Aristotle's Megarian Manoeuvres

Abstract: Towards the end of Theta. 4 of the Metaphysics, Aristotle appears to endorse the obviously invalid modal principle that the truth of A will entail the truth of B if the possibility of A entails the possibility of B. I attempt to show how Aristotle's endorsement of the principle can be seen to arise from his accepting a non-standard interpretation of the modal operators and I indicate how the principle and its interpretation are of independent interest, quite apart from their role in understanding Aristotle.

Towards the end of Theta.4 of the Metaphysics (1047b14-b30), Aristotle attempts to establish two modal principles. The passage (based on the revised Oxford translation but with my paragraphing and square bracketing) goes as follows:

[Principle 1] At the same time it is clear that if, when A is B must be, then, when A is possible B also must be possible.

[Argument for Principle 1] For if B need not be possible, there is nothing to prevent its not being possible. Now let A be supposed possible. Then, when A is possible, nothing impossible would follow if A were supposed to be; and then B must be. But we supposed B to be impossible. Let it be impossible, then. If, then, B is impossible, A also must be. But the first was supposed possible; therefore the second is also. If, then, A were possible, B also will be possible, if they were so related that if A is B must be. If, then, A and B being thus related, B is not possible in this way, A and B will not be related as was supposed.

[Principle 2] And if when A is possible B must be possible, then if A is B must also be.

[Argument for Principle 2]For to say that B must be possible if A is possible means that if A is both at the same time when and in the way in which it was supposed capable of being, B also must then and in that way be.

This passage raises severe exegetical problems. One of these problems is that the second principle seems obviously to be incorrect; and so it is not clear why Aristotle would have wanted to endorse it. For suppose that a fair coin is tossed and turns up heads. It is then plausible to maintain that when it is possible that the coin is fair and turns up heads it must be possible that it turn up tails and hence not turn up heads. By the principle it follows that when the coin is fair and turns up heads then it must not turn up heads; and from this it follows that it is not true that it is both fair and turns up heads, contrary to our original supposition.[1]

There are also difficulties in understanding the arguments themselves. For although the first principle is plausible, the argument for it appears to be blatantly circular (as well as being highly repetitious); and the argument for the second principle seems simply to rest upon a confusion of what is possible with what is actual.

Aristotle was a great thinker; and it is a safe general rule that great thinkers do not commit egregious errors. We are therefore under an obligation of coming to a better understanding of what Aristotle might have meant. It is my aim in this paper to suggest that Aristotle may have had a highly novel conception of modality in mind in this passage, one that not only clears him of error but that is also of considerable independent interest. From a proof-theoretic point of view, it requires that one give up the T-axiom $\Box A \supset A$ in its full generality; and from a semantic point of view, it requires that one make a distinction, in evaluating a modal formula at a world, between treating that world as actual and as possible.

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[1] As a referee pointed out, the premiss of this inference already appears to be in conflict with Aristotle's denial that $\neg A$ can necessarily follow from A (*Prior Analytics* 57b13-14) and so the supposition that a fair coin is tossed and turns up heads is not even required. Let me also note that we can give a highly abstract counter-example to the principle, based simply on the assumption that there is a description of how things are (theorem 5 of the appendix).

I begin by considering the different ways in which Aristotle's two principles might be formalized within the framework of propositional modal logic (§1). I then consider the deductive and semantic consequences of the different ways in which these principles might be formalized, using the apparatus of contemporary modal logic (§2). It is shown that the difficulties confronting Aristotle are even greater than might have been thought, since the second principle leads to 'modal collapse', the collapse of possibility to actuality, which is something that Aristotle had previously argued explicitly against. Three recent attempts to get Aristotle 'off the hook' - those of Brennan 1994, Makin 1999 & 2006, and Nortmann 2006 - are considered and found wanting (§3). I then propose an alternative solution, which rests upon distinguishing between a world as the *locus* and as the *witness* of possibilities (§4). Once the semantics for Aristotle's use of the modalities is understood in this way, it becomes perfectly explicable why he would have wanted to endorse the converse principle and how he can avoid modal collapse. I defend this interpretation of Aristotle against some objections and try to indicate why it is of independent interest (§5). I conclude with an attempt to vindicate Aristotle's argument for the first principle (§6).

§1 Formalization of the Principles

I begin by considering how the two principles might be formalized within the symbolism of propositional modal logic.

Suppose we read:

(i) '--- > …' as 'if --- then …';

(ii) '--- ⇒ …' as 'when --- then it is necessary that … '; and

(iii) '$\Diamond$---' as ' it is possible that ---'.

(Of course, what I really have in mind is the corresponding Greek). Then it seems clear that the two principles should symbolized as:

P1. $(A \Rightarrow B) > (\Diamond A \Rightarrow \Diamond B)$; and

P2. $(\Diamond A \Rightarrow \Diamond B) > (A \Rightarrow B)$.

In providing these symbolizations, I am assuming nothing about the interpretation of the connectives $\Rightarrow$, $>$, and $\Diamond$ beyond their reading in English (or Greek); and viewed in this way, they amount to little more than a transcription of the English (or Greek) into symbols, with some incidental changes in grammar and word-order.

I can think of only one way, with any degree of plausibility, in which this rudimentary form of symbolization might be challenged. For it might be thought that the bracketing in the formulas is not faithful to the scope of the second 'necessity' in each principle, which should be taken to have wide scope over the whole conditional (with '$>$') rather than over the immediate conditional (with '$\Rightarrow$'). Thus the proper symbolizations would then be:

P1'. $\Box(\Box(A \rightarrow B) > (\Diamond A \rightarrow \Diamond B))$; and

P2'. $\Box(\Box(\Diamond A \rightarrow \Diamond B) > (A \rightarrow B))$,

where ' $>$ ' and '$\Diamond$' are read as before, '--- $\rightarrow$ …' is read as 'when --- then …' (with the necessity in the consequent dropped), and '$\Box$ ---' is read as 'it is necessary that ---'.

But it is hard to take this alternative seriously. As a reading of the second 'necessity' in each principle, it is highly strained; and it fails to do justice to the sense of form that the wording of the principles so naturally conveys. For we naturally take the consequent in the first principle to be the result of substituting $\Diamond A$ for A and $\Diamond B$ for B in the antecedent, thereby delivering $\Box(\Diamond A$

$\rightarrow \Diamond B$), rather than ($\Diamond A \rightarrow \Diamond B$), as the consequent (and similarly, but with the role of antecedent and consequent reversed, in the second principle); and we naturally take the two theses to be converses, obtained simply by interchanging antecedent and consequent (as remarked in Brennan 1994, p. 163).

I stress these points since several earlier discussions of the passage have failed to take them into account. Thus Hintikka (1973, pp. 185-90) and Frede (as reported in Brennan 1994, fn. 2) symbolize the first principle by:

$\Box(A \supset B) \supset (\Diamond A \supset \Diamond B)$[2];

which is not of the required general form, as specified in P1; and Frede symbolizes the second principle by:

$\Box(\Diamond A \supset \Diamond B) \supset (A \supset B)$,

which is not of the required general form, as specified in P2.

Moreover, as we shall see, these subtle issues of symbolization will later be critical in arriving at a satisfactory interpretation of Aristotle's views.[3]

The formalization of the theses as P1 and P2 still leaves open how we should understand the two forms of conditional operator that they involve, the inner operator '$\Rightarrow$' and the outer operator '$>$'. It seems relatively harmless, in the context of the discussion, to read the outer

[2]Hintikka's misreading may have been motivated by the desire to see Aristotle as an early proponent of the characteristic K-axiom.

[3]I might add that Makin (1999, p. 117) takes P1 to be of the form $(A \Rightarrow B) \Rightarrow (\Diamond A \Rightarrow \Diamond B)$ (and similarly for P2). But this is to treat the outer conditional operator '$>$' and the inner conditional operator '$\Rightarrow$' as the same and is almost certainly a mistake since it takes no account of the necessity that appears as a qualification in the statement of the inner conditional. Furth (1985, note to 47b14-16) makes a similar mistake, which is compounded by his treating the principle, when so formalized, as equivalent to $\Diamond A \Rightarrow \sim((A \Rightarrow B) \ \& \sim\Diamond B)$.

operator '>' in each principle as the material conditional '$\supset$', even if that is not how Aristotle would have understood it. For our current interest is in seeing how the first principle is acceptable while the second is not. But the reasons we have for rejecting the second principle apply to P2 under a material reading of the conditional and so, will apply to P2 under any other, stronger, reading of the conditional. And given that P1 is acceptable under a material reading of the conditional, it is hard to see how it might plausibly fail to be acceptable under any other reading of the conditional, even if stronger than the material reading.[4]

The understanding of the inner operator '$\Rightarrow$' is another matter. It is not altogether implausible that '$A \Rightarrow B$' (when A then it is necessary that B) should be be capable of an analysis in terms of a nonmodal conditional $\rightarrow$ (when --- then …) and a necessity operator $\Box$. And of the two possible scopes for the necessity operator, it is only the wide scope reading (in which it governs the whole conditional and not just the consequent) that makes any reasonable sense. We therefore arrive at the previous analysis of '$A \Rightarrow B$' as '$\Box(A \rightarrow B)$'.

There remains the question of how we should understand the innermost conditional

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[4]Some remarks of Patzig 1968 suggest a reading of these principles for which these connections may not hold. He appears to read the first principle as claiming that 'the validity of "If A then B" entails the validity of "If possibly A then possibly B"' and comments that 'this is specious but false - on the normal interpretation of the operator' (pp. 85-6, fn. 21). The comment is in error. The validity (or, more generally, the theoremhood) of $A \supset B$ entails the validity (theoremhood) of $\Diamond A \supset \Diamond B$ in all normal modal logics. Moreover, the theoremhood of $\Box(A \supset B)$ will entail the theoremhood of $\Box(\Diamond A \supset \Diamond B)$ in all normal modal logics even though $\Box(A \supset B) \supset \Box(\Diamond A \supset \Diamond B)$ may not itself be a theorem. Somewhat more interestingly, there are modal logics (such as K) for which the theoremhood of $\Box(\Diamond A \supset \Diamond B)$ will entail the theoremhood of $\Box(A \supset B)$, even though $\Box(\Diamond A \supset \Diamond B) \supset \Box(A \supset B)$ is not itself a theorem. We shall later consider an interpretation in Nortmann 2006 along these lines.

operator '→'.[5] And again, it is not altogether implausible that this should be understood as the material conditional, the whole strength of the outer conditional 'A $\Rightarrow$ B' residing in the application of the necessity operator to the material conditional 'A ⊃ B'. The two principles are then purely modal theses whose acceptability does not depend upon some peculiarity in our understanding of the conditional.

Either or both of the last two steps in the formalization (the account of A ⇒ B as $\Box$(A → B) and the account of A → B as A ⊃ B) might not have been acceptable to Aristotle. But this may not much matter in the present context. For many of the counterexamples to the second principle seem to work under any reasonable understanding of the inner conditional '⇒'. In the case of the fair coin, for example, it seems plausible, however we read the conditional, that when it is possible that the coin is fair and turns up heads then it must be possible that it not turn up heads. Thus the difference in strength between the modal and non-modal reading of the conditional appears to be irrelevant to the force of the counter-examples; and this makes it hard to see how one might find the principle acceptable under the one reading but not under the other.

We are therefore left with the following straightforward formalization of the theses within the contemporary idiom of modal logic:

P_1. $\Box(A \supset B) \supset \Box(\Diamond A \supset \Diamond B)$;

P_2. $\Box(\Diamond A \supset \Diamond B) \supset \Box(A \supset B)$.

And it is mainly these formalizations that we shall consider in what follows, although we shall also look at some variants of them for the purposes of comparison. For convenience, let us call

[5] Though, as Gavin Lawrence has pointed out, Aristotle later in the passage feels free to use the 'if'-construction in place of the 'when'-construction, suggesting that he would not have wished to make a distinction between the two forms of conditional.

$\Box(A \supset B)$ (or its Greek counterpart) the 'necessity clause' and $\Box(\Diamond A \supset \Diamond B)$ (or its Greek counterpart) the 'possibility clause'.

§2 The Logical Situation

I now wish to consider the deductive and semantic implications of these various formalizations of Aristotle's principles.[6]

The minimal system of modal logic, K, has the following axioms and rules[7]:

Axioms

A1. All tautologous formulas

A2. $\Box(A \supset B) \supset (\Box A \supset \Box B)$

Rules

Modus Ponens. $A, A \supset B / B$ (from A and $A \supset B$ one may infer B)

Necessitation. $A / \Box A$ (from A, given that it is a theorem, one may infer $\Box A$).

The system T is the result of adding the axiom:

T. $\Box A \supset A$

to K; and the system D is the result of adding the axiom:

D. $\Diamond\top$

[6]Some related results have been stated in Brennan 1993, but they concern the semantic rather than the deductive implications. Given the 'incompleteness phenomenon', an axiom may not have the deductive import one would expect it to have on the basis of its semantic import; and so it is the former rather than the latter that is most directly relevant to evaluating its acceptability.

[7] Strictly speaking, we have an axiom-scheme (all substitution-instances of the scheme are axioms) rather than a single axiom. But I shall not be careful about such niceties except where it matters.

to K (for $\top$ any tautology). The system T is normally taken for granted in interpreting '$\Box$' as some form of alethic necessity.

Let KP_1 be the result of adding P_1 above as an additional axiom to K; and similarly for KP_2 and KP_1P_2 and other cases of this sort. In order to ascertain the strength of the resulting systems, it will be helpful to consider the following further axioms:

4. $\Box A \supset \Box\Box A$

M. $A \equiv \Diamond A$

$M_\Box$ $\Box(A \equiv \Diamond A)$.

4 is the characteristic axiom for the system S4 (S4 being the system KT4). I call M the 'Megarian axiom'; it states the collapse of possibility to truth. $M_\Box$ is the necessity of the Megarian axiom; it states that it is necessary that possibility collapses to truth. In the presence of the axiom T, it yields the unqualified Megarian axiom M as a theorem; but it should not be assumed to yield M in the absence of T. I call KM the 'Megarian system' and $KM_\Box$ the 'quasi-Megarian system'.

Say that two systems are *deductively equivalent* if their theorems are the same. We can then establish the following results concerning the addition of one or both of P_1 and P_2 to K:

KP_1 is deductively equivalent to K4,

KP_2 to $KM_\Box$, and

KP_1P_2 to KP_2.

Thus in the context of K, the adoption of P_1 is equivalent to the adoption of axiom 4, the adoption of P_2 equivalent to the adoption of $M_\Box$, the quasi-Megarian axiom, and the adoption of both P_1 and P_2 equivalent to the adoption of P_2 (which is to say that the second principle commits one to the first).[8]

There are similar results concerning the addition of one or both of P_1 and P_2 to the system T, with 'T' substituted for 'K'. Of course, within the context of T, the adoption of $M_\Box$ commits one to the adoption of M and so, in this case, TP_2 and TP_1P_2 will both be deductively equivalent to the Megarian system TM.

The second and third of the results for T are deeply awkward for Aristotle. For surely, one might think, he is committed to the T-axiom. But in the presence of T, P_2 is equivalent to the Megarian axiom M, stating the coincidence of the possible with the actual. Yet just before (in Theta.3-4), he had vigorously argued against the Megarian position. And so how can he now (in the latter half of Theta.4) be propounding a principle that commits him to a position that he had previously rejected? It is not just, as some commentators have supposed, that the principle is itself implausible; it is also at odds with things he asserts in the very context of the discussion.

Nor does it appear to help to suppose that he might somehow not be committed to the T-axiom. He might then admit the *necessity* of the coincidence of the possible with the actual without admitting its *actual* coincidence. But we are at loss to know this notion of necessity might be or how the necessity of the Megarian position might somehow be more palatable than its truth.

In the face of these difficulties, it is worth seeing whether there is any variation in how these principles might be formalized that could rescue Aristotle from inconsistency. Even if these variants are not so close to what Aristotle appears to say, the mere fact that they spared him

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[8]The proofs are for the most part straightforward. The difficult directions of the second and third equivalences are proved under theorem 1 of the appendix.

from inconsistency would provide some degree of support in their favor. Unfortunately, the prospects in this direction do not look good. The variants of P_2 that come to mind are these:

$P_{2,1}$. $(\Diamond A \supset \Diamond B) \supset \Box(A \supset B)$.

$P_{2,2}$. $\Box(\Diamond A \supset \Diamond B) \supset (A \supset B)$.

$P_{2,3}$. $(\Diamond A \supset \Diamond B) \supset (A \supset B)$.

$P_{2,4}$. $\Box(\Box(\Diamond A \supset \Diamond B) \supset (A \supset B))$.

Some of these variants are bruited in the literature. Frede, as I have mentioned, proposes $P_{2,2}$ and G. E. L. Owen (as reported in Burnyeat et al. 1984, p. 109) proposes $P_{2,3}$.

In order to ascertain the deductive strength of the resulting systems, it will be helpful to consider some variants on the axiom M:

M_1 $\Box A \equiv \Diamond A$;

$M_{1\Box}$ $\Box(\Box A \equiv \Diamond A)$

$M_{\Box\Box}$ $\Box\Box(A \equiv \Diamond A)$.

M_1 is a Megarian axiom to the effect that necessity coincides with possibility; it means, in effect, that there is at most one possibility, though it leaves open whether the one possibility, if it exists, is given by what is actual. $M_{1\Box}$ is to the effect that the new Megarian axiom is necessary, and $M_{\Box\Box}$ is to the effect that the original Megarian axiom M is necessarily necessary.

We may then establish the following further results concerning the addition of one of $P_{2,1}$ $P_{2,2}$ $P_{2,3}$ or $P_{2,4}$ to K:

$KP_{2,1}$ is deductively equivalent to KM_1,

$KP_{2,2}$ to $KM_{1\Box}$,

$KP_{2,3}$ to KM, and

$KP_{2,4}$ to $KM_{\Box\Box}$.

However, these results are no more helpful than the original results. For, in the presence of T, the distinctions between all five axioms - M, $M_{\Box}$, M_1,$M_{1\Box}$, and $M_{\Box\Box}$ - disappears; and so we are again left with a commitment to the original Megarian position. And, even in the absence of T, it is still unclear how Aristotle might find these variants on the Megarian position any more palatable than the position itself.[9]

In the face of these difficulties, one might wonder whether the underlying modal reasoning should be questioned. I take it that the theorems of the minimal modal logic K are not in dispute. However, it can be shown that the possibility of deriving $M_{\Box}$ from P_2 requires that we assume not only the actual truth of P_2 but also its necessary truth (see theorem 2 of the appendix). Is it conceivable that Aristotle might have accepted P_2 as an actual truth though not as a necessary truth? Unfortunately, this way out is of no help if we assume that Aristotle is committed to the system T or to the system K4 (which he might appear to be given the fore-mentioned equivalence between KP_1 and K4). For then M can be derived from the actual truth of P_2 alone (theorem 3 of the appendix).

A perhaps more promising approach is to suppose that Aristotle was only committed to the truth of P_2 for nonmodal formulas and not also for modal formulas. The derivation of M, even in the context of the system S4 and assuming the necessary truth of P_2, will not then go through (theorem 4 of the appendix). I shall later criticize this proposal but let us note here that, even in this case, the derivation of M can be reinstated if we assume that there is a complete non-

[9] The interested reader may verify that the more extreme scopal variants - such as $\Diamond A \supset \Box\Diamond B \supset \Box(A \supset B)$ - are also of no help in this regard.

modal description of the actual world (theorem 5 of the appendix). All in all, we see that it is very difficult for Aristotle to wriggle out of the embarrassing consequences of adopting some form of the second principle.

Let us turn to the semantic import of the various axioms. We can interpret a formula $\Box A$ to be true in a given world w iff A is true in all worlds 'accessible from' (or 'possible relative to') w. Under such an interpretation, any axiom can then be taken to correspond to a condition on the accessibility relation, the condition whose satisfaction is both necessary and sufficient for the axiom to be valid. The T-axiom, for example, corresponds to the condition that the accessibility relation be reflexive (with each world accessible from itself).[10]

Call a world w *Megarian* if it is accessible from itself and is the only world accessible from itself ($\forall v(wRv \equiv w = v)$); and call a world w *deterministic* if exactly one world is accessible from it ($\exists u \forall v(wRv \supset u = v)$). We then have the following correspondences:

4 (and hence P_1) corresponds to the accessibility relation being transitive;

M (and hence $P_{2,3}$) corresponds to each world being Megarian;

$M_{\Box}$ (and hence P_2) corresponds to each accessible world being Megarian;

$M_{\Box\Box}$ (and hence $P_{2,4}$) corresponds to each accessible world from an accessible world being Megarian;

M_1 (and hence $P_{2,1}$) corresponds to each world being deterministic;

$M_{1\Box}$ (and hence $KP_{2,2}$) corresponds to each accessible world being deterministic.

When the accessibility relation is assumed to be reflexive, each of the conditions corresponding

[10]A system need not be complete with respect to the conditions to which its axioms correspond. But all of the systems we consider will, in fact, be complete.

to $M_{\Box}$, M_1,$M_{1\Box}$, and $M_{\Box\Box}$ simply reduces to the condition that each world be Megarian.

These results serve to reinforce the impression that there is something amiss with axiom P_2 and its variants. For surely, one is inclined to think, each world should be possible relative to itself; and surely some worlds should be possible relative to other worlds. But not, it seems, if P_2 or its variants are to be believed.

§3 Some Previous Solutions

We face a formidable problem in making sense of Aristotle's endorsement of the second principle. It is open to obvious counter-example; it appears to lead, by impeccable modal reasoning, to a thesis from which he explicitly dissents; and it can straightforwardly be seen to commit him to the view that only the actual world is possible. Is there anything to be said on his behalf?

I know of only three philosophers who have attempted to come to Aristotle's defense - Brennan (1994), Nortman (2006) and Makin (1999, 2006). Brennan makes two main claims in his interpretation of Aristotle, one negative and the other positive. The negative claim is that Aristotle should not be understood as stating something that can be formalized in terms of boxes and diamonds (pp. 164 & 173). The positive claim is that he should be understood as stating something corresponding to the standard semantic clauses for $\Box$. Thus P1 is to be 'translated' as:

(*) if $\Box(A \supset B)$ is true, then for all worlds α, if A is true at α then B is true at α,

and P2 is to be 'translated' by its converse (pp. 168-9).

Brennan perhaps overstates his case.[11] Of course, P1 can be symbolized using boxes and diamonds as long as no particular assumptions are made about how the boxes and diamonds are to be understood. Indeed, he presupposes as much when he maintains that ''P' and 'it is possible that P' can be substituted for one another with impunity' in the two principles (p. 170). It is also clear that it is only in a very loose sense of the term that the consequent of (*) can be said to 'translate' the consequent of Aristotle's first principle, since the one is explicitly quantificational while the other is not. We may grant that the former provides truth-conditions for the latter. But, we still require some explanation as to how Aristotle's modal formulation is capable of having the stated truth-conditions rather than the truth-conditions that one would expect from the standard semantics.[12]

Brennan's essential point, I believe, is better stated as follows. It may be conceded that Aristotle's two principles can be formalized in terms of boxes and diamonds. But it will be denied that the boxes and diamonds, in the principles as so formalized, can be subject to a standard or even a uniform semantic treatment. The box in the necessity-clause may indeed be understood in the standard way as implicitly quantifying over all possible (or relevantly possible)

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[11]One might also be a little worried about how Brennan appears to mix up use and mention. Since it is clear that Aristotle does not mention any sentences, it would be preferable to translate P1 as $\Box(A \supset B) \supset \forall\alpha(TA\alpha \supset TB\alpha)$, where T is a suitable connective connecting sentences and terms for worlds).

[12] We might see Brennan's remarks (in Brennan 1994], pp. 165-6) on the flexible use of modal language as gesturing in the direction of the desired explanation. Linguists have developed a 'dynamic' approach to modal discourse, under which a modal statement can serve to 'update' the possible situation under consideration (as in Veltman 1996, for example). I had originally thought that a dynamic semantics of this sort might provide a technical tool for understanding Aristotle's use of modal terms in the given passage, but I now think that the key to understanding him lies elsewhere.

worlds. But the diamonds in the possibility-clause cannot be understood in a comparable way as implicitly quantifying over some possible world, since this leads to Megarian collapse. The diamonds must therefore be seen to have a different function, as somehow serving to index the worlds within the implicit range of the outermost box to the statements taken to *be* possible rather than to the statements of possibility themselves.

This is an interesting proposal but, even in the absence of further detail, it is subject to some serious objections. First, it is prima facie implausible, given Aristotle's quasi-formal treatment of the topic and his general logical proclivities, that he was not intending to use the modal operators in a uniform way. Second, if the modal operators are not being used in a uniform way then the normal canons of modal reasoning cannot be expected to hold. Indeed, we know that the second principle leads by normal modal reasoning to the Megarian thesis; and so this reasoning must break down somewhere. But where it breaks down will be impossible to say since it will depend upon how exactly we carry through the anomalous reading of the boxes and diamonds from one step to another. This difficulty in evaluating such reasoning would not be so disturbing if Aristotle had confined himself to an isolated assertion of each principle. But immediately after stating the first principle, he attempts to state an argument, using standard canons of modal reasoning, in its favor. That he feels free to do this makes it even more plausible that he had a uniform reading of the modal terms in mind.

The final objection is a general difficulty for any account, though one which is perhaps especially acute for Brennan's. The arguments in support of the first and second principles are strikingly different. The first is a piece of modal reasoning (we will later attempt to say what it comes to) but the second is simply a statement of equivalence between the possibility clause (

$\Box(\Diamond A \supset \Diamond B)$) and its truth-conditional meaning, that 'if A is both at the same time when and in the way in which it is supposed capable of being, B also must then and in that way be' (something which seems roughly corresponds to B being true in any world in which A is true). It is therefore clear that Aristotle takes it to be evident that the truth-conditional equivalent of the possibility-clause implies the necessity-clause ($\Box(A \supset B)$). But should it not be equally evident (perhaps even more evident) that the necessity-clause implies the truth-conditional equivalent? And given that this is so, then why did Aristotle not give a similar justification for the first principle rather than the somewhat involved argument that he actually does give? Brennan, who is unwilling to see any significant difference between the possibility clause and its truth-conditional equivalent, is not in a good position to answer this question.

Nortmann (2006) follows Patzig 1968 in interpreting Aristotle's two principles (or, at least, the second) as rules of proof rather than as rules of deduction:

P1. $\Diamond A \supset \Diamond B$ is provable if $A \supset B$ is provable

P2. $A \supset B$ is provable if $\Diamond A \supset \Diamond B$ is provable.

He then observes that P1 will be correct for any normal system of modal logic and that P2 will be correct for many standard systems of modal logic under the restriction (which he takes Aristotle implicitly to adopt) that the formulas A and B are non-modal.

Nortmann's case can in fact be strengthened. First, the restricted form of P2 will be correct for *any* consistent (normal or non-normal) extension of the modal system D.[13] Second,

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[13]Here is a proof (much simpler than Nortmann's). Suppose $A \supset B$, for A and B non-modal, is not a theorem. By substituting $\top$ and $\bot$ for the sentence letters, we can find substitution-instances A' and B' of A and B which are respectively provably equivalent to $\top$ and $\bot$. Suppose, for reductio, that $\Diamond A \supset \Diamond B$ is provable. Then $\Diamond A' \supset \Diamond B'$ is provable and so $\Diamond\top$

there is no need to insist that the principles be interpreted as rules of proof rather than as rules of deduction. For we can add the axioms $P_1 = (\Box(A \supset B) \supset \Box(\Diamond A \supset \Diamond B))$ and $P_2 = (\Box(\Diamond A \supset \Diamond B) \supset \Box(A \supset B))$ to T without essential mishap and even allow the unrestricted application of the rule of necessitation as long as A and B in P_2 are required to be non-modal. Indeed, it is to be expected that one should be able to push a meta-logical notion of provability 'downwards' into the object language.[14]

There are, to my mind, three main problems with Nortmann's account. In the first place, it is based on an implausibly strong interpretation of the entailment relation that is expressed by the necessity clause (B must be when A is). On his account, the relation can take no heed of the meaning of the component statements A and B, merely of their logical form. But would not Aristotle be willing to say that someone's being a man entails that he was an animal or that something's being a horse entails that it was not a man? And if the relation is not constrained by logical form, then it is no longer so clear why Aristotle would not want to admit cases in which $\Diamond A$ entails $\Diamond B$ but A does not entail B.

In the second place, the restriction to non-modal formulas does not appear to be well motivated. We might express the first principle in the form:

$\supset\Diamond\bot$ is provable. But $\Diamond\top$ is an axiom of D; so $\Diamond\bot$ is provable and the system is inconsistent.

The second rule is also correct for certain modal logics, such as K, without the restriction to nonmodal formulas. However, it cannot be correct for systems containing axiom 4 (which Aristotle seems to accept) without $A \supset \Diamond A$ also being a theorem.

[14] See theorem 4 of the appendix. Alternatively, we may give a meta-logical interpretation in the style of Meyer 1971 and Fine under which $\Box A$ is taken to be true if A is true and a theorem of the system in question. I should also note that there is a difficulty in interpreting the first principle as a rule if the argument Aristotle gives for the principle is to be seen to be non-circular.

if A entails B then $\Diamond$A entails $\Diamond$B.

Suppose A entails B. Then $\Diamond$A entails $\Diamond$B. Are we now to suppose that Aristotle does not envisage the possibility of re-applying the principle to this new entailment in order to derive the result that $\Diamond\Diamond$A entails $\Diamond\Diamond$B? Or again, one might well think that A entails $\Diamond$A. Are we to suppose that Aristotle does not envisage the possibility of applying the principle to this case in order to derive the result that A entails $\Diamond\Diamond$A? After all, this is a context in which the embedded use of modal claims is already in play. Whence the squeamishness against a further form of embedding?

But if $\Diamond$A and $\Diamond$B can be substituted for A and B in the first principle then why not in the second, which appears, after all, to be a converse of the first?

Finally, the justification of the restricted form of P2 is far from obvious (even under my own version of the proof). Nortmann (2006, p. 393) suggests that 'owing to an exceptional instinct for affairs of logic and, maybe, to a portion of luck, Aristotle comes to advance and defend what is, in effect a true equivalence'. Maybe. But Aristotle did not simply conjecture P2. He took it to have an obvious and straightforward justification. Why?

Finally, let us briefly discuss Makin's defense of Aristotle. Makin (1999, p. 114)) claims that 'the argument for [2] [the converse principle] is invalid, though plausible' and that although '[2] appears to be false', 'the argument provided for [2] explains why Aristotle might nevertheless have asserted it.'[15] Makin's reconstruction of Aristotle's argument for the converse principle is highly conjectural and the error that he attributes to Aristotle strikes me as egregious, no matter how prettily Makin may dress it up. But no matter, for, by Makin's own lights, the

[15]A similar line of argument is considered at pp. 93-4 of Makin 2006.

second principle is open to obvious counter-example and so it is hard to see how Aristotle could have been taken in by this argument, even if it had in fact occurred to him.

§4. Ways of Being Possible

I wish to propose an alternative solution to the exegetical problem. Rather than supposing, with Brennan, that Aristotle did not even intend to formulate a modal thesis and rather than supposing, with Makin, that he did but was taken in by a fallacious argument for its truth, I shall suggest that he was indeed attempting to formulate a modal thesis, and a correct one at that, but that he was using the modal locutions by which the thesis was formulated in a somewhat peculiar, though not altogether unnatural, way.

This peculiar use is may be understood by means of an analogy with tense. Consider the following sentence (similar remarks apply to the Greek):

(1) In the past, Pete was drunk

Regimenting in a familiar way, we obtain:

(1′) PPp

thinking of 'Pete was drunk' as the result of applying the past-tense operator P to p = 'Pete is drunk'.

Suppose now we subject the operator P to the standard truth-conditions:

(P) PA is true at t iff A is true at some time $u < t$

We then obtain that (1′) above is true at t iff P[Pete is drunk] is true at some time $u < t$ (by (P)), which holds iff Pete is drunk at u' for some times u' and u for which $u' < u < t$ (by (P) again). But this is not correct since, if there is only one time earlier than t, then (1) will still be true even though the truth-conditions are not satisfied.

To make the point more vivid, consider the sentence:

(2) In the past, Pat was drunk and Quentin was drunk.

We may regiment this as:

(2′) P(Pp & Pq).

Using the truth-conditions for P, we then obtain that (2′) is true t iff P[Pete is drunk] is true at u and P[Quentin is drunk] is true at u for some u < t, which holds iff 'Pete is drunk' is true at u′ and 'Quentin is drunk' is true at u′′ for some u, u′ and u′′ for which u′ < u, u′′ < u, and u < t. But again these truth-conditions are incorrect, since they do not require that Pete should be drunk at the same past time as Quentin.

The following question now arises. Is there some way of accepting the standard syntactic regimentation of (1) and (2) as (1′) and (2′) respectively and yet modifying the semantic regimentation of PAST in (P) so that the truth-conditions for (1) and (2) are correct? Here is one possible response. The standard clause (P) is correct when t is the present time t_0 but needs to be modified when t is past (< t_0) or future (> t_0). In that case, the tense operator is simply anaphorically frozen, so to speak, to the time t. We are thereby led to the following modified clause for PAST:

(P′) PA is true at t iff either (i) t = t_0 and A is true at some time u < t

or (ii) t < t_0 and A is true at t.

This clause will then result in the correct predictions. For P(P[Pete is drunk] & P[Quentin is drunk]), for example, will be true at t_0 iff P[Pete is drunk] and P[Quentin is drunk] are both true at u for some u < t_0 (by (P′)(i)), which holds iff 'Pete is drunk' at u and 'Quentin is drunk' at u for some $u < t_0$ (by (P′)(ii)).

However, even this is not quite right. For let us bring NOW into the picture. The natural clause for this operator (at least when t is the present time t_0) is:

(N) NA is true at t iff A is true at t.

But then NPp (Now, Pete was drunk) is true at the present time t_0 iff P[Pete is drunk] is true at t_0, which holds iff 'Pete is drunk' is true at some time $u < t_0$ (by (P′)). So 'Now, Pete was drunk' will have the same truth-conditions as 'Pete was drunk'; and yet surely we want 'Now, Pete was drunk' to be false (or, at least, not true).

To get round this difficulty, it looks as if we must distinguish two difference roles the present time t_0 can play: one is *as* the present; and the other is as a possible *alternative* to the present (but which happens to coincide with the present). The first role is invoked when an unembedded tense operator is evaluated at the time of utterance. The second role is invoked when an embedded tensed operator is evaluated at a time that has been thrown up through the evaluation of another tensed operator within which it has been embedded. Normally, this time will be different from the present but it might, as with the evaluation of NOW, be the same as the present.

Let us use the symbol '●' in conjunction with the designation of a time to indicate that the time is being considered as the present; and let us use '○' in conjunction with the designation of a time to indicate that the time is being considered as an alternative to the present. We then have the following clauses for PAST and NOW:

(P●) PA is true at t,● iff $t = t_0$ and A is true at u,○ for some $u < t$

(P○) PA is true at t,○ iff $t < t_0$ and A is true at t,○

(N●) NA is true at t,● iff $t = t_0$ and A is true at t,○

(N○) NA is true at t,○ iff $t = t_0$ and A is true at t,○

When a tensed statement is evaluated at the present time, considered as the present, we look to se if the component clause is true at an 'alternative' time suitably related to the present time but, when the tensed sentence is evaluated at an alternative time, we look to see whether that time already is suitably related to the present time and is one at which the component clause is true. This then gives the desired result that NP[Pete is drunk]] is not true at t_0. For the truth of this sentence would require that P[Pete is drunk] be true at t_0,○, that is, at t_0 considered as an alternative to the present - which is impossible given that $t_0 \nless t_0$.

POSSIBLY behaves in a similar way to PAST. Consider the following modal analogue to (1):

(3) Possibly, Pam might be drunk

Regimenting in a familiar way, we obtain:

(3′) $\Diamond\Diamond p$

thinking of 'Pam might be drunk' as the result of applying the possibility operator $\Diamond$ to 'Pam is drunk'.

Suppose now we subject $\Diamond$ to the standard truth-conditions:

($\Diamond$) $\Diamond$A is true at w iff A is true at some world v accessible from w.

We then obtain that (3′) is true at w iff 'Pam is drunk' is true at some world accessible from a world accessible from w; and yet surely we want (3), like the simple 'Pam might be drunk', to be true at w iff 'Pam is drunk' is true at some world accessible from w.

Again, the discrepancy in truth-conditions comes out more vividly if we consider the modal analogue to (2):

(4) Possibly, Pam might be drunk and Quentin might be drunk

which gets regimented as:

(4′) $\Diamond(\Diamond p \,\&\, \Diamond q)$.

(4′) will be true at a world w iff $\Diamond p$ and $\Diamond q$ are true at some world v accessible from w, which holds iff p is true at some world accessible from u and M is true at some world accessible from u for some world u accessible from w. But again, the truth-conditions are incorrect since what we want is that both p and q should be true at some world accessible from w.

The semantic regimentation required to obtain the desired results is the same as before. Just as we need to distinguish between a time considered as present and considered as an alternative to the present, so we need to distinguish between a world considered as actual or considered as possible, that is, as an alternative to the actual; and just as a past tensed statement is subject to the standard semantics when evaluated at a time considered as present but to a non-standard 'freeze' semantics when evaluated at a time considered as an alternative to the present, so a possibility statement is subject to the standard semantics when evaluated at a world considered as actual but to a non-standard freeze semantics when evaluated at a world considered as possible.

We therefore arrive at the following clauses for '$\Diamond$':

($\Diamond\bullet$) $\Diamond A$ is true at $w,\bullet$ iff $w = w_0$ and A is true at $v,\circ$ for some world v;

($\Diamond\circ$) $\Diamond A$ is true at $w,\circ$ iff A is true at $w,\circ$.

And there will, of course, be corresponding clauses for $\Box$:

($\Box\bullet$) $\Box A$ is true at $w,\bullet$ iff $w = w_0$ and A is true at $v,\circ$ for all worlds v;

(□○) □A is true at w,○ iff A is true at w,○.

In stating these clauses, I have presupposed the semantic framework of S5 in which all worlds are taken to be accessible from one another and there is therefore no need to indicate that the world v in the first clause is accessible from w or that the world w in the second clause is accessible from the actual world.[16]

We see that there are two different ways in which a possibility-statement ◊A might be true at a world. In the first case, the statement is rendered true *in* the world, that is, the world is the *locus* of the possibility. This is the standard way for a possibility statement ◊A to be true at a world w; there is some alternative world v at which A is true. In the second case, the statement is rendered true *by* the world, that is, the world bears *witness* to the possibility. This is the non-standard anaphoric way for a possibility statement ◊A to be true at the world w; w is an alternative to the actual world at which A itself is true. In considering a world as actual, we are taking it to be the locus of possibility; and in considering a world as possible, we are taking it to be a witness of possibility. Statements of necessity are not so naturally interpreted in this way, since the truth of A at w does not bear witness to the truth of □A at w. But if we understand □A as ¬◊¬A, then we can think of the truth of A at w as witnessing the failure of w to bear witness

[16]In the formal development of the semantics, one should take a model (W, R, φ, w_0) to be equipped with an actual world w_0. Truth in a model will then amount to truth in the model at w_0 considered as actual and validity will be truth in all models.

The present semantics should not be confused with the two-dimensional semantics advocated by Kamp 1971, Chalmers 1996, Stalnaker 2001 and others. For us, there are two ways to evaluate a sentence at a given world, depending upon whether it is considered as actual or as possible. Thus even if we were given a double index (w, v), we would still not know how to evaluate a sentence at the index until we knew which of the component worlds w or v to take into account. Another difference, of course, is that we give a non-standard anaphoric clause for the evaluation of modal (or tensed) sentences at worlds (or times) considered as possible (or as alternatives to the present).

to the truth of $\Diamond\neg A$.

We have so far not considered constructions corresponding to Aristotle's 'when ... then must ...'. The tense-logical analogue of such a construction would be 'whenever ... then ...' (with quantification over times replacing quantification over worlds). Consider now the sentence:

(5) Whenever Pam was drunk Quentin was drunk

The obvious regimentation of this sentence is:

(5′) W(Pp, Pq),

for W the whenever-operator.[17] The standard semantic clause for WHENEVER is:

W(A, B) is true at t iff B is true at any time u at which A is true.

But if we combine the standard clause for WHENEVER with the standard clause for PAST, we do not obtain the correct truth-conditions for (5). For (5) will then turn out to be true at a given time iff any time at which there is an earlier time at which Pam is drunk is a time at which there is an earlier time at which Quentin is drunk. But what we want is that any time, earlier than the given time, at which Pam is drunk is a time at which Quentin is drunk.

However, if we employ the 'freeze' semantics, then we do obtain the required result. For let us adopt the following clause for WHENEVER:

(W●) W(A, B) is true at t,● iff $t = t_0$ and B is true at any u,○ at which A is true

(W○) W(A, B) is true at t,○ iff B is true at any u,○ at which A is true.

[17] Two referees for the journal independently suggested that (5) be regimented as PW(p, q). But this does not deliver the correct truth-conditions, at least under the standard clause for P, and the strategy does not, in any case, work for such sentences as 'whenever Pam was or will be drunk Quentin was or will be drunk'.

Then W(Pp, Pq) will be true at t_0 considered as present if Pq is true at any time considered as an alternative to the present at which Pp is true. But Pp will be true at a time considered as an alternative to the present iff p is true at that time and the time is earlier than the present time; and similarly for Pq. So the truth-condition for the truth of W(Pp, Pq) at t_0 is that q should be true at any time earlier than the present time at which p is true - which is what we required.

The modal analogue:

(6) when Pam might be drunk then, necessarily, Quentin might be drunk

of (5) will be assigned similar truth-conditions. Upon assigning 'necessarily' wide scope, we obtain the following regimentation:

(6′) $\Box(\Diamond p \supset \Diamond q)$.

And applying the freeze semantics for $\Box$ and $\Diamond$ then give us that $\Box(\Diamond p \supset \Diamond q)$ is true just in case q is true at any world at which p is true.

What I would now like to suggest is that we see Aristotle in the cited passage from Theta.4 as implicitly adopting - and perhaps even, to some extent, as explicitly proposing- the previous freeze semantics for the modal operators. The main argument *against* such a view is that sentences such as (6) - or their Greek equivalents - are not naturally read in conformity with the semantics. There is a striking difference between (6) and its tense-logical analogue (5) in this regard. For (5) *is* most naturally read in conformity with the semantics - for it to be true that whenever Pam was drunk Quentin was drunk is for every past time at which Pam is drunk to be a past time at which Quentin is drunk; and the reading in conformity with the standard semantics, under which there may be past times at which Pam is drunk but Quentin is not, is strained in the extreme. On the other hand, (6) is most naturally read in conformity with the standard semantics and it is only with the utmost strain that we can get a reading in conformity with the freeze semantics.

It is not clear to me how seriously to take this piece of counter-evidence. For one thing, just as it is possible to massage (5) so as to get a reading in conformity with the standard semantics, it is possible to massage (6) so as to get a reading in conformity with the freeze semantics. For to say that when Pam might be drunk then, necessarily, Quentin might be drunk is to say that in any possible circumstance when Pam might be drunk then Quentin might be drunk, which is just to say that in any possible circumstance when Pam is drunk then Quentin is drunk.[18]

But more importantly, it is not clear that Aristotle is concerned to state logical principles that are valid in ordinary Greek. Any systematic attempt to state logical principles is likely to involve some deviation from ordinary use. For what we do, for logical purposes, is to pick up on certain critical features in our ordinary use of logical locutions and then attempt to ascertain what the inferential and semantical behavior of locutions with those features would be, even though other aspects of our ordinary use may have been ignored. Sometimes, the deviation from ordinary use may be drastic. Thus it is that we find that modus ponens is not valid for ordinary 'if-then' or that Prior's tense logic is unable to account for the semantic behavior of simple embeddings like (1) or (2) or (5) above. But we do not hold it against Prior that he cannot deliver the correct interpretation of these sentences, as long as he has genuinely picked up on a logically

[18]Waterlow (1982, p. 159) has suggested that Aristotle's modalities are always time-relative. Certainly, this is how they appear in the passage under discussion; and this perhaps makes it even more plausible that he would have wanted to use the modal and tense-logical locutions in an analogous way.

significant aspect of our use of the tenses; and no more should we hold it against Aristotle if, in his picking up on another logical aspect of our use of modal or tensed discourse, we find that it is also at odds with ordinary use.[19]

One central argument in *favor* of the freeze interpretation is that it validates Aristotle's two principles and thereby explains why he would have wanted to accept them.[20] But it also accounts for his brief and enigmatic justification of the second principle:

for to say that B must be possible if A is possible means that if A is both at the same time when and in the way in which it was supposed capable of being, B also must then and in that way be' (1047b27-30).

He is here asserting some kind of equivalence between:

(1) B must be possible if A is possible (which we have rendered as $\Box(\Diamond A \supset \Diamond B)$)

and:

(2) if A is both at the same time when and in the way in which it was supposed capable of being, B also must then and in that way be (rendered as $\Box(A \supset B)$).

As Makin (2006, pp. 90-1) has pointed out, there appear to be two ways to understand (2):

(2)(i) $\Diamond B$ must obtain at any world at which $\Diamond A$ obtains.

(2)(ii) B must obtain at any world at which A obtains

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[19] According to Morison (2011), Aristotle's logical project is to give a semantical *description* of the various valid forms of argument and not a *presentation* of those arguments in ordinary Greek or even in a stylized version of Greek. This then makes it even more likely that there might be some distance between the ordinary presentation of an argument and the intended semantical description.

[20] For $\Box(\Diamond A \supset \Diamond B)$ will be true at the actual world w_0 considered as actual iff $\Diamond A \supset \Diamond B$ is true at all worlds v considered as possible, which holds iff $A \supset B$ is true at all worlds considered as possible, which is to say that $\Box(A \supset B)$ is true at w_0 considered as actual.

However, under the first reading, we do not obtain the desired implication of $\Box(A \supset B)$ while, under the second, we do not obtain the desired equivalence with $\Box(\Diamond A \supset \Diamond B)$.

The problem would be solved if (2)(i) was obviously equivalent to (2)(ii). For it could be supposed that Aristotle was implicitly appealing to the obvious equivalence of (2) to (2)(i) and then presupposing the obvious equivalence of (2)(i) to (2)(ii) in stating the equivalence of (2) to (2)(ii). But how could Aristotle plausibly be presupposing the equivalence of (2)(i) to (2)(ii) - let alone the obvious equivalence - given that they are obviously not equivalent?

It is here that the freeze interpretation can help (and I do not see how else to solve the problem). For Aristotle may be using the various modal locutions in conformity with such an interpretation. Indeed, in proposing the equivalence of (2) (or (2)(i)) to (2)(ii), he may not merely be presupposing the interpretation but stipulating that it, or certain aspects of it, should hold. For he may be aware that there is a problem over the interpretation of embedded possibilities, even given his previous criterion for possibility at Theta.3, 1047a24-29; and so he may here be making a proposal as to how they are to be understood.

Either way, once the present interpretation is adopted, $\Diamond B$'s obtaining at a world w will evidently be equivalent to B's obtaining at w in any context in which the world is regarded as 'possible' rather than 'actual'. But in the current modal context (provided by 'if ...must') the world thrown up by the evaluation *is* to be regarded as possible; and so the equivalence of (2)(i) to (2)(ii) will be immediately forthcoming. (There remains the problem, which we shall later consider, of why Aristotle did not provide a similar simple justification for the first principle).

If the present interpretation can be sustained, then we see, contra Brennan (1994), that both of Aristotle's principles may be correctly formalized as purely modal principles, without any explicit appeal to worlds. We may agree that the formalization (*) (from §3 of Brennan 1994) embodies the correct truth-conditions for the possibility-clause. Indeed, it is hard to see how it could fail to do so, given the equivalence between the necessity- and possibility-clauses and given Aristotle's explicit remarks to that effect. But the possibility-clause is itself no more a statement of these truth-conditions than is the necessity-clause. In both cases, these conditions are derived from an underlying semantics for the modal operators; and it is only by supposing a difference in the logical form of the principles and the statement of the truth-conditions that we can make any sense of how such a semantical explanation might proceed.

We see, contra Nortmann, that Aristotle's entailments need not have an implausibly strong interpretation. Indeed, in application to non-modal statements their interpretation can be as broad or as narrow as we like. Nor is there any need to restrict the application of the second principle to non-modal statements or to treat its justification as intolerably indirect.

Finally, we see, contra Makin, that Aristotle's converse principle is both correct and correctly argued for. We can perhaps accuse Aristotle of using the idiom of possibility in a somewhat unusual way. But, as we shall see, even this charge against him may possibly be mitigated once we take into account his larger purpose in stating the two principles.

§5. Some Semantic and Deductive Implications

Aristotle's endorsement of the converse principle appears to have unacceptable semantical and deductive consequences. There is a standard interpretation of the modal operators in terms of possible worlds and, under this interpretation, the converse principle appears to commit Aristotle to treating the actual world as the one and only possible world.

Should we assume that Aristotle's position commits him to rejecting the standard semantics? And if we do, then which aspect of the semantics should it lead him to reject? Or again, there is a derivation of the Megarian thesis from the converse principle. Should we assume that he would reject the derivation? And if we do, then which step of the derivation should we take him to reject? Although the focus of recent discussion has not been on these questions, an answer to them is required if we are to have a full understanding of Aristotle's position.

It may seem evident on the basis of the semantics that we have provided on Aristotle's behalf that his position would require him to reject the standard semantics for modality. But there are two aspects of the standard semantics that need to be distinguished. On the one hand, there is its general structure, as given by the formal conditions on the accessibility relation and the form of the semantic clauses for the modal operators. On the other hand, there is the specific understanding of the 'worlds' of the semantics as possible worlds and of the accessibility-relation as the relation that holds between a given world and the worlds that are possible from it.

We certainly should not take Aristotle to be subscribing to both aspects of the semantics. But we can reasonably take him to be subscribing to the first as long as it is divorced from the second. For there is a way of understanding the freeze semantics as a form of the standard semantics. We take a 'world' or 'index' of the semantics to be an ordered pair (w, π) consisting of a possible world w in the ordinary sense of the term and an 'aspect' π (which is either $\bullet$ or $\circ$); and we take the accessibility-relation to be a relation that holds between two such ordered pairs, (w, π_1) and (v, π_2), when either π_1 is $\bullet$ and π_2 is $\circ$ (in the shift from a world considered as actual) or when π_1 is $\circ$, π_2 is $\circ$, and $w = v$ (in the shift from the world considered as possible). It is then readily verified that the clauses that we gave before for the modal operators translate into the standard clauses; possibility is truth in some accessible 'world'. It is also readily verified that the accessibility relation will conform to the condition that was seen to correspond to axiom P_2, viz. that every accessible 'world' w is accessible from w and is the only world accessible from w. For it is only 'worlds' of the form (w, ○) that are accessible from some world; and any such 'world', on the present understanding of the accessibility-relation, will be the one and only world accessible from itself .

It is this feature of the semantics that appeared to commit Aristotle to the view that every possible world is Megarian, that is, one in which the possible and the actual coincide. But it only has this metaphysical consequence under the standard construal of the worlds of the semantics as possible worlds. If they are taken as worlds under an 'aspect', then there is no metaphysical mystery as to why an accessible 'world' of the semantics should only be accessible from itself. For such a 'world' is regarded as a witness rather than as a locus of possibilities; and so it is only the 'world' itself that is relevant to the possibilities that it verifies. Of course, the world is also the *locus* of possibilities and, indeed, of possibilities that are not actually realized; it is just that the semantics does not take these possibilities into account when the world is considered as possible rather than as actual.

There is also no mystery as to why the actual 'world' should *not* be accessible from itself. For this only has the metaphysical consequence that the actual world is not genuinely possible under the standard construal of the 'worlds' and the accessibility-relation. Take the actual 'world' of the semantics to be the actual world considered as actual and take the accessibility-relation to encode the aspect under which a world is being considered; and we see that the failure of reflexivity simply arises from our regarding the actual world under two different aspects once we make the shift from considering it as actual to considering it as possible.

Similar palliative remarks may be made in regard to the deductive consequences of the principle. Just as there is no need for Aristotle to reject the standard semantic framework, so there is no need for him to reject the standard principles of modal reasoning as embodied in the minimal system K. Given that the quasi-Megarian thesis $\Box(\Box A \equiv \Diamond A)$ follows from P_2 within K, he should accept the quasi-Megarian thesis.

But this is not to say that he should accept the Megarian thesis, $\Box A \equiv \Diamond A$, for he may reject the inference of this thesis from the quasi-Megarian thesis. However, this is only on the cards if he is willing to reject the T-principle $\Box A \supset A$. Indeed, he is required to reject it in a strong way, not merely as an axiom but as a rule of proof, since $\Box(\Box A \equiv \Diamond A)$ will be a theorem (or 'valid') even though $\Box A \equiv \Diamond A$ is not.

Again, once we understand the underlying interpretation of $\Box$ there is no mystery as to why he should reject the T-principle. For in asserting a statement of the form $\Box A$, we are asserting that A holds in each possible world considered as possible while, in asserting A itself, we are only asserting that it holds in the actual world considered as actual. But from the fact that A holds in the actual world considered as possible it does not follow that it holds in the actual world considered as actual.

Still, it might be thought odd that Aristotle would be willing to countenance a notion of necessity that did not conform to the T-principle. For is it not built into the very notion of necessity that it should imply truth? It is, of course, very likely that Aristotle was not aware that it was only by rejecting the T-principle that one could avoid a commitment to the Megarian thesis. The derivation of the Megarian thesis from the T-principle is far from obvious (it requires making a modal substitution in the T-axiom, see theorem 4 of the appendix). Moreover, none of Aristotle's commentators seem to have appreciated that the one was derivable from the other; and one hopes that it is not uncharitable to Aristotle to suppose that he might have possessed no more logical acumen in this respect than his commentators.

It is also conceivable, if the counter-example to the T-principle had been presented to him, that he would regarded it with equanimity. After all, we ourselves face a similar situation in regard to the putative counter-examples to modus ponens. Consider McGee's (1985) example concerning the election of 1980:

A Republican wins the election

If a Republican wins the election, then if it is not Reagan who wins then it will be Anderson

∴ If it is not Reagan who wins then it will be Anderson.

We are inclined to think that the inference is an instance of modus ponens (from A and 'if A then B' to B) with true premises and a false conclusion. But rather than reject the putative counter-example, we might simply regard it as an oddity arising from the presence of 'side effects' within the embedded conditional and take our very clear intuition of the validity of modus ponens to concern those cases in which the side-effects are not present. And similarly, one might regard the counter-example to the inference from □A to A as an oddity arising from side-effects within the embedded modality and take our very clear intuition of the validity of the inference to concern those cases in which the side effects are not present.

Indeed, the analogy between the two cases is much closer than this rough comparison might suggest. For suppose that □A constitutes a counter-example to the inference from □A to

A and understand 'if B then C' as the strict implication $\Box(B \supset C)$. Then where $\top$ is any tautology, $\top$ and 'if $\top$ then A' will constitute a counter-example to modus ponens. Conversely, suppose A and 'if A then B' constitute a counter-example to modus ponens. In that case, the inference from 'if A then B' to the material implication $A \supset B$ cannot be valid (since the inference from $A \supset B$ and A to B is valid); and so, as long as 'if A then B' implies $\Box(A \supset B)$ for some suitable notion of necessity $\Box$, the inference from $\Box(A \supset B)$ to $(A \supset B)$ will also not be valid.

There would appear to be an underlying semantic explanation for this close connection between the counter-examples. Under the semantics for the conditional developed in McGee 1985, p. 469, formulas are evaluated at a world relative to a set of hypotheses Γ - or, equivalently, relative to a truth-set V (intuitively, the set of worlds at which all of the hypotheses are true). In evaluating the conditional 'if A then B' relative to a truth-set V, there is then a change in the set as we move from A to B so that B is evaluated relative to the set of worlds of V at which A is true.

But the evaluation of modal formulas under the freeze semantics can be regarded in a similar way. To evaluate a formula A at w,○ - that is, at w considered as actual - is to evaluate it at w relative to the universal (least informative) truth-set W; and to evaluate the formula A at w, ● - that is, at w considered as possible - is to evaluate it at w relative to the singleton (most informative) truth-set {w}. The evaluation of a modal formula $\Box$A at w relative to a truth-set V is then a matter of relativizing the modality to V (only those worlds in V are taken to be possible); and in evaluating the modal formula $\Box$A, we shift from the given truth-set V to those of the form {v} for each v in V. Thus the present relativized semantics seems to be as much a part of our understanding of modality and tense as it is of our understanding of the conditional.

I also believe that Aristotle may have had deeper reasons for wanting to use the modalities in this way. As Brennan (1998, p. 170) has observed, the current passage appears to be essentially concessive in character.[21] Aristotle had previously rejected the Megarian thesis; and now he wants to show how something like it might, all the same, be true. For the principle and its converse provide a context, $\Box(A \supset B)$, in which what is actually the case (A or B) comes to the same thing as what is possibly the case ($\Diamond A$ or $\Diamond B$); one may be substituted for the other salve veritate even though they are not strictly equivalent. Aristotle may even be suggesting that the Megarians might have fallaciously inferred their coming to the same thing in general from their coming to the same thing in this special case.

Of course, Aristotle might have made this dialectical point in even more dramatic form. For he might have remarked that it is necessary that the actual and the possible should be equivalent ($\Box(A \equiv \Diamond A)$), thereby coming as close to a full endorsement of the Megarian thesis as one could reasonably hope to get. The fact that he did not make the point in this more dramatic form is yet further evidence that he did not recognize the validity of the quasi-Megarian thesis or the fact that it was a consequence of the converse principle.[22]

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[21]Nortmann (2006, p. 392) has also picked up on this theme.

[22]The reader might wonder how Aristotle could have implicitly adopted the present interpretation while failing to recognize the validity of the quasi-Megarian thesis $\Box(A \equiv \Diamond A)$. One possible explanation is that he focused on the interpretation of embedded modalities within sentences of the form $\Box(\Diamond A \supset \Diamond B)$ without appreciating the more general implications of such an interpretation. Another is that he was not willing to interpret $\Box$ as the dual $\neg\Diamond\neg$ of $\Diamond$ (though this would still have left him with the validity of $\neg\Diamond\neg(A \equiv \Diamond A)$).

Another motivation relates to the question of embedding and provides a reason for adopting the second principle that is quite independent of any desire he might have had to be concessive towards the Megarian position. Let us say that a formula of propositional modal logic is *non-iterative* if it contains no embedded modalities, that is, no occurrences of one modal operator within the scope of another. A natural philosophical view is that it is only the non-iterative formulas that genuinely express what is or is not possible and that iterative formulas are some kind of artifice whose meaning should be given in terms of its non-iterative equivalents. Under such a view, we should therefore attempt to find a system of modal logic that is *reductive* in the sense of making every iterative formula provably equivalent to a non-iterative formula.

One might reasonably require that such a system not foreclose any of the genuine possibilities. Let $A_1, A_2, \ldots, A_n$ be a sequence of n non-modal formulas, n > 0, any one of which is truth-functionally consistent and any two of which are truth-functionally inconsistent (a simple example is given by the formulas p&q, ~p&q, p & ~q). Since we wish to leave open that each of $A_1, A_2, \ldots, A_n$ should be a possibility, we should therefore require that our system not make the formula:

$$M_n\colon\ \neg(\Diamond A_1 \,\&\, \Diamond A_2 \,\&\, \ldots \,\&\, \Diamond A_n)$$

a theorem for any n = 1, 2, 3, ... We might call such a system *strongly anti-Megarian*, since it rejects not only the Megarian axiom but also any other axiom which sets some finite bound on the number of incompatible possibilities.[23]

It may now be shown that the familiar system S5 (obtained by adding the axiom A ⊃

[23]The idea of such a system goes back to Scroggs (1951) and is used in his well-known characterization of the extensions of S5.

$\Box\Diamond A$ to T) and the Aristotelian system KP_2 (equivalently, KP_1P_2) are the only reductive, strongly anti-Megarian normal extensions of the modal system D. If one wants to eliminate the iterative possibilities and respect the non-iterative possibilities, then these two systems constitute the only plausible options.[24]

Indeed, in certain respects, the reductive procedure embodied in the Aristotelian system KP_2 is superior to that for S5. The procedure for the Aristotelian system essentially works by dropping innermost modalities, while the procedure for S5 essentially works by dropping outermost modalities. Thus $\Box\Diamond A$ will be equivalent to $\Box A$ in KP_2 but to $\Diamond A$ in S5. However, there is a complication in executing the procedure in the case of S5; $\Box(A \supset \Diamond B)$, for example, will be equivalent to $\Diamond A \supset \Diamond B$, rather than to $A \supset \Diamond B$. No such complication arises in the case of KP_2; $\Box(A \supset \Diamond B)$ is simply equivalent to $\Box(A \supset B)$. There is a similar advantage on the semantic side. Suppose that we are evaluating the truth-value of $\Box\Diamond A$ at a given world w_0 using the semantics for S5. This requires that we evaluate $\Diamond A$ at each world w which, in its turn, requires that, for each such world w, we evaluate A at each world v. Thus each world will be 'visited' n + 1 times, for n the number of worlds. In the case of the semantics for KP_2, it will be required, as before, that we evaluate $\Diamond A$ at each world w, but this will then only require, for each such world w, that we evaluate A at that very world. Thus each world will only be visited twice and the redundancy in the semantic evaluation of a formula will be avoided.

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[24] I prove this result in an unpublished textbook on modal logic, jointly authored with Steven Kuhn. The proof was by 'brute force'; I systematically considered how the provable equivalence to non-iterative formulas might be achieved. Tomaz Kowalski, using algebraic techniques, has proved the slightly more general result that there will be four such systems extending K - the two extensions of D and their intersections with the system $K\Box\bot$.

We might conjecture that, in so far as Aristotle might have been inclined to adopt a reductive stance towards modality, he may well have been tempted in the direction of KP_2 rather than S5. We might also note that there are perhaps special reasons for favoring the KP_2 -option in the particular case that he considers. For there are only two non-iterative formulas that might reasonably be taken to be equivalent to $\Box(\Diamond A \supset \Diamond B)$: $\Box(A \supset B)$ and $(\Diamond A \supset \Diamond B)$. But we naturally construe $\Box(\Diamond A \supset \Diamond B)$ as a form of necessary implication; and if this feature of it is to be preserved under the reduction, then we should opt for the Aristotelian equivalent $\Box(A \supset B)$ over the standard S5-equivalent $(\Diamond A \supset \Diamond B)$.

§6. The Argument for the First Principle

We have shown how to reconstruct Aristotle's argument for the converse principle, but it remains to consider his argument for the first principle.

Commentators have been no more impressed with Aristotle's argument for the first principle than with his endorsement of its converse; they have found his argument to be both circular and repetitious.[25] Brennan (1994) has attempted to extenuate the charge of circularity. He writes (p. 161) 'It is in some sense perfectly forgivable that Aristotle's argument in support of it in T1 [1047b16-26] should be circular. There is no more basic principle to which he could appeal, which could still be a principle of *metaphysical* modality'. I suspect that the question of whether or not the modality is metaphysical is irrelevant to the issue. But it is, in any case,

[25] See, for example, the judgement of Burnyeat et al. (1984, p. 110) that 'the whole argument is of little value as a *proof* of "what follows from the possible is itself possible", since all it does is to derive the truth of this from the principle "nothing impossible follows from what is possible"'

highly implausible that Aristotle is not here attempting to provide an *argument*, that is, a non-circular justification, in support of the principle. Nor does it seem correct to suppose that 'there is no more basic principle to which he could appeal'. The principle in its most obvious formulation, that is, as $\Box(A \supset B) \supset \Box(\Diamond A \supset \Diamond B)$, is not plausibly taken to be *axiomatically* basic, since it can be derived from the K4 axiom, $\Box A \supset \Box\Box A$; and the principle in Brennan's preferred formulation can be derived from more basic semantic clauses for '$\Box$' and '$\supset$'. Even if the principle were plausibly taken to be axiomatically basic, as with $\Box(A \supset B) \supset (\Box A \supset \Box B)$, there would still be the possibility of deriving it from natural deduction rules for the modal operators, just as axiomatically basic tautologies can be derived from natural deduction rules for the truth-functional connectives.

Indeed, I suspect that this is no idle possibility in the case at hand. It is characteristic of natural deduction derivations that they often involve repeated occurrences of the very same formula, the difference in these occurrences lying in the suppositions upon which the formula rests; and what principally accounts for the apparently repetitious character of Aristotle's argument, I believe, is its shifting suppositional structure. Thus it is only by making this structure explicit within something like a system of natural deduction that we can properly understand what the argument is.

To this end, let me first present a formal reconstruction of Aristotle's argument within a system of natural deduction.[26] I shall then try to indicate how the argument Aristotle actually

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[26]The reconstruction employs a rule of reductio, in keeping with the accounts of Aristotle's theory of the syllogism developed in Corcoran 1972 and Smiley 1973; and it also employs a rule of $\Diamond$-introduction. Malink & Rosen 2011 details Aristotle's extensive use of this rule throughout the whole of his corpus.

gives corresponds to the formal reconstruction. We shall find it most helpful to use a Fitch-style system of 'subordinate' proof. The exact formal details do not matter, but it will be important to distinguish between two kinds of supposition. *Straight* supposition corresponds to supposing that a statement is true in the actual world, while *modal* supposition corresponds to supposing that it is true in some possible world. We use the notation:

$\mid \underline{A}$ (Straight Supposition) $\quad$ ${}_{\Diamond}\mid \underline{A}$ (Modal Supposition)

$\mid$ $\quad$ $\mid$

to distinguish between the two. 'Iteration' into straight supposition is allowed:

$A \; \ldots \; \mid \underline{B}$

$\mid$

$\mid A$ (Iteration)

while iteration into modal supposition is not. We need, however, a form of □-Elimination which allows us to transfer A from □A into a modal supposition:

$\Box A \; \ldots \; {}_{\Diamond}\mid \underline{B}$

$\mid$

$\mid A$ (□-Elimination)

A can here be transferred across several modal suppositions (this corresponds to adopting the K4 axiom $\Box A \supset \Box\Box A$). We also need forms of □- and ◊-Introduction:

${}_{\Diamond}\mid \underline{A}$

$\mid$

$\mid B$

$\Box(A \supset B)$ (□-Introduction)

$$
\begin{array}{l}
\Diamond A \\
\quad {}_{\Diamond}\mid \underline{A} \\
\quad\;\; \mid \\
\quad\;\; \mid B \\
\Diamond B \qquad\qquad\qquad\qquad (\Diamond\text{-Introduction})
\end{array}
$$

Finally, for the truth-functional connectives ⊃ and ~, we shall use modus ponens, conditional proof and forms of modus tollens and reductio.

Some of these rules can be derived from more basic rules, but the ones we give are the ones best suited to making the comparison with Aristotle. It is worth noting that we have no rules corresponding to the T-axiom $\Box A \supset A$. Indeed, our rules can all be seen to be valid within the system K4.

Here now is the formal reconstruction of Aristotle's argument:

$$
\begin{array}{lllllll}
1. & \mid \underline{\Box(A \supset B)} & & & & & \text{(Straight Supposition)} \\
2. & \mid & \mid \underline{\sim\Box(\Diamond A \supset \Diamond B)} & & & & \text{(Straight Supposition)} \\
3. & \mid & \mid & {}_{\Diamond}\mid \underline{\sim\Diamond B} & & & \text{(Modal Supposition)} \\
4. & \mid & \mid & \mid & \mid \underline{\Diamond A} & & \text{(Straight Supposition)} \\
5. & \mid & \mid & \mid & \mid & {}_{\Diamond}\mid \underline{A} & \text{(Modal Supposition)} \\
6. & \mid & \mid & \mid & \mid & \mid A \supset B & (\Box\text{-Elimination from 1)} \\
7. & \mid & \mid & \mid & \mid & \mid B & \text{(Modus Ponens from 5\& 6)} \\
8. & \mid & \mid & \mid & \mid \Diamond B & & (\Diamond\text{-Introduction from 7)}
\end{array}
$$

9.	$\mid \quad \mid \quad \mid \quad \mid \sim\Diamond B$	(Iteration from 3)
10.	$\mid \quad \mid \quad \mid \sim\Diamond A$	(Reductio from 4, 8 & 9)
11.	$\mid \quad \mid \Box(\sim\Diamond B \supset \sim\Diamond A)$	($\Box$-Introduction from 3 & 10)
12.	$\mid \quad \mid \quad {}_{\Diamond}\mid \underline{\Diamond A}$	(Modal Supposition)
13.	$\mid \quad \mid \quad \mid \sim\Diamond B \supset \sim\Diamond A$	($\Box$-Elimination from 11)
14.	$\mid \quad \mid \quad \mid \Diamond B$	(Modus Tollens from 12 & 13)
15.	$\mid \quad \mid \Box(\Diamond A \supset \Diamond B)$	($\Box$-Introduction from 12 & 14)
16.	$\mid \Box(\Diamond A \supset \Diamond B)$	(Reductio from 2 & 15).

The following commentary indicates how the various steps in Aristotle's argument correspond to each line of the formal reconstruction[27]:

a. '… if, when A is B must be, then, when A is possible B also must be possible.' A statement of the principle. Its antecedent becomes an implicit supposition (line 1 above) of the argument to follow.

b. 'for if B need not be possible, there is nothing to prevent its not being possible'. The claim is preparatory to a derivation of a contradiction from the supposition of $\sim\Box(\Diamond A \supset \Diamond B)$ (line 2). Aristotle is claiming that if $\Box(\Diamond A \supset \Diamond B)$ does not hold, then it should not be possible to derive a contradiction from its not holding. Thus if we *can* derive a contradiction from $\Box(\Diamond A \supset \Diamond B)$ not

[27] I have followed Burnyeat et al. (1984, 109-10), in supposing that ' let A be supposed possible' at 1047b17-18 is a corruption of 'let it [B] be supposed impossible' (the Greek is similar) and that 'let it be impossible' at 1047b20 is a misplaced correction to 1047b17-18. I should emphasize that these emendations were proposed quite independently of anything like the present reconstruction. See p. 121 of Makin 1999 and also the commentary at pp. 94-5 and the notes to 47b17-18, 47b21 at p. 271 of Makin 2006 for further discussion.

holding, then $\Box(\Diamond A \supset \Diamond B)$ must hold.

c. 'Now let B be supposed impossible'. Corresponds to the modal supposition of $\sim\Diamond B$ at line 3.

d. 'Then when A is possible'. Corresponds to the straight supposition of $\Diamond A$ at line 4.

e. 'If A were supposed to be'. Corresponds to the modal supposition of A at line 5.

f. 'Nothing impossible would follow'. A reference to the $\Diamond$-Introduction rule, to be invoked at line 8.

g. 'And then B must be'. Corresponds to the derivation of B at lines 6 and 7.

h. 'But we supposed it [B] to be impossible.' Corresponds to line 9.

i. 'If, then, B is impossible, A also must be.' Corresponds to the derivation of $\Box(\sim\Diamond B \supset\sim\Diamond A)$ at lines 10 and 11.

j. 'But the first [B] was supposed impossible; therefore the second [A] is also.' A forward reference to the inference of $\sim\Diamond B \supset \sim\Diamond A$ at line 13.

k. 'If, then, A were possible' Corresponds to the modal supposition of $\Diamond A$ at line 12.

l. 'Then B will also be possible'. Corresponds to the derivation of $\Diamond B$ at lines 14.

m. 'if they were so related that if A is B must be'. Reminder that the supposition of $\Box(A \supset B)$ at line 1 is in play.

n. 'If, then, A and B being thus related [$\Box(A \supset B)$], B is not possible in this way [$\sim\Box(\Diamond A \supset \Diamond B)$], A and B will also not be related as was supposed [$\sim\Box(A \supset B)$].' Given the supposition of $\Box(A \supset B)$ and of $\sim\Box(\Diamond A \supset \Diamond B)$, we can infer a contradiction (line 15) and hence show that $\Box(A \supset B)$ is false. It is this that prevents $\sim\Box(\Diamond A \supset \Diamond B)$ from holding and thereby justifies the inference of

$\Box(\Diamond A \supset \Diamond B)$ by reductio at line 16.[28]

The correspondence between the formal and the textual argument is remarkably close; and we should note that the argument, as so reconstructed, is neither circular nor repetitious. It *is* inelegant, since it makes two unnecessary appeals to proof by reductio. But one should perhaps see Aristotle here as engaged in something like the strategy involved in proofs by semantic tableaux, in which one attempts to establish a proposition by systematically ruling out the different ways in which it might be false.[29]

We are now in a position to see why Aristotle might have given such different arguments in defence of his two principles. His argument for the first principle is relatively uncontentious; it appeals to no modal assumptions beyond those of the modal system K4. His argument for the converse principle on the other hand, is more problematic and appeals, in effect, to some special assumptions concerning the semantic behavior of possibility-statements within modal contexts.

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[28]Malink & Rosen (2011) present a somewhat different reconstruction of the argument, though from the same basic materials.

[29]Makin's reconstruction (Makin 1999, p. 122 and Makin 2006, 95-6) of the first part of Aristotle's argument, up to line 10 above, is in some ways quite close to my own. However, his reconstruction of the latter part of the argument (from lines 11 to 16) strikes me as bizarre, since it makes no sense for Aristotle to consider different ways in which the contradiction derived at lines 8 and 9 might be resolved. I am also inclined to think that Aristotle's argument is best regarded as a purely *formal* argument and that Makin is mistaken in thinking of it as involving an appeal to semantical considerations.

The present reconstruction of Aristotle's argument serves to illustrate, in miniature, a hermeneutic principle of which I am perhaps inordinately fond. It is that it is often only by thinking through a philosopher's ideas for oneself that one can understand what he or she is saying. I came to the present reconstruction by attempting to work out for myself how a natural deduction style of argument for the second principle might go; and I doubt that it could have been discerned simply from an examination of the text.

Aristotle could have appealed to these more problematic assumptions in support of the first principle but he would clearly have wished to provide a less contentious argument, given that such an argument was to be had.[30]

Appendix

We establish various results on what can and cannot be derived from P_2 or its variants.

Theorem 1 The following are theorems of KP_2:

(i) $\Diamond A \supset \Diamond\Diamond A$

(ii) $\Box(A \supset \Diamond A)$

(iii) $\Box(\Diamond A \supset A)$

(iv) $\Diamond\Diamond A \supset \Diamond A$

Proof (i) Substituting ~B for B in P_2, we have that $\Box(\Diamond A \supset \Diamond{\sim}B) \supset \Box(A \supset {\sim}B)$ is a theorem (of KP_2). Taking the contrapositive and using K, we have that:

(a) $\Diamond(A \ \& \ B) \supset \Diamond(\Diamond A \ \& \ \Box B)$ is a theorem.

Letting B be A and using K, we then have that $\Diamond A \supset \Diamond\Diamond A$ is a theorem.

(ii) Substituting $\Diamond A$ for B in P_2, we have that $\Box(\Diamond A \supset \Diamond\Diamond A) \supset \Box(A \supset \Diamond A)$ is a theorem. Applying the rule of necessitation to (i) and using Modus Ponens, we therefore have that $\Box(A \supset \Diamond A)$ is a theorem.

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[30] This paper was presented at a one-day conference on Aristotle, held during the Summer of 2004 at Edinburgh University and organized by Theodore Scaltsas and Anna Marmadoro. I am grateful to the participants of the conference, Patricia Curd, Ulrich Nortmann, Jacob Rosen, and Nicholas White for helpful comments. I owe a special debt of thanks to the editor and referees for Mind. I have never before received such an abundance of good comments from a journal and the paper is much better as a result. Further discussion of some of the formal issues arising from the paper can be found in the blog for March 5, 2005 at tar.weatherson.org.

(iii) Substituting A for B in (a) and using K, we have that:

(b) $\Diamond A \supset \Diamond(\Diamond A \,\&\, \Box A)$

is a theorem. The negation of $\Box(\Diamond A \supset A)$ is provably equivalent in K to $\Diamond({\sim}A \,\&\, \Diamond A)$. Substituting ${\sim}A \,\&\, \Diamond A$ for A in (b), we have that:

(c) $\Diamond({\sim}A \,\&\, \Diamond A) \supset \Diamond(\Diamond({\sim}A \,\&\, \Diamond A) \,\&\, \Box({\sim}A \,\&\, \Diamond A))$

is a theorem and hence, by K (and the rule of necessitation), that

(d) $\Box(\Diamond({\sim}A \,\&\, \Diamond A) \supset \Diamond\Box({\sim}A \,\&\, \Diamond A))$

is a theorem.

From (c) and (d), we obtain that $\Diamond({\sim}A \,\&\, \Diamond A) \supset \Diamond(\Diamond\Box({\sim}A \,\&\, \Diamond A) \,\&\, \Box({\sim}A \,\&\, \Diamond A))$. But $\Diamond\Box({\sim}A \,\&\, \Diamond A) \,\&\, \Box({\sim}A \,\&\, \Diamond A)$ implies $\Diamond\Box{\sim}A$ and $\Box\Diamond A$ in K and hence is refutable in K. It follows that $\Diamond({\sim}A \,\&\, \Diamond A)$ is refutable (in KP_2) and that $\Box(\Diamond A \supset A)$ is therefore a theorem.

(iv) $\Box(\Diamond A \supset A)$ is a theorem by (iii) and so $\Diamond\Diamond A \supset \Diamond A$ is a theorem by K.

The previous derivations of (ii), (iii) and (iv) use the rule of necessitation in application to P_2. It can be shown that the use of the rule is indispensable. For any normal modal system L, let L<A> be the non-normal extension of L obtained by adding the axiom-scheme A to L but not allowing the rule of necessitation to apply to theorems derived with the help of A. We then have:

<u>Theorem 2</u> $\Box(p \supset \Diamond p)$, $\Box(\Diamond p \supset p)$ and $\Diamond\Diamond p \supset \Diamond p$ are not theorems of the non-normal system $K\langle P_2\rangle$.

<u>Proof</u> (Sketch) It can be shown that $A \supset B$ is a theorem of K whenever $\Diamond A \supset \Diamond B$ is a theorem of K. For each consistent formula A_i of K, i = 1, 2, ..., choose a model $M_i = (W_i, R_i, \varphi_i, w_i)$ in which

A_i is true and in such a way that all of the models M_i are disjoint from one another. Choose an element w_0 not in any W_i and let M be the model (W, R, φ, w_0), where:

$W = \{w_0\} \cup \bigcup\{W_i: i = 1, 2, ...\}$,

$R = \{(w_0, w_i): i = 1, 2, ...\} \cup \bigcup\{R_i: i = 1, 2, ...\}$, and

$\varphi = \bigcup\{\varphi_i: i = 1, 2, ...\}$.

Each instance $\Box(\Diamond A \supset \Diamond B) \supset \Box(A \supset B)$ of P_2 is true in M. For suppose $\Box(\Diamond A \supset \Diamond B)$ is true in M. Then $\Diamond A \supset \Diamond B$ is true at each w_i in M and so is a theorem of K. So by the meta-theorem above, $A \supset B$ is a theorem of K; and so $\Box(A \supset B)$ is also be true in M. It follows that each theorem of $K\langle P_2\rangle$ is true in M.

Since $p \,\&\, \neg\Diamond p$ is consistent in K, there is model M_i in which it is true. But $p \,\&\, \neg\Diamond p$ is then true at the world w_i in M and so $\Box(p \supset \Diamond p)$ is not true at the world w_0 of M and hence is not a theorem of $K\langle P_2\rangle$. Similarly for $\Box(\Diamond p \supset p)$ but using now the fact that $\Diamond p \,\&\, \neg p$ is consistent in K and hence is true at some world of M.

The case of $\Diamond\Diamond p \supset \Diamond p$ is trickier. We redo the previous proof but using models for the consistent formulas of $K\langle\Diamond\top\rangle$ instead of models for the consistent formulas of K. The resulting model M can then be shown in the same way as before to verify the theorems of $K\langle P_2\rangle$. Now $\Diamond\Box\bot$ is a consistent formula of $K\langle\Diamond\top\rangle$ and so there is a world w_i of M at which it is true. But it is then clear that $\Diamond\Diamond\Box\bot$ is true at the world w_0 of M but that $\Diamond\Box\bot$ is not.

We needed the rule of necessitation to derive the results (ii) - (iv) within $K\langle P_2\rangle$. But we can dispense with the rule if we can make use of the system T or the system K4 in place of K.

Theorem 3 (i) - (iv) above are theorems in the non-normal extension T<P_2> of T and in the non-normal extension K4<P_2> of K4.

Proof The derivation of (i) works within K<P_2> and so will also work within T<P_2> and K4<P_2>. (iv) can be derived form (iii) just using K-theorems. This leaves (ii) and (iii).

Let us first consider how they might be derived within T<P_2>:

(ii) is obtained in the same way as before but using the fact that $\Box(\Diamond A \supset \Diamond\Diamond A)$ is a theorem of T.

(iii) requires a different derivation. From the theoremhood of (b) above (which requires P_2 but not the application of Necessitation to theorems depending upon P_2), we obtain the theoremhood of $\Diamond A \supset \Diamond\Box A$. It follows, upon the substitution of $\neg A$ & $\Diamond A$ for A, that $\Diamond(\neg A \,\&\, \Diamond A) \supset \Diamond\Box(\neg A \,\&\, \Diamond A)$ is a theorem. But $\Box(\neg A \,\&\, \Diamond A)$ implies $\Box{\sim}A$ and $\Diamond A$ in T and hence is refutable in T, making $\Diamond(\neg A \,\&\, \Diamond A)$ refutable in T<P_2>.

Let us now consider how (ii) and (iii) might be derived within K4<P_2>:

(ii) Consulting the proof of theorem 1, we see that:

$$\text{(c) } \Diamond(\neg A \,\&\, \Diamond A) \supset \Diamond(\Diamond(\neg A \,\&\, \Diamond A) \,\&\, \Box(\neg A \,\&\, \Diamond A))$$

may be derived within K<P_2>. But $\Diamond(\neg A \,\&\, \Diamond A) \,\&\, \Box(\neg A \,\&\, \Diamond A)$ implies $\Diamond A$ and $\Box\neg A$ within K4. So $\Diamond(\Diamond(\neg A \,\&\, \Diamond A) \,\&\, \Box(\neg A \,\&\, \Diamond A))$ and hence $\Diamond(\neg A \,\&\, \Diamond A)$ are refutable within K4<P_2>.

(iii). From the instance $\Box(\Diamond\Diamond A \supset \Diamond A) \supset \Box(\Diamond A \supset A)$ of P_2 and the fact that $\Box(\Diamond\Diamond A \supset \Diamond A)$ is a theorem of K4.

Use P_2^* to signify the nonmodal substitution instances of P_2. Note that the system $S4P_2^*$ is still closed under necessitation though not under arbitrary substitution.

<u>Theorem 4</u> (i) $\Diamond A \supset A$ is not a theorem of the system $S4P_2^*$

(ii) $\Box(\Diamond A \supset A)$ is not a theorem of the system $K4P_2^*$

(iii) the system $S4P_2^*$ is anti-Megarian.

<u>Proof</u> Call $M = (W, R, \varphi)$ a *Nortmann* model if if wRv implies that, for some v' in W, wRv', v' is Megarian, and v' agrees with v on sentence-letters. It is then easy to show that $K4P_2^*$ will be sound for the class of transitive Nortmann models and that $S4P_2^*$ will be sound for the class of reflextive and transitive Nortmann models. To establish (i), (ii) and (iii), we then show that there exist appropriate Nortmann models in which $\Diamond p \supset p$ or $\Box(\Diamond p \supset p)$ are not true or in which any one of the anti-Megarian formulas M_n is false.

These negative results will no longer hold in the presence of a constant α for the actual world.

<u>Theorem 5</u> Let $S4P_2^*[\alpha]$ be the non-normal extension of $S4P_2^*$ obtained by adding the axioms:

Truth: α

Completeness: $A \supset \Box(\alpha \supset A)$ for any formula A

to $S4P_2^*$. Then M is a theorem of $S4P_2^*[\alpha]$.

<u>Proof</u> From Completeness it follows that $A \supset (\Box\alpha \supset \Box A)$ and hence that:

(d) $\Box\alpha \supset (A \supset \Box A)$

is a theorem. Substituting $\Diamond\neg\alpha$ for A in Completeness, it follows that $\Diamond\neg\alpha \supset \Box(\alpha \supset \Diamond\neg\alpha)$ is a theorem. By the properties of S4, $\Diamond\neg\alpha \supset \Box(\Diamond\alpha \supset \Diamond\neg\alpha)$ is a theorem and, by P_2^*, $\Box(\Diamond\alpha \supset \Diamond\neg\alpha) \supset \Box(\alpha \supset \neg\alpha)$ and hence $\Box(\Diamond\alpha \supset \Diamond\neg\alpha) \supset \neg\alpha$ is a theorem. Given Truth, $\Box(\Diamond\alpha \supset \Diamond\neg\alpha)$ is refutable and hence $\Diamond\neg\alpha$ is refutable. But then:

(e) $\Box\alpha$

is a theorem; and so from (d) and (e), it follows that $A \supset \Box A$ is a theorem.

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