta \leq \{A, B\}}$$

There is a corresponding rule for disjunction. What we would like to say is that the grounds for a disjunction $A \vee B$ should be mediated through its disjuncts. But when the grounds for the disjunction are strict, we should allow the grounds for the disjuncts to be weak; and given the possibility of amalgamation, we should also allow that the ground for the disjunction may be a distributive ground for its disjuncts. We are therefore led to the following principle:

If Δ is a strict full ground for $A \vee B$, then either Δ is a weak full ground for A or a weak full ground for B or a weak distributive ground for A and B. Or more formally:

$$\vee E. \qquad \frac{\Delta < A \vee B}{\Delta \leq A;\ \Delta \leq B;\ \Delta \leq \{A, B\}}$$

[17] The formulation calls for a further extension of our framework. For once we spell out the notion of distributive ground, we see that $\Delta \quad \{A, B\}$ is equivalent to: $(\Delta_{11} \quad A \wedge \Delta_{12} \quad B) \vee (\Delta_{21} \quad A \wedge \Delta_{22} \quad B) \vee ...$, where $\{\Delta_{11}, \Delta_{12}\}$, $\{\Delta_{21}, \Delta_{22}\}$, ... run through all the divisions of Δ into pairs. But we may wish to express the same conclusion without embedding the statements of ground within the truth-functional connectives. To this end, note that this disjunction of conjunctions is equivalent to a conjunction of disjunctions. Thus we can state that $\Delta \quad \{A, B\}$ is a consequence of $\Delta < A \wedge B$ by stating that each of the conjoined disjunctions $D_1 \vee D_2 \vee$ is a consequence; and this may then be expressed as a multiple-conclusion inference: .

$$\frac{\Delta < A \wedge B}{D_1, D_2, ...}$$

(where the semicolons are used to indicate the disjunctive character of the conclusion).

For the universal quantifier, we wish to say that a set of truths will be a strict full ground for $\forall xB(x)$ if it distributively grounds the totality claim and all of its instances. That is:

$$\forall E. \qquad \frac{\Delta < \forall xB(x)}{\Delta \leq \{T(a_1, a_2, \ldots), B(a_1), B(a_2), \ldots\}}$$

where $a_1, a_2, \ldots$ are names, as before, for all of the individuals of the domain.

For the existential quantifier (under the variant approach I suggested), we shall wish to say that a set of truths will be a strict full ground for $\exists xB(x)$ if it distributively grounds the totality claim and some of its true instances. That is:

$$\exists E. \qquad \frac{\Delta < \exists xB(x)}{\Delta \leq \{T(a_1, a_2, ..., ...),\ B(a_{11}), B(a_{12}), \ldots\}; \Delta \leq \{T(a_1, a_2, ..., ...), B(a_{21}), B(a_{22}), \ldots)\}; \ldots}$$

where the $a_{k1}, a_{k2}, \ldots$ run through all of the non-empty subsets of the a for which B(a) is true.

For the different kinds of negative statement, we have the following elimination rules:

$$\neg\wedge E. \qquad \frac{\Delta < \neg(A \wedge B)}{\Delta \leq \neg A; \Delta \leq \neg B; \Delta \leq \{\neg A, \neg B\}}$$

$$\neg\vee E. \qquad \Delta < \neg(A \vee B)$$

$$\frac{}{\Delta \leq \{\neg A, \neg B\}}$$

¬¬E.

$$\frac{\Delta < \neg\neg A}{\Delta \leq A}$$

¬∀E.

$$\frac{\Delta < \neg\forall xFx}{\Delta \leq \{T(a_1, a_2, ..., ...), \neg Fa_{11}, \neg Fa_{12}, \ldots\};\ \Delta \leq \{T(a_1, a_2, ..., ...),\ \neg Fa_{21}, \neg Fa_{22}, \ldots\};\ \ldots}$$

¬∃E.

$$\frac{\Delta < \neg\exists xFx}{\Delta \leq \{T(a_1, a_2, ..., ...), \neg Fa_1, \neg Fa_2, \ldots\}}$$

From the introduction and elimination rules together, we can establish inferential counterparts of the following biconditionals, which relate the strict ground for a logically complex truth on the left to the weak grounds for its simpler constituents on the right (in the biconditionals for the quantifiers, we use $a_1, a_2, \ldots$ as names for all of the individuals in the domain and $b_1, b_2, \ldots$ as names for some of the individuals in the domain):

∧IE. $\Delta < A \wedge B$ iff there are Δ_1 and Δ_2 for which $\Delta_1 \cup \Delta_2 = \Delta$, $\Delta_1 \leq A$ and $\Delta_2 \leq B$;

$\neg\wedge$IE. $\Delta < \neg(A \wedge B)$ iff $\Delta \leq \neg A$ or $\Delta < \neg B$ or there are Δ_1 and Δ_2 for which $\Delta_1 \cup \Delta_2 = \Delta$,

$\Delta_1 \leq A$ and $\Delta_2 \leq B$;

$\vee$IE. $\Delta < A \vee B$ iff $\Delta \leq A$ or $\Delta \leq B$ or there are Δ_1 and Δ_2 for which $\Delta_1 \cup \Delta_2 = \Delta$,

$\Delta_1 \leq A$ and $\Delta_2 \leq B$;

$\neg\vee$IE. $\Delta < \neg(A \vee B)$ iff there are Δ_1 and Δ_2 for which $\Delta_1 \cup \Delta_2 = \Delta$, $\Delta_1 \leq \neg A$ and

$\Delta_2 \leq \neg B$;

$\neg\neg$IE. $\Delta < \neg\neg A$ iff $\Delta \leq A$;

$\forall$IE. $\Delta < \forall x B(x)$ iff there are $\Delta_0, \Delta_1, \Delta_2, \ldots$ for which $\Delta = \Delta_0 \cup \Delta_1 \cup \Delta_1 \cup \ldots$ and

$\Delta_0 \leq T(a_1, a_2, \ldots)$, $\Delta_1 \leq B(a_1)$, $\Delta_2 \leq B(a_2)$, …;

$\neg\forall$IE. $\Delta < \neg\forall x B(x)$ iff there are $\Delta_0, \Delta_1, \Delta_2, \ldots$ for which $\Delta = \Delta_0 \cup \Delta_1 \cup \Delta_2 \cup \ldots$,

and $\Delta_0 \leq T(a_1, a_2, \ldots)$, $\Delta_1 \leq \neg B(b_1)$, $\Delta_2 \leq \neg B(b_2)$, …;

$\exists$IE. $\Delta < \exists x B(x)$ iff there are $\Delta_0, \Delta_1, \Delta_2, \ldots$ for which $\Delta = \Delta_0 \cup \Delta_1 \cup \Delta_2 \cup \ldots$

and $\Delta_0 \leq T(a_1, a_2, \ldots)$, $\Delta_1 \leq B(b_1)$, $\Delta_2 \leq B(a_2)$, …;

$\neg\exists$IE. $\Delta < \neg\exists x B(x)$ iff there are $\Delta_0, \Delta_1, \Delta_2, \ldots$ for which $\Delta = \Delta_0 \cup \Delta_1 \cup \Delta_2 \cup \ldots$ and

$$\Delta_0 \leq T(a_1, a_2, \ldots), \Delta_1 \leq \neg B(a_1), \Delta_2 \leq B(a_2), \ldots.$$

Suppose that Δ in the IE rules above is confined to 'simple' truths - those that are of the form $Fa_1a_2, \ldots a_n$ or $\neg Fa_1a_2, \ldots a_n$ for some atomic predicate F or are of the form Ea or $T(a_1, a_2, \ldots)$. Then by using the introduction and elimination rules for weak grounding, it can be shown that the strict grounding statement on the left can be replaced with a weak grounding statement, since the only way some simple truths can be a weak ground for a complex truth is by being a strict ground for the complex truth. Thus in place of $\wedge$IE, we have:

$\wedge$IE* $\Delta \leq A \wedge B$ iff there are Δ_1 and Δ_2 for which $\Delta_1 \cup \Delta_2 = \Delta$, $\Delta_1 \leq A$ and $\Delta_2 \leq B$;

with weak grounding on both left and right; and similarly for the other cases. This reformulation is significant because of its recursive character: the weak grounding of a complex truth (via simple truths) will successively reduce to the weak grounding of simpler truths.

There is one lacuna in the above account. For it might be hoped that one could say more about when two truths are ground-theoretic equivalents. Two very plausible principles of this sort concern alphabetic variance:

$\forall xB(x) \leq \forall yB(y)$ $\exists xB(x) \leq \exists yB(y)$.

The ground-theoretic import of a quantified truth is not effect by a change in variables. Some other principles, though less obvious, might also be adopted. For example:

$A \wedge B \leq B \wedge A$

$A \wedge (B \wedge C) \leq (A \wedge B) \wedge C$

There are, however, definite limits on how far one can go in laying down such principles compatibly with the other rules. One cannot adopt:

$A \wedge A \leq A$,

for example, given $A < A \wedge A$, since then $A < A$. I do not pretend to have a full understanding of how much leeway actually exists.

Given a stock of rules for weak grounding, one might then hope to provide elimination rules for weak grounding. Suppose, for example, that we wish to provide necessary conditions for when Δ is a weak full ground for the conjunction $A \wedge B$. One case, already covered by the rules, is when Δ is a strict full ground for $A \wedge B$. So let us suppose that Δ is a weak but not a strict full ground for $A \wedge B$. Then even though Δ is not itself a strict full ground for $A \wedge B$, it may contain subsets Γ that are strict full grounds for $A \wedge B$. By Amalgamation, the union Γ^* of all such subsets will also be a strict full ground for $A \wedge B$. Let Δ' be the result of removing Γ^* from Δ. Then in this case it may be maintained that Δ' must consist of ground-theoretic equivalents of $A \wedge B$. And so, once we know what these are, we will be done.

§9 Lambda-Abstraction

The lambda operator may be used in two different ways, which are not normally distinguished. Given an open sentence A(x) (such as ‘x is unmarried and x is a man), we may use a lambda operator, call it ‘λx’, to form the predicate expression $\lambda x A(x)$ (‘is an x such that x is unmarried and x is a man) or we may use a lambda operator, call it ‘Λx’, to form the property expression $\Lambda x A(x)$ (‘the property of being an x such that x is unmarried and x is a man’). The resulting expressions will differ syntactically; for the first will be an expression that occupies predicate position while the second will be one that occupies nominal position. They will also, plausibly, differ semantically; for the first will play a descriptive role, enabling us to say how things are, while the second will play a designative role, enabling us to pick out things to be described.[18]

[18]An analogous distinction may be drawn for the functional reading of the λ-notation. For $\lambda x t(x)$ may be understood either as a functional expression or as a term for a function.

Given a monadic predicate expression P (say 'is wise') and a nominal expression t (say 'Socrates'), let us use P(t) for the result of predicating P of t ('Socrates is wise); and given a property term Π (say 'the property of being wise') and a nominal expression t (say 'Socrates'), let us use $\Pi[t]$ to indicate that the object designated by t has the property designated by Π ('Socrates has the property of being wise'). The natural view is that $\Pi[t]$ itself is the result $H(t, \Pi)$of predicating the 'has' predicate H of t and Π.

The two forms of abstraction (we might call them *predicate* and *property* abstraction, respectively) are plausibly taken to conform to the following introduction rules:

ΛI $\quad \lambda xA(x)(c) < \Lambda xA(x)[c]$ $\qquad$ $\neg\Lambda$I $\quad \neg\lambda xA(x)(c) < \neg\Lambda xA(x)[c]$

λI $\quad A(c) < \lambda xA(x)(c)$ $\qquad$ $\neg\lambda$I $\quad \neg A(c) < \neg\Lambda xA(x)[c]$

According to ΛI ($\neg\Lambda$I is analogous) John's having the property of being an unmarried man, say, is grounded in the fact that Charles is an unmarried man; and according to λI (again, $\neg\lambda$I is analogous), John's being an unmarried man is grounded in the fact that Charles is unmarried and Charles is a man.

The two rules are very different. ΛI effects a reduction in ontological complexity; properties are eliminated in favor of predicates. λI, by contrast, effects a reduction in logical complexity; complex predication is eliminated in favor of simple predication. ΛI and λI are often merged:

$$A(c) < \Lambda xA(x)[c].$$

But we see the merger as the product of two separate ground-theoretic connections.

There are corresponding rules of elimination:

ΛE $\quad \dfrac{\Delta < \Lambda xA(x)[c]}{\Delta \leq \lambda xA(x)(c)}$ $\qquad$ $\neg\Lambda$E $\quad \dfrac{\Delta < \neg\Lambda xA(x)[c]}{\Delta \leq \neg\lambda xA(x)(c)}$

$$\lambda E \quad \frac{\Delta < \lambda x A(x)(c)}{\Delta \leq A(c)} \qquad\qquad \neg\lambda E \quad \frac{\Delta < \neg\lambda x A(x)[c]}{\Delta \leq \neg A(c)}$$

Thus, according to λE, any strict ground for $\lambda xA(x)(c)$ must be mediated through a weak ground for $A(c)$.

There are also some rules for weak ground-theoretic implication (or equivalence) that one may wish to lay down. Thus corresponding to the rule of alphabetic variance for the quantifiers, we have:

$$\Lambda xA(x)[c] \leq \Lambda yA(y)[c] \qquad\qquad \lambda xA(x)(c) \leq \lambda yA(y)(c).$$

One might, in addition, adopt the following rule of equivalence for λ:

$$P(c) \leq \lambda xP(x)(c) \qquad\qquad \lambda xP(x)(c) \leq P(c)$$

The predicate $\lambda xP(x)$, where what follows λx is a simple predication $P(x)$, is to be treated the same as P. The second of the two principles is, of course, incompatible with λI above, since this requires that $P(c) < \lambda xP(x)[c]$; and so, if we adopt the equivalence, we must restrict λI to the case in which $A(x)$ is not a simple predication.

The original introduction principles, ΛI and λI, raise some troubling issues, related to the paradox of analysis. For according to λI, $A(c) < \lambda xA(x)(c)$. But there surely must then be a sense of 'proposition', in which the proposition expressed by $A(c)$ can be taken to stand in a relation of ground to the proposition expressed by $\lambda xA(x)(c)$. Using <A> to signify the proposition expressed by A, we therefore have that <A(c)> grounds <λxA(x)(c)>. In the case in which c is 'Charles', for example, and A(x) is the open sentence 'x is unmarried & x is a man', we will have that the proposition that Charles is an unmarried man will ground the proposition that he is a bachelor.

But how can that be? For both propositions are predicatively composed of the subject Charles and the property of being a bachelor; the property is predicated of the subject to form the proposition. And so how can the propositions be different, as would be required for the one to ground the other?

Likewise, according to ΛI, $\lambda xA(x)(c) < \Lambda xA(x)[c]$. So $<\lambda xA(x)(c)>$ grounds $<\Lambda xA(x)[c]>$ - the proposition that Charles is bachelor, for example, will ground the proposition that Charles has the property of being a bachelor. But again, how can that be given that both propositions are predicatively composed of Charles and the property of being a bachelor?

I suggest that we solve this puzzle by distinguishing between different ways in which a proposition may be predicatively composed of a subject and a property. It can be *straight* predication. Thus if the subject is Charles and the property is the property of being an unmarried man (Λx(x is unmarried & x is a man)), then the resulting proposition is the one expressed by the sentence 'Charles is an unmarried man' ('λx(x is a man & x is unmarried)(Charles)'). Thus the property occurs as a property (or predicatively) in the resulting proposition.

The predication can also be *upward*. Thus if the subject is Charles and the property is the property of being an unmarried man (Λx(x is unmarried & x is a man)), then the resulting proposition is the one expressed by the sentence 'Charles has the property of being an unmarried man' ('Λx(x is a man & x is unmarried)[Charles]'). Here the property occurs as an object (or nominally) in the resulting proposition. As I have mentioned, a plausible view is that the upward predication of P of x is the same as the straight predication of the relation of having of P and x; and if this is so, then upward predication is directly reducible to straight predication.

Finally, the predication can be *downward*. If the subject is Charles and the property is the property of being an unmarried man (Λx(x is unmarried & x is a man)), then the resulting proposition is the one expressed by the sentence 'Charles is a man & Charles is unmarried'. Here the property occurs as an abstract (or as the result of abstraction) in the resulting proposition. It is seen to be present in the proposition through a process of abstraction whereby the subject (Charles) is removed and the property remains. We cannot give a direct definition of downward predication in terms of straight predication but the result of downward predication can always be seen to be the result of straight predication. The proposition that Charles is unmarried & Charles is a man, for example, can also be seen to be the result of conjoining the straight predication of the property of being unmarried of Charles with the straight predication of the property of being a man of Charles.

So even though the propositions expressed by 'Λx(x is unmarried & x is a man)[Charles]', 'λx(x is unmarried & x is a man)(Charles)' and 'Charles is unmarried & Charles is a man' are each the result of predicating the property of being an unmarried man of the subject Charles, they are each this result by means of a different manner of predication; and there is therefore no difficulty in distinguishing between the propositions or in allowing the various grounding relations between them may to obtain.[19]

§10 The Semantics of Ground

There is a standard 'possible worlds' semantics for logical consequence or entailment. Under this semantics, each sentence A of the language under consideration is associated with the 'truth-set' |A| of possible worlds in which it is true, and it is then supposed that:

---

[19] Rosen [2010], p. 125, is of the view that the proposition that *a* is grue (where to be grue is to be red or green) and the proposition that *a* is red or green will differ in that (i) the former but not the latter will contain *grue* and (ii) the latter but not the former will contain *green*. Thus where I see a difference in the manner of composition (or predication), he sees a difference in the constituents. The first of his two claims is much more plausible than the second, but only the first is required to distinguish between the two propositions. I could perhaps agree with him on the first claim (even if not on the second) under a construal of composition in which composition through abstraction is not allowed. However, I believe that the downward form of composition should not simply be ignored but should be recognized as a genuine form of composition in its own right. For more on the underlying conception of composition, see Fine [2010a].

C is a consequence of A_1, A_2, … iff a world *w* verifies C (i.e. is a member of the truth-set |A|) whenever it verifies each of A_1, A_2, … (i.e. whenever it belongs to each of the truth-sets $|A_1|$, $|A_2|$, …).

This semantics is not suited to the notion of ground since it yields Weakening. If C is a consequence of A_1, A_2, … then it is a consequence of A_1, A_2, … along with any other sentences B_1, B_2, …. But is there any alternative account of the semantics of sentences and of the connection of ground that might be made to work?

It turns out that one can provide a very natural semantics of this sort in terms of the idea of truth-making. I have said some harsh things about truth-maker theory. But even though it might not amount to much as an approach to ontology, it provides an ideal framework within which to set up a semantics for ground. For in setting up such a semantics, we would like to be able to appeal to something analogous to the relationship between a sentence and the worlds within which it is true; and it turns out that the relationship between a sentence and the facts that make it true will exactly fit the bill.

Just as we previously supposed that each sentence A was associated with a truth-set |A|, the set of possible worlds in which it is true, we may now suppose that each (true) sentence is associated with a verification-set [A], the set of facts which make it true. Facts differ from worlds in two respects. First, they are actual and not also possible. Second, they need not be complete, i.e. they need not settle the truth-value of every proposition. Facts, on this conception, are *parts* of the actual world.

There is a natural sense in which facts may be *fused*. So, for example, given the fact *f* that this ball is red and the fact *g* that it is round, there will be a *fused* fact *f.g* to the effect that the ball is both red and round; and in general, given any facts *f*, *g*, *h*, …, there will exist a fusion *f.g.h.* … of those facts. It will be supposed that the verification set [A] for each true sentence is closed under fusion. In other words, the fusion of facts that verify a sentence also verifies the sentence (this is a kind of semantic counterpart of Amalgamation).

We may now adopt the following semantical clause for weak ground:

(i) $A_1, A_2, \ldots$ is a weak full ground for C ($A_1, A_2, \ldots \leq C$ is true) iff $f_1.f_2.f_3....$ verifies C (i.e. $f_1.f_2.f_3....$ is a member of the verification-set [C]) whenever f_1 verifies A_1, f_2 verifies A_2, f_3 verifies A_3 … (i.e. whenever each of $f_1, f_2, \ldots$ is a member of the respective verification-sets $[A_1], [A_2], [A_3], \ldots$). Note that the fact $f_1.f_2.f_3....$ that is to verify C is the fusion of the facts $f_1, f_2, f_3 \ldots$ that verify the antecedents $A_1, A_2, A_3, \ldots$; they cooperate, so to speak, in verifying the consequent C. It is because of this difference in the clause for consequence that Weakening no longer holds. For suppose that A is a weak full ground for C, so that any fact that verifies A will verify C. There is then no guarantee that A, B is also a weak full ground for C. For given that f verifies A and that g verifies B, we will know that f verifies C but not that the fusion $f.g$ will verify C.

Similar clauses can be given for the other notions of ground:

(ii) A is a weak partial ground for C ($A \preceq C$ is true) iff for some sentences $A_1, A_2, \ldots$ (and assignment of verification-sets to them) $A, A_1, A_2, \ldots$ is a weak full ground for C;

(iii) $A_1, A_2, \ldots$ is a strict full ground for C ($A_1, A_2, \ldots < C$) iff $A_1, A_2, \ldots$ is a weak full ground for C and C is not a weak partial ground for any of $A_1, A_2, \ldots$;

(iv) A is a strict partial ground for C ($A \prec C$ is true) iff A is a weak partial ground for C but C is not a weak partial ground for A.

Using these clauses, we can then establish soundness and completeness for the pure logic of ground, as set out above. This provides some kind of vindication both for the system and for the semantics.

There is a natural extension of the above semantics to the connectives and the quantifiers. To allow for the presence of negation in the language, we now associate with each sentence C both a set $[C]^+$ of verifiers and a set $[C]^-$ of falsifiers. One of these sets is non-empty, depending upon whether the sentence is true or false, while the other is empty.[20] Let us use τ for the totality fact. Then using the recursive rules at the end of §10 as our guide, we are led to adopt the following semantical clauses for when a fact will verify or falsify a logically complex sentence:

(I) $\wedge$T f verifies A $\wedge$ B iff there are f_1 and f_2 such that $f_1.f_2 = f$, f_1 verifies A and f_2 verifies B;

$\wedge$F f falsifies A $\wedge$ B iff f falsifies A or f falsifies B or there are f_1 and f_2 such that $f = f_1.f_2$, f_1 falsifies A and f_2 falsifies B;

(ii) $\vee$T f verifies A $\vee$ B iff f verifies A or f verifies B or there are f_1 and f_2 such that $f = f_1.f_2$, f_1 verifies A and f_2 verifies B;

$\vee$F f falsifies A $\vee$ B iff there are f_1 and f_2 such that $f = f_1.f_2$, f_1 falsifies A and f_2 falsifies B;

(iiii) $\neg$T f verifies $\neg$A iff f falsifies A;

$\neg$F f falsifies $\neg$A iff f verifies A;

---

[20] Bas van Fraassen [69] has developed some related ideas but his semantical clauses and the logic he gets out of them are somewhat different from my own.

(iv) $\forall$T f verifies $\forall x B(x)$ iff there are $f_1, f_2, \ldots$ such that $f = \tau.\ f_1.\ f_2.\ \ldots$, and f_1 verifies $B(a_1)$, f_2 verifies $B(a_2)$, … (with a_1, a_2, ... running through all of the individuals);

$\forall$F f falsifies $\forall x Fx$ iff there are $f_1, f_2, \ldots$ such that $f = \tau.\ f_1.\ f_2.\ \ldots$, and f_1 falsifies $B(b_1)$, f_2 falsifies $B(b_2)$, ... (with b_1, b_2, ... running through some of the individuals);

(v) $\exists$T f verifies $\exists x Fx$ iff there are $f_1, f_2, \ldots$ such that $f = \tau.\ f_1.f_2.\ \ldots$, and f_1 verifies $B(b_1)$, f_2 verifies $B(b_2)$, ...;

$\exists$F f falsifies $\exists x Fx$ iff there are $f_1, f_2, \ldots$ such that $f = \tau.\ f_1.\ f_2.\ \ldots$, and f_1 falsifies $B(a_1)$, f_2 falsifies $B(a_2)$, ….

The clause for $\wedge$T, for example, corresponds to rule $\wedge$IE* above, but with the with the facts f, f_1 and f_2 in place of the grounding sets Δ, Δ_1 and Δ_2 and with the fusion $f_1 . f_2$ of the facts f_1 and f_2 in place of the union $\Delta_1 \cup \Delta_2$ of the grounding sets Δ_1 and Δ_2. It should be noted that, under this semantics, the 'factual content' of $A \vee \neg A$ and of $B \vee \neg B$ will not in general be the same, since the verifiers for $A \vee \neg A$ will be the facts that either verify or falsify A while the verifiers for $B \vee \neg B$ will be the facts that either verify or falsify B; and similarly for A and $(A \wedge B) \vee (A \wedge \neg B)$ and many other truth-functionally equivalent formulas.

I believe that the factualist semantics has numerous other applications - to the semantics of counterfactuals, for example, to confirmation theory and the theory of verisimilitude, to the frame problem in AI, and to a number of problems in linguistics; and I find it remarkable that the semantics should have an independent 'purely metaphysical' motivation in terms of the inferential behavior of ground.[21]

§11 Essence and Ground

Given an object or some objects, we may say that it lies in the nature of those objects that such and such should hold – that it lies, in the nature of singleton Socrates, for example, that it should have Socrates as a member. But what then is the connection between statements of nature or essence and statements of ground?[22]

A natural view is this. Given that the fact F is grounded in the facts $G_1, G_2, \ldots$, then it lies in the nature of the fact F (or of the items that it involves) that it should be so grounded given that the facts $G_1, G_2, \ldots$ do indeed obtain. So, for example, given that the fact that the ball is red and round is grounded in the fact that it is red and the fact that it is round, it will lie in the nature of the fact that the ball is red and round that this fact will be grounded in the fact that the ball is red and the fact that the ball is round (given that the ball is in fact red and is in fact round).[23]

Unfortunately, this view will not quite do as it stands. The fact that someone is a philosopher, we may suppose, is grounded in the fact that Socrates is a philosopher (and perhaps also that he exists and is a person). But it does not lie in the nature of the fact that someone is a philosopher that

[21]An application to counterfactuals is developed in Fine [2011b]

[22] The concept of essence is further discussed in Fine [1994].

[23] I say that it lies in the nature of the *fact* that the ball is red and round and thereby treat the fact as an object (perhaps identical to the *proposition* that the ball is red and round). But there is something to be said for allowing it to lie in the nature of what it is for the ball to be red and round, where this is represented by a sentential rather than by a nominal complement to the essentialist operator.

the fact is so grounded given that Socrates is indeed a philosopher. The fact, so to speak, knows nothing of Socrates. Or again, the fact that the ball is colored is grounded, we may suppose, in the fact that it is red. But it does not lie in the nature of the fact that the ball is colored that it is so grounded given that the ball is indeed red. The fact and *color*, in particular, know nothing of the specific colors.

The difficulty in these cases arises from the grounds G_1, G_2, … being merely an instance of the grounds that the given fact F is capable of possessing. Thus the fact that someone is a philosopher could equally well be grounded in the fact that Plato is a philosopher and the fact that the ball is colored could equally well be grounded in the fact that it is blue. But suppose that we generalize the statement of ground. We say that the fact that someone is a philosopher is, for any person x, grounded in the fact that x is a philosopher given that x is indeed a philosopher and that the fact that the ball is colored is, for any color c, grounded in the fact that the ball is of color c given that the ball is indeed of color c. It does then seem plausible to say that these generalized statements of ground will hold in virtue of the nature of the grounded fact – that it lies in the nature of the fact that someone is a philosopher, for example, that this fact will, for any person x, be grounded in the fact that x is a philosopher given that x is indeed a philosopher.

Let us state the point more generally. Suppose that the truth C is grounded in B_1, B_2, …. Then the grounds B_1, B_2, … will concern certain existing items a_1, a_2, …and so may be stated in the form $B_1(a_1, a_2, \ldots)$, $B_2(a_1, a_2, \ldots)$, …. A generalization of this particular connection of ground will therefore take the form:

$B_1(x_1, x_2, \ldots)$, $B_2(x_1, x_2, \ldots)$, … is a ground for C whenever $A(x_1, x_2, \ldots)$, where $A(x_1, x_2, \ldots)$ is a condition that in fact holds of $a_1, a_2, \ldots$.[24] Thus given that $A(x_1, x_2, \ldots)$ in fact holds of the existing items $a_1, a_2, \ldots$, the particular connection of ground will logically follow from the general connection.

What we may now claim is that whenever a given truth C is grounded in other truths, then there is a generalization of the particular connection of ground that will hold in virtue of the nature of C (or of the items it involves). Thus the particular explanatory connection between the fact C and its grounds may itself be explained in terms of the nature of C.

It should be noted that what explains the ground-theoretic connection is something concerning the nature of the fact that C (or of what it is for C to be the case) and not of the grounding facts themselves. Thus what explains the ball's being red or green in virtue of its being red is something about the nature of what it is for the ball to be red or green (and about the nature of disjunction in particular) and not something about the nature of what it is for the ball to be red. It is the fact to be grounded that 'points' to its grounds and not the grounds that point to what they may ground.

One might hold that the ground-theoretic connection holds in virtue of the nature of its grounds and the general nature of ground in addition to the nature of the fact to be grounded. But this is a far weaker and far less interesting claim. For it might be held as a general thesis that every necessary truth is grounded in the nature of certain items (Fine [1994]); and, as a rule, these will be the items involved in the necessary truth itself. But given that C is grounded in $B_1, B_2, \ldots$, it will be necessary that C is grounded in $B_1, B_2, \ldots$ if $B_1, B_2, \ldots$ are the case; and so it will follow from the general thesis that it lies in the nature of certain items – presumably those involved in C and $B_1, B_2, \ldots$ and *ground* itself - that this is so. Claiming that the fact to be grounded bears full responsibility,

---

[24] Strictly speaking, we should also require that it is a *necessary* truth that $B_1(x_1, x_2, \ldots), B_2(x_1, x_2, \ldots), \ldots$ is a ground for C whenever $A(x_1, x_2, \ldots)$ and $B_1(x_1, x_2, \ldots), B_2(x_1, x_2, \ldots), \ldots$ are the case.

so to speak, for the ground-theoretic connection is to make an essentialist claim that goes far beyond the assertion of a general link between necessity and nature.

Part of the interest of the stronger thesis lies in its bearing upon the methodology of metaphysics. For investigation into ground is part of the investigation into nature; and if the essentialist locus of ground-theoretic connections lies in the fact to be grounded and not in the grounds, then it is by investigating the nature of the items involved in the facts to be grounded rather than in the grounds that we will discover what grounds what. Thus the asymmetry supports a top-down approach in which we start with the facts to be grounded and work our way down to their grounds, rather than the other way round.

Part of the interest of the stronger thesis also lies in its bearing upon the general nature of objects. If we were merely given a general link between necessity and nature, then this would be perfectly compatible with ground-theoretic connections always holding partly in virtue of the nature of *ground*. Thus the nature of *being colored* might have nothing to do with *ground*; and so whereas it might lie in the nature of *being colored* that anything colored was of a particular color, it would not lie in the nature of *being colored* that anything colored was colored *in virtue of* being a particular color. What then accounted for the fact that anything colored was colored *in virtue of* being a particular color would be something about the nature of *being colored* and something about the nature of *ground*. The nature of *ground* would somehow 'feed off' the nature of *being colored* to give us this particular ground-theoretic connection.

But what is being claimed is that this is not so and that ground-theoretic connections will be inextricably involved in the nature of certain things. It is not just that they must, by their very nature, behave in a certain way but that there must, by their very nature, be a certain ground-theoretic basis for their behavior. Thus it may well be thought to be essential to the nature of being colored not merely that anything colored is of a specific color but that anything colored is colored *in virtue of* being a specific color; the ground-theoretic basis for being colored is built into the very identity of what it is to be colored.

Rosen ([2010], 132-3) has suggested some counter-examples to the proposed link between ground and essence (and even to weaker versions of the link). Suppose, for example, that something's being right or good is grounded in certain naturalistic features of the object. Then on a nonreductive view of normativity, it will not lie in the nature of *right* or *good* that it is grounded in these particular features. And similarly for the case of a nonreductive materialist, who thinks that facts about pain are grounded in facts about our brain or the like and yet does not think that it lies in the nature of pain that it should be so grounded.

My own view is that the apparent plausibility of these counter-examples depends upon conflating different conceptions of ground. Corresponding to the concepts of normative and natural necessity will be normative and natural conceptions of ground, which are to be distinguished from the purely metaphysical conception. The view that the normative is grounded in the natural is only plausible for the normative conception of ground and the view that the mental is grounded in the physical is only plausible for the natural conception. Since the grounding relation in these cases is not metaphysical, there is no need for there to be explanation of its holding in terms of the essentialist nature of the items involved. What may be plausible, though, is that it should lie in the nature of *goodness*, say, that it should have *some* ground in what is natural and that it should lie in the nature of *pain* that it should have *some* ground in what is physical; and if the respective conceptions of ground here are normative and natural, then we see that these other conceptions of ground may have an important role to play in delineating the nature of certain essentially 'realized' properties or features.[25]

---

[25] I have benefitted from Rosen's ([2010], §13) discussion of these issues and our views are somewhat alike. But note that whereas I have one thesis, that any ground-theoretic connection can be generalized to one that flows from the nature of the items involved in the given fact, he has two theses: that any ground-theoretic connection can be

Given the proposed connection between essence and ground, it might be wondered whether it might somehow be converted into a definition of ground.[26] For given that $B_1, B_2, \ldots$ is a ground for C, there will be some generalization of this statement of ground that will hold in virtue of the nature of C. Now this generalization will be a general statement of what grounds what. But corresponding to this ground-theoretic generalization will be a ground-free generalization in which the notion of material implication replaces the notion of ground. Thus instead of saying 'C is grounded in $B_1, B_2, \ldots$ (or the like) if $B_1, B_2, \ldots$ (or the like) hold', we simply say 'C if $B_1, B_2, \ldots$'. It may now be suggested that we define $B_1, B_2, \ldots$ to ground C just in case some ground-free generalization of the statement of ground holds in virtue of the nature of C.

However, there are a number of things wrong with this definition. One is that it does not enable us to distinguish between the plural ground $B_1, B_2, \ldots$ and the single conjunctive ground $B_1 \wedge B_2 \wedge \ldots$, since the ground-free generalization will be the same in each case. Another is that it will predict the result that $A \wedge A$ grounds A, since it will be true in virtue of the nature of A that A if $A \wedge A$. Thus the proposed definition will not even provide a sufficient condition for weak ground.

To this last objection, it may be responded that it will only be true in virtue of the nature of A that A if $A \wedge A$ under a 'consequentialist' conception of essence, one in which the essentialist truths are taken to be closed under some notion of logical consequence. But it might be thought that underlying any consequentialist conception of essence is a 'constitutive' conception, which will not

generalized; and that any general ground-theoretic connection will flow from the nature of the items involved in the given fact and its grounds. Thus my view traces the source of the ground-theoretic connection to the nature of the items involved in the given fact while his also appeals to the nature of the items involved in its grounds. I should add that there are problems with his formulation of *Formality* (p. 131). Merely saying that a statement of ground has a generalization is not enough. The ball is perhaps red or round because it is red. But the required level of generality is not achieved by saying that anything will be red or round because it is red. Moreover, the appropriate generalization cannot in general be stated simply through appeal to propositional forms. One might think, for example, that the species Dog exists because there are dogs. But how are we to generalize this without appealing to the connection between Dog and being a dog? Our own formulation avoids these problems.

[26] E. J. Lowe considers and points out some difficulties in providing an essentialist account of truth-making in chapter 11 of Lowe and Rami [2008].

be automatically closed under logical consequence and for which it will not be true in virtue of the nature of A that A if $A \wedge A$.[27]

There is no doubt that appeal to a constitutive conception of essence will enable us to approximate more closely to the notion of ground. But how are we to understand the relationship between constitutive and consequentialist essence? One view is that we understand the latter in terms of the former. Roughly, to belong to the consequentialist essence of something is to be a logical consequence of what belongs to the constitutive essence. But another view, to which I am more inclined, is that we understand the former in terms of the latter. One statement of consequentialist essence may be partly grounded in others. The fact that it lies in the nature of a given set to be a set or a set, for example, is partly grounded in the fact that it lies in the nature of the set to be a set. The *constitutive* claims of essence can then be taken to be those consequentialist statements of essence that are not partly grounded in other such claims. This way of conceiving the distinction enables us to 'factor out' the purely essentialist aspect of the concept of essence from the partly explanatory aspect. But it means that the constitutive concept of essence is then of no help to us in understanding the concept of ground.

But there is perhaps a more serious objection to the proposed definition, which may arise even when we make use of the constitutive conception of essence. For certain statements of essence appear to be symmetric between ground and what is grounded. It might be thought, for example, that there is a distinction between existing *at a time* and existing *simpliciter* and that it is essential to any object that exists in time that it exists simpliciter iff it exists at a time. But the definition will then give us an equal right to say that the object exists simpliciter in virtue of existing at a time and that it exists at a time in virtue of existing simpliciter. But compatibly with the essentialist claim, we might want to make the first of these ground-theoretic statements to the exclusion of the other and we

[27] The distinction between the two conceptions of essence is further discussed in Fine [2003].

would certainly not want to make both statements under a strict conception of ground or even under a weak conception of ground, given that existing at a time essentially involves the notion of time while existing simpliciter does not.

I think it should be recognized that there are two fundamentally different types of explanation. One is of identity, or of what something is; and the other is of truth, or of how things are. It is natural to want to reduce them to a common denominator - to see explanations of identity as a special kind of explanation of the truth or to see explanations of truth as a special kind of explanation of identity or to see them in some other way as instances of a single form of explanation. But this strikes me as a mistake.

Carnap distinguished some time ago between formation and transformation rules. The former were for the construction of formulas and the latter for the construction of proofs. The formation rules provide an explanation of identity, of what the formulas are, while the transformation rules help provide an explanation of truth, of when a formula is true (or valid). It would clearly be an error to think of the one kind of rule as an instance of the other or to see them as falling under a common rubric. And it seems to me that there is a similar error - but writ large over the whole metaphysical landscape - in attempting to assimilate or unify the concepts of essence and ground. The two concepts work together in holding up the edifice of metaphysics; and it is only by keeping them separate that we can properly appreciate what each is on its own and what they are capable of doing together.

References

Armstrong J., [1997] 'A World of States of Affairs', Cambridge: Cambridge University Press.

Armstrong J., [2004] 'Truth and Truthmakers', Cambridge: Cambridge University Press.

Audi Paul [2010?] ‘Grounding’, to appear.

Batchelor R. [2009] ‘Grounds and Consequences’, to appear in Grazer Philosophische Studien.

Blackburn S., [1987] ‘Morals and Modals’, in ‘Fact, Science and Value: Essays in Honor of A. J. Ayer’s ‘Language, Truth and Logic’’ (eds. C. Wright and G. MacDonald), Oxford: Blackwell, reprinted in ‘Essays in Quasi-Realism’, Oxford: Oxford University Press.

Cameron R., [2011] ‘Truth-makers’, to appear in ‘Oxford Handbook of Truth’ (ed. M. Glanzberg), Oxford: Oxford University Press.

Correia F., [2005] ‘Existential Dependence and Cognate Notions’, Philosophica Verlag.

Fine K., [1975] Review of ‘Counterfactuals’ by David Lewis, Mind 84, 451-8; reprinted in ‘Modality and Tense: Philosophical Papers’, Oxford: Clarendon Press (2005).

Fine K., [1994] ‘Essence and Modality’, Philosophical Perspectives 8, 1-16.

Fine K., [2001] ‘The Question of Realism’, Imprint, vol. 1, no. 1, 2001; reprinted in ‘Individuals, Essence and Identity: Themes of Analytic Philosophy’ (ed. A. Bottani, M Carrara, P. Giaretta), Dordrecht: Kluwer 2002, 3- 41.

Fine K., [2003] ‘Senses of Essence’, in ‘Modality, Morality and Belief: Essays in Honor of Ruth Barcan Marcus’ (ed. W. Sinnott-Armstrong), Cambridge: Cambridge University Press.

Fine K., [2005] ‘The Varieties of Necessity’, in ‘Modality and Tense: Philosophical Papers’, Oxford: Oxford University Press.

Fine [2010a] ‘Towards a Theory of Part’, Journal of Philosophy CVII, no. 11.

Fine [2010b] ‘Some Puzzles Concerning Ground’, Notre Dame Journal of Formal Logic 51(1), 97-118.

Fine [2011a] ‘The Pure Logic of Ground’, to appear in the Review of Symbolic Logic.

Fine [2011b] ‘Counterfactuals Without Possible Worlds’, to appear in Journal of Philosophy.

Fox J. F. [1987] ‘Truthmaker’, Australasian Journal of Philosophy 65, 188-207.

Hawthorne J., [2006] ‘Metaphysical Essays’, Oxford: Oxford University Press.

Horwich P., [2008] ‘Being and Truth’, Midwest Studies in Philosophy 32, 258-73.

Lowe E. J., and Rami A., (eds) [2008] ‘Truth and Truth-making’, UK.: Acumen Publishing.

Mulligan K., Simons P., Smith B., [1984] ‘Truth-Makers’, Philosophy and Phenomenological Research 44, 210-55.

Raven M., [2009] ‘Ontology, From a Fundamentalist Point of View’, Ph. D., New York University.

Rosen G., [2010] ‘Metaphysical Dependence: Grounding and Reduction’, to appear in a collection of papers with OUP.

Russell B., [1918], ‘The Philosophy of Logical Atomism’, reprinted with an introduction by D. Pears [1985], Open Court: La Salle, Illinois.

Schaffer J., [2009] ‘On What Grounds What’, in ‘Metametaphysics (eds. Chalmers, Manley, Wasserman), pp. 347-83, Oxford: Oxford University Press.

Schnieder B., [2006] ‘Troubles with Truth-making: Necessitation and Projection’, Erkenntnis 64, 61-74.

Schnieder B., [2011?] ‘A Logic of ‘Because’, to appear in Bulletin of Symbolic Logic (?)

Terese [2003] ‘Term Rewriting Systems’, Cambridge: Cambridge Tracts in Computer Science.

Van Fraassen B. [1969] ‘Facts and Tautological Entailments’, Journal of Philosophy 66, 477-87.

Williamson T., [2000] ‘Knowledge and its Limits’, Clarendon Press: Oxford.