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Relatively Unrestricted Quantification

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There are four broad grounds upon which the intelligibility of quantification over absolutely everything has been questioned—one based upon the existence of semantic indeterminacy, another on the relativity of ontology to a conceptual scheme, a third upon the necessity of sortal restriction, and the last upon the possibility of indefinite extendibility. The argument from semantic indeterminacy derives from general philosophical considerations concerning our understanding of language. For the Skolem–Lowenheim Theorem appears to show that an understanding of quantification over absolutely everything (assuming a suitably infinite domain) is semantically indistinguishable from the understanding of quantification over something less than absolutely everything; the same first-order sentences are true and even the same first-order conditions will be satisfied by objects from the narrower domain. From this it is then argued that the two kinds of understanding are indistinguishable tout court and that nothing could *count* as having the one kind of understanding as opposed to the other.

The second two arguments reject the bare idea of an object as unintelligible, one taking it to require supplementation by reference to a conceptual scheme and the other taking it to require supplementation by reference to a sort. Thus we cannot properly make sense of quantification over *mere* objects, but only over objects of such and such a conceptual scheme or of such and such a sort. The final argument, from indefinite extendibility, rejects the idea of a *completed* totality. For if we take ourselves to be quantifying over all objects, or even over all sets, then the reasoning of Russell's paradox can be exploited to demonstrate the possibility of quantifying over a more inclusive domain. The intelligibility of absolutely unrestricted quantification, which should be free from such incompleteness, must therefore be rejected.

The ways in which these arguments attempt to the undermine the intelligibility of absolutely unrestricted quantification are very different; and each calls for extensive discussion in its own right. However, my primary concern in the present paper is with the issue of indefinite extendibility; and I shall only touch upon the other arguments in so far as they bear upon this particular issue. I myself am not persuaded by

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the other arguments and I suspect that, at the end of day, it is only the final argument that will be seen to carry any real force. If there is a case to be made against absolutely unrestricted quantification, then it will rest here, upon logical considerations of extendibility, rather than upon the nature of understanding or the metaphysics of identity.

2.1 THE EXTENDIBILITY ARGUMENT

Let us begin by reviewing the classic argument from indefinite extendibility. I am inclined to think that the argument is cogent and that the intelligibility of absolutely unrestricted quantification should therefore be rejected. However, there are enormous difficulties in coming up with a cogent formulation of the argument; and it is only by going through various more or less defective formulations that we will be in a position to see how a more satisfactory formulation might be given. I shall call the proponent of the intelligibility of absolute quantification a 'universalist' and his opponent a 'limitavist' (my reason for using these unfamiliar labels will later become clear).

The extendibility argument, in the first instance, is best regarded as an ad hominem argument against the universalist. However, I should note that if the argument works at all, then it should also work against someone who claims to have an understanding of the quantifier that is *compatible* with its being absolutely unrestricted. Thus someone who accepted the semantic argument against there being an interpretation of the quantifier that was *determinately* absolutely unrestricted might feel compelled, on the basis of this further argument, to reject the possibility of there even being an interpretation of the quantifier that was *indeterminately* absolutely unrestricted.

Let us use '$\exists$' and '$\forall$' for those uses of the quantifier that the universalist takes to be absolutely unrestricted. The critical step in the argument against him is that, on the basis of his understanding of the quantifier, we can then come to another understanding of the quantifier according to which there will be an object (indeed, a set) whose members will be all those objects, in *his* sense of the quantifier, that are not members of themselves. Let us use $\exists^+$ and $\forall^+$ for the new use of the quantifier. Then the point is that we can so understand the new quantifiers that the claim:

(R) $\exists^+ y[\forall x(x \in y \equiv \sim(x \in x))]$

is true (using $\exists^+ y$ with wide scope and $\forall x$ with narrow scope).

The argument to (R) can, if we like, be divided into two steps. First, it is claimed that on the basis of our opponent's understanding of the quantifier $\exists$, we can come to an understanding of the quantifier $\exists'$ according to which there is an object (indeed, a set) of which every object, in his sense of the quantifier, is a member:

(U) $\exists' z \forall x(x \in z)$.

It is then claimed that, on the basis of our understanding of the quantifier $\exists'$, we can come to an understanding of the quantifier $\exists^+$ according to which there is an object whose members, in the sense of $\forall$, are all those objects that belong to some selected object, in the sense of $\forall'$, and that satisfy the condition of not being self-membered:

(S) $\forall' z \exists^+ y \forall x\, [(x \in y \equiv (x \in z \,\&\, {\sim}(x \in x))]$.

From (U) and (S), (R) can then be derived by standard quantificational reasoning.

(S) is an instance of 'Separation', though the quantifier $\exists^+$ cannot necessarily be identified with $\exists'$ since the latter quantifier may not be closed under definable subsets. (S) is relatively unproblematic, at least under the iterative conception of set, since we can simply take $\exists^+$ to range over all subsets of objects in the range of $\exists'$. Thus granted the relevant instance of Separation, the existence of a Russell set, as given by (R), will turn upon the existence of a universal set, as given by (U).

There is also no need to assume that the membership-predicate to the left of (R) is the same as the membership-predicate to its right. Thus we may suppose that with the new understanding $\exists^+$ of the quantifier comes a new understanding $\in^+$ of the membership predicate, so that (R) now takes the form:

(R′) $\exists^+ y[\forall x(x \in^+ y \equiv\ {\sim}(x \in x))]$.

It is plausible to suppose that $\in^+$ 'conservatively' extends $\in$:

(CE) $\forall x \forall y(x \in^+ y \equiv x \in y))$.[1]

But we may then derive:

(R^+) $\exists^+ y[\forall x(x \in^+ y \equiv\ {\sim}(x \in^+ x))]$,

which is merely a 'notational variant' of (R), with $\in^+$ replacing $\in$.

The rest of the argument is now straightforward. From (R) (or (R^+)), we can derive the 'extendibility' claim:

(E) $\exists^+ y \forall x(x \neq y)$.

For suppose, for purposes of reduction, that $\forall^+ y \exists x(x = y)$. Then (R) yields:

(R^*) $\exists y[\forall x(x \in y \equiv\ {\sim}(x \in x))]$,

which, by the reasoning of Russell's paradox, leads to a contradiction.

But the truth of (E) then shows that the original use of the quantifiers $\exists$ and $\forall$ was not absolutely unrestricted after all.

Even though we have stated the argument for the particular case of sets, a similar line of argument will go through for a wide range of other cases—for ordinal and cardinal numbers, for example, or for properties and propositions. In each of these cases, a variant of the paradoxical reasoning may be used to show that the original quantifier was not absolutely unrestricted. Thus in order to resist this conclusion, it is not sufficient to meet the argument in any particular case; it must be shown how in general it is to be met.

[1] (CE) might be doubted on the grounds that $\in^+$ may have the effect of converting urelements according to $\in$ into sets. But even this is not on the cards, if it is insisted that the initial quantifier $\forall$ should only range over sets.

Indeed, even this is not enough. For there are cases in which objects of *two* kinds give rise to paradox (and hence to a paradoxically induced extension) even though each kind of object, when considered on its own, is paradox-free. For example, there would appear to be nothing to prevent the arbitrary formation of singletons or the arbitrary formation of mereological sums, but the arbitrary formation of both gives rise to a form of Russell's paradox (given certain modest assumptions about the mereological structure of singletons).[2] These cases create a special difficulty for the proponent of absolutely unrestricted quantification, even if he is content to block the automatic formation of new objects in those cases in which a single kind of object gives rise to paradox. For it might appear to be unduly restrictive to block the arbitrary formation of both kinds of objects in those cases where two kinds of object are involved and yet invidious to block the formation of one kind in preference to the other. Thus we do not want to block the arbitrary formation of both singletons and mereological sums. And yet why block the formation of one in preference to the other? Rather than have to face this awkward choice, it might be thought preferable to 'give in' to the extendibility argument and allow the arbitrary extension of the domain by objects of either kind.

There are various standard set-theoretic grounds upon which the transition to (R) might be questioned, but none is truly convincing. It has been suggested, for example, that no set can be 'too big', of the same size as the universe, and that it is this that prevents the formation of the universal or the Russell set. Now it may well be that no understanding of the quantifier that is subject to reasonable set-theoretical principles will include sets that are too big within its range. But this has no bearing on the question of whether, given such an understanding of the quantifier, we may come to an understanding of the quantifier that ranges over sets that would have been too big relative to the original understanding of the quantifier. For surely, given any condition whatever, we can so understand the quantifier that it ranges over a set whose members are all those objects (according to the original understanding of the quantifier) that satisfy the condition; and the question of *how many* objects satisfy the condition is entirely irrelevant to our ability to arrive at such an understanding of the quantifier.

Or again, it has been suggested that we should think of sets as being constructed in stages and that what prevents the formation of the universal or the Russell set is there being no stage at which its members are all constructed. We may grant that we should think of sets as being constructed at stages and that, under any reasonable process by which might take them to be constructed, there will be no stage at which either the universal or the Russell set is constructed. But what is to prevent us from so understanding the quantifier over stages that it includes a stage that lies after all of the stages according to the original understanding of the quantifier ($\exists^{+}\alpha\forall\beta(\alpha > \beta)$)?

[2] The matter is discussed in Lewis (1991), Rosen (1995) and Fine (2005a) and in Uzquiano's paper in the present volume. A similar problem arises within an ontology of properties that allows for the formation both of arbitrary disjunctions (properties of the form: $P \vee Q \vee \ldots$) and of arbitrary identity properties (properties of the form: *identical to P*); and a related problem arises within the context of Parsons' theory of objects (Parsons, 1980), in which properties help determine objects and objects help determine properties.

And given such a stage, what is to prevent us from coming to a correlative understanding of a quantifier over sets that will include the 'old' universal or Russell set within its range? The existence of sets and stages may be linked; and in this case, the question of their extendibility will also be linked. But it will then be of no help to presuppose the inextendibility of the quantifier over stages in arguing for the inextendibility of the quantifier over sets.

Or again, it has been supposed that what we get is not a universal or a Russell *set* but a universal or Russell *class*. But I have stated the argument without presupposing that the universal or Russell object is either a set or a class. What then can be the objection to saying that we can so understand the quantifier that there is something that has all of the objects previously quantified over as members? Perhaps this *something* is not a class, if the given objects already includes classes. But surely we can intelligibly suppose that there is something, be what it may, that has all of the previously given objects as its members (in a sense that conservatively extends our previous understanding of membership).

Thus the standard considerations in support of ZF or the like do nothing to undermine the argument from extendibility. Their value lies not in showing how the argument might be resisted but in showing how one might develop a consistent and powerful set theory within a given domain, without regard for whether than domain might reasonably be taken to be unrestricted.

But does not the extendibility argument take the so-called 'all-in-one' principle for granted? And has not Cartwright (1994) shown this principle to be in error? Cartwright states the principle in the following way (p. 7):

> to quantify over certain objects is to presuppose that those objects constitute a 'collection' or a 'completed collection'—some one thing of which those objects are members.

Now one might indeed argue for extendibility on the basis of the all-in-one principle. But this is not how our own argument went. We did not argue that our understanding of the quantifier $\forall$ *presupposes* that there is some one thing of which the objects in the range of $\forall$ are members ($\exists^+ y \forall x(x \in y)$). For this would mean that the quantifier $\forall$ was to be understood in terms of the quantifier $\forall^+$. But for us, it is the other way round; the quantifier $\forall^+$ is to be understood in terms of the quantifier $\forall$. It is through a *prior* understanding of the quantifier $\forall$ that we come to appreciate that there is a sense of the quantifier $\forall^+$ in which it correct to suppose that some one thing has the objects in the range of $\forall$ as members. Thus far from presupposing that the all-in-one principle is true, we presuppose that it is false!

Of course, there is some mystery as to how we arrive at this new understanding of the quantifier. What is the extraordinary mental feat by which we generate a new object, as it were, merely from an understanding of the quantifier that does not already presuppose that there is such an object? I shall later have something to say on this question. But it seems undeniable that we *can* achieve such an understanding even if there is some difficulty in saying how we bring it off. Indeed, it may plausibly be argued that the way in which we achieve an understanding of the quantifier $\forall^+$ is the same as the way in which we achieve a more ordinary understanding of the set-theoretic quantifier. Why, for example, do we take there to be a set of all natural numbers? Why not simply assume that the relevant portion of the 'universe' is exhausted by the *finite* sets of natural numbers? The obvious response is that we can intelligibly quantify over all the natural numbers and so there is nothing to prevent us from so understanding the set-theoretic quantifier that there is a set whose members are all the natural numbers ($\exists x \forall n(n \in x)$). But then, by parity of reasoning, such an extension in our understanding of the quantifier should always be possible. The great stumbling block for the universalist, from this point of view, is that there would appear to be nothing short of a prejudice against large infinitudes that might prevent us from asserting the existence of a comprehensive set in the one case yet not in the other.[3]

FN:3

2.2 GENERALIZING THE EXTENDIBILITY ARGUMENT

The extendibility argument is not satisfactory as it stands. If our opponent claims that we may intelligibly understand the quantifier as absolutely unrestricted, then he is under some obligation to specify a particular understanding of the quantifier for which this is so. And once he does this, we may then use the extendibility argument to prove him wrong. But what if no opponent is at hand? Clearly, it will not do to apply the extendibility argument to our *own* interpretation of the quantifier. For what guarantee will we have that our opponent would have regarded it as absolutely unrestricted?

Clearly, what is required is a generalization of the argument. It should not be directed at this or that interpretation of the quantifier but at any interpretation whatever. Now normally there would be no difficulty in generalizing an argument of this sort. We have a particular instance of the argument; and, since nothing special is assumed about the instance, we may generalize the reasoning to an arbitrary instance and thereby infer that the conclusion generally holds. However, since our concern is with the very nature of generality, the attempt to generalize the present argument gives rise to some peculiar difficulties.

The general form of the argument presumably concerns an arbitrary interpretation (or understanding) of the quantifier; and so let us use I, J, ... as variables for interpretations, and I_0 and J_0 and the like as constants for particular interpretations. I make no particular assumptions about what interpretations are and there is no need, in particular, to suppose that an interpretation of a quantifier will require the specification of some 'object' that might figure as its domain. We shall use $\exists_I x\varphi(x)$, with I as a subscript to the quantifier, to indicate that there is some x under the interpretation I for which $\varphi(x)$. Some readers may baulk at this notation. They might think that one should use a meta-linguistic form of expression and say that the sentence '$\exists x\varphi(x)$' is true under the interpretation I rather than that $\exists_I x\varphi(x)$. However, nothing in what follows will turn on such niceties of use-mention and, in the interests of presentation, I have adopted the more straightforward notation.

[3] A somewhat similar line of argument is given by Dummett [1991], pp. 315–16.

Let us begin by reformulating the original argument, making reference to the interpretations explicit. Presumably, our opponent's intended use of the quantifier will conform to a particular interpretation I_0 of the quantifier. We may therefore assume:

(1) $\forall x \exists_{I\downarrow 0} y(y = x) \& \forall_{I\downarrow 0} y \exists x(x = y)$.

We now produce an 'extension' J_0 of I_0 subject to the following condition:

(2) $\exists_{J\downarrow 0} y \forall_{I\downarrow 0} x(x \in y \equiv \sim x \in x)$.

From (2) we may derive:

(3) $\exists_{J\downarrow 0} y \forall_{I\downarrow 0} x(x \neq y)$.

Defining $I \subseteq J$ as $\forall_I x \exists_J y(x = y)$, we may write (3) as:

(3)′ $\sim(J_0 \subseteq I_0)$.

Let us use UR(I) for: I is absolutely unrestricted. There is a difficulty for the limitavist in explaining how this predicate is to be understood since, intuitively, an absolutely unrestricted quantifier is one that ranges over absolutely everything. But let us put this difficulty on one side since the present problem will arise even if the predicate is taken to be primitive. Under the intended understanding of the predicate UR, it is clear that:

(4) $UR(I_0) \supset J_0 \subseteq I_0$.

And so, from (3) and (4), we obtain:

(5) $\sim UR(I_0)$.

From this more explicit version of the original argument, it is now evident how it is to be generalized. (2) should now assume the following more general form:

$(2)^G$ $\forall I \exists J \exists_J y \forall_I x(x \in y \equiv \sim x \in x)$.

This is the general 'Russell jump', taking us from an arbitrary interpretation I to its extension J. (We could also let the interpretation of ∈ vary with the interpretation of the quantifier; but this is a nicety which we may ignore.) By using the reasoning of Russell's paradox, we can then derive:

$(3)^G$ $\forall I \exists J[\sim(J \subseteq I)]$.

Define an interpretation I to be *maximal*, Max(I), if $\forall J\ (J \subseteq I)$. Then $(3)^G$ may be rewritten as:

$(3)^{G\prime}$ $\forall I \sim Max(I)$.

Step (4), when generalized, becomes:

$(4)^G$ $\forall I[UR(I) \supset Max(I)]$.

And so from $(3)^{G\prime}$ and $(4)^G$, we obtain:

$(5)^G$ $\forall I \sim UR(I)$,

i.e. no interpretation of the quantifier is absolutely unrestricted, which would appear to be the desired general conclusion.

But unfortunately, things are not so straightforward. For in something like the manner in which our opponent's first-order quantifier over objects was shown not to be absolutely unrestricted, it may also be shown that *our* own second-order quantifier over interpretations is not absolutely unrestricted; and so $(5)^G$ cannot be the conclusion we are after. For we may suppose, in analogy with (1) above, that there is an interpretation M_0 to which the current interpretation of the quantifiers over interpretations conforms:

(6) $\forall I \exists_{M\downarrow 0} J(J = I) \,\&\, \forall_{M\downarrow 0} J \exists I(I = J)$.[4]

Now associated with any 'second-order' interpretation M is a first-order interpretation I, what we may call the 'sum' interpretation, where our understanding of $\exists_I x\varphi(x)$ is given by $\exists_M J \exists_J x\varphi(x)$. In other words, something is taken to φ (according to the sum of M) if it φ's under some interpretation of the quantifier (according to M). The sum interpretation I is maximal with respect to the interpretations according to M, i.e.$\forall_M J(J \subseteq I)$; and so there will be such an interpretation according to M_0 if M_0 is absolutely unrestricted:

(7) $UR(M_0) \supset \exists_{M\downarrow 0} I \forall_M J(J \subseteq I)$.

Given (6), (7) implies:

(8) $UR(M_0) \supset \exists I[Max(I)]$.

And so $(3)^{G,}$ above yields:

(9) $\sim UR(M_0)$.

The second-order interpretation of the first-order quantifier is not absolutely unrestricted.[5] FN:5

In this proof, we have helped ourselves to the reasoning by which we showed the universalist's first-order quantifier not to be absolutely unrestricted. But it may be shown, quite regardless of how $(5)^G$ might have been established, that its *truth* is not compatible with its quantifier being absolutely unrestricted. For it may plausibly be maintained that if a second-order interpretation M is absolutely unrestricted then so is any first-order interpretation that is maximal with respect to M (or, at least, if the notion is taken in a purely extensional sense). Thus in the special case of M_0, we have:

(10) $UR(M_0) \supset \forall_{M\downarrow 0} I[\forall_{M\downarrow 0} J(J \subseteq I) \supset UR(I)]$.

So from (7) and (10), we obtain:

(11) $UR(M_0) \supset \exists_{M\downarrow 0} I[UR(I)]$.

[4] Instead of appealing to the notion of identity between interpretations in stating this assumption, we could say $\forall I \exists_{M\downarrow 0} J[\forall_I x \exists_J y(x = y) \& \forall_J y \exists_I x(y = x)]$; and similarly for the second conjunct.

[5] An argument along these lines is also to be found in Lewis (1991), p. 20, McGee (2000), p. 48, and Williamson (2003), and also in Weir's contribution to the present volume.

But given (6), we may drop the subscript M_0. And contraposition then yields:

(12) $\forall I \sim UR(I) \supset \sim UR(M_0)$.

In other words, if it is true that no interpretation of the quantifier is absolutely unrestricted, then the interpretation of the quantifier 'no interpretation' is itself not absolutely unrestricted.[6]

Of course, it should have been evident from the start that the limitavist has a difficulty in maintaining that all interpretations of the quantifier are absolutely unrestricted, since it would follow from the truth of the claim that the interpretation of the quantifier in the claim itself was not absolutely unrestricted and hence that it could not have its intended import. What the preceding proof further demonstrates is the impossibility of maintaining a mixed position, one which grants the intelligibility of the absolutely unrestricted 'second-order' quantifier over all interpretations but rejects the intelligibility of the absolutely unrestricted 'first-order' quantifier over all objects. If we have the one then we must have the other.[7]

The resulting dialectical situation is hardly satisfactory. The universalist seems obliged to say something false in defense of his position. For he should say what the absolutely unrestricted interpretation of the quantifier is—or, at least say that there is such an interpretation; and once he does either, then we may show him to be in error. The limitavist, on the other hand, can say nothing to distinguish his position from his opponent's—at least if his opponent does not speak. For his position (at least if true) will be stated by means of a restricted quantifier and hence will be acceptable, in principle, to his opponent. Both the universalist and the limitavist would like to say something true but, where the one ends up saying something indefensible, the other ends up saying nothing at all.

The situation mirrors, in miniature, what some have thought to hold of philosophy at large. There are some propositions that are of interest to assert if true but of no interest to deny if false. Examples are the proposition that there is no external world or the proposition that I alone exist. Thus it is of interest to be told that there is *no* external world, if that indeed is the case, but not that there *is* an external world. Now some philosophers of a Wittgensteinian persuasion have thought that philosophy consists entirely of potentially interesting propositions of this sort and that *none of them is true*. There is therefore nothing for the enlightened philosopher to assert that is both true and of interest. All he can sensibly do is to wait for a less enlightened colleague to say something false, though potentially of interest, and then show him to be wrong. And similarly, it seems, in the present case. The proposition that some particular interpretation of the quantifier is absolutely unrestricted is of interest only if true; and given that it is false, all we can sensibly do, as enlightened limitavists, is to hope that our opponent will claim to be in possession of an absolutely

[6] We should note that, for the purpose of meeting these arguments, it is of no help to draw a grammatical distinction between the quantifiers $\forall I$ and $\forall x$.

[7] It is perhaps worth remarking that there are not the same compelling arguments against a position that tolerates the intelligibility of unrestricted first-order quantification but rejects the intelligibility of unrestricted second-order quantification (see Shapiro, 2003).

unrestricted interpretation of the quantifier and then use the Russell argument to prove him wrong!

2.3 GOING MODAL

The previous difficulties arise from our not being able to articulate what exactly is at issue between the limitavist and the universalist. There seems to be a well-defined issue out there in logical space. But the universalist can only articulate his position on the issue by saying something too strong to be true, while the limitavist can only articulate his position by saying something too weak to be of interest. One gets at his position from above, as it were, the other from below. But what we want to be able to do is to get at the precise position to which each is unsuccessfully attempting to approximate.

Some philosophers have suggested that we get round this difficulty by adopting a schematic approach. Let us use r(I) for the interpretation obtained by applying the Russell device to a given interpretation I. Then what the limitavist wishes to commit himself to, on this view, is the *scheme*:

(ES)$\exists_{r(I)} y \forall_I x \sim (x = y)$ (something under the Russell interpretation is not an object under the given interpretation).

Here ‘I’ is a schematic variable for interpretations; and in committing oneself to the scheme, one is committing oneself to the truth of each of its instances though not to the claim that each of them is true.[8]

The difficulty with this view is to see how it might be coherently maintained. We have an understanding of what it is to be committed to a scheme; it is to be committed to the truth of each of its instances. But how can one understand what it is to be committed to the truth of each of its instances without being able to understand what it is for an arbitrary one of them to be true? And given that one understands what it is for an arbitrary one of them to be true, how can one be willing to commit oneself to the truth of each of them without also being willing to commit oneself to the claim that each of them is true? But once one has committed oneself to this general claim, then the same old difficulties reappear. For we can use the quantifier ‘every instance’ (just as we used the quantifier •‘every interpretation) to construct an instance that does not fall within its range.

The schematist attempts to drive a wedge between a general commitment to particular claims and a particular commitment to a general claim. But he provides no plausible reason for why one might be willing to make the one commitment and yet not both able and willing to make the other. Indeed, he appears to be as guilty as the universalist in not being willing to face up to the facts of intelligibility. The universalist thinks that there is something special about the generality implicit in our

[8] Lavine and Parsons advocate an approach along these lines in the present volume; and it appears to be implicit in the doctrine of ‘systematic ambiguity’ that has sometimes been advocated—by Parsons (1974, p. 11) and Putnam (2000, p. 24), for example—as a solution to the paradoxes.

understanding of a certain form of quantification that prevents it from being extended to a broader domain, while the schematist thinks that there is something special about the generality implicit in a certain form of schematic commitment that prevents it from being explicitly rendered in the form of a quantifier. But in neither case can either side provide a plausible explanation of our inability to reach the further stage in understanding and it seems especially difficult to see why one might baulk at the transition in the one case and yet not in the other.

I want in the rest of the chapter to develop an alternative strategy for dealing with the issue. Although my sympathies are with the limitavist, it is not my *principal* concern to argue for that position but to show that there is indeed a position to argue for. The basic idea behind the strategy is to adopt a modal formulation of the theses under consideration. But this idea is merely a starting-point. It is only once the modality is properly understood that we will be able to see how a modal formulation might be of any help; and to achieve this understanding is no small task. It must first be appreciated that the relevant modality is 'interpretational' rather than 'circumstantial'; and it must then be appreciated that the relevant interpretations are not to be understood, in the usual way, as some kind of restriction on the domain but as constituting a genuine form of extension. It has been the failure to appreciate these two points, I believe, that has prevented the modal approach from receiving the recognition that it deserves.[9]

FN:9

Under the modal formulation of the limitavist position, we take seriously the thought that any given interpretation *can* be extended, i.e. that we can, in principle, come up with an extension. Thus in coming up with an extension we are not confined to the interpretations that fall under the current interpretation of the quantifier over interpretations. Let us use $I \subset J$ for 'J (properly) extends I' (which may be defined as: $I \subseteq J \sim (J \subseteq I)$). Let us say that I is *extendible*—in symbols, E(I)—if possibly some interpretation extends it, i.e.$\Diamond\exists J(I \subset J)$. Then one formulation of the limitavist position is:

(L) $\forall I E(I)$.

But as thorough-going limitavists, we are likely to think that, whatever interpretation our opponent *might* come up with, it will be possible to come up with an interpretation that extends it. Thus a stronger formulation of the limitavist's position is:

$(L)^+$ $\Box\forall I E(I)$(i.e. $\Box\forall I\Diamond\exists J(I \subset J)$).

It should be noted that there is now no longer any need to use a primitive notion of being absolutely unrestricted (UR) in the formulation of the limitativist's position.

The theses (L) and $(L)^+$ are intended to apply when different delimitations on the range of the quantifier may be in force. Thus the quantifier might be understood, in a generic way, as ranging over *sets*, say, or *ordinals*, but without it being determined which sets or, which ordinals, it ranges over. Thesis (L) must then be construed as saying that any interpretation of the quantifier over sets or over ordinals can be

[9] The approach is briefly, and critically, discussed in §5 of Williamson (2003); and it might be thought to be implicit in the modal approach to set theory and number theory, though it is rarely advocated in its own right.

extended to another interpretation of the quantifier over sets or over ordinals. Thus the extension is understood to be possible within the specified range of the quantifier. We might say that the concept by which the quantifier is delimited is *extendible* if (L) holds and that it is *indefinitely extendible* if $(L)^+$ holds. We thereby give precise expression to these familiar ideas.

It is essential to a proper understanding of the two theses that the interpretations be taken to be modally 'rigid'. Whatever objects an interpretation picks out or fails to pick out, it must necessarily pick out or fail to pick out; its range, in other words, must be constant from world to world.[10] Without this requirement, an interpretation could be extendible through its range contracting or inextendible through its range expanding, which is not what we have in mind. We should therefore distinguish between the concept, such as *set* or *ordinal*, by which the range of the quantifier might be delimited and an interpretation of the quantifier, by which its range is fixed. The latter is constant in the objects it picks out from world to world, even if the former is not.

It will also be helpful to suppose that (necessarily) each interpretation picks out an object within the *current* range of the first-order quantifier ($\Box\forall I \forall_I x \exists y(y = x)$). This is a relatively harmless assumption to make, since it can always be guaranteed by taking the interpretations within the range of '∀I' to include the 'sum' interpretation and then identifying the current interpretation with the sum interpretation. It follows on this approach that there is (necessarily) a maximal interpretation ($\Box\exists I \forall J(J \subseteq I)$) but there is no reason to suppose, of course, that it is necessarily maximal ($\Box\exists I \Box\forall J(J \subseteq I)$). Given this simplifying supposition, the question of whether the current interpretation I_0 is extendible (i.e. of whether $\Diamond\exists J(I_0 \subset J)$) is simply the question of whether it is possible that there is an object that it does not pick out (something we might formalize as $\forall I(\forall x \exists_I y(y = x) \supset \Diamond\exists x \sim\exists_I y(y = x))$), where the condition $\forall x \exists_I y(y = x)$ serves to single out the current interpretation I_0).

However, the critical question in the formulation of the theses concerns the use of the modalities. Let us call the notions of possibility and necessity relevant to the formulation 'postulational'. How then are the postulational modalities to be understood? The familiar kinds of modality do not appear to be useful in this regard. Suppose, for example, that '□' is understood as metaphysical necessity. As limitavists, we would like to say that the domain of pure sets is extendible. This would mean, under the present proposal, that it is a metaphysical possibility that some pure set is not actual. But necessarily, if a pure set exists, then it exists of necessity; and so it is not possible that some pure set is not actual. Thus we fail to get a case of being extendible that we want. We also get cases of being extendible that we do not want. For it is presumably metaphysically possible that there should be more atoms than there actually are. But we do not want to take the domain of atoms to be extendible—or, at least, not for this reason.

[10] I might add that all we care about is which objects are in the range, not how the range is determined, and so, for present purpose, we might as well take '∀I' to be a second-order extensional quantifier.

Suppose, on the other hand, that '$\Box$' is understood as logical necessity (or perhaps as some form of conceptual necessity). There are, of course, familiar Quinean difficulties in making sense of first-order quantification into modal contexts when the modality is logical. Let me here just dogmatically assume that these difficulties may be overcome by allowing the logical modalities to 'recognize' when two objects are or are not the same.[11] Thus $\Box\forall x\Box\ (x = y \supset \Box x = y)$ and $\Box\forall x\Box\ (x \neq y \supset\ \Box\ x \neq y)$ will both be true though, given that the modalities are logical, it will be assumed that they are blind to any features of the objects besides their being the same or distinct.

There is also another, less familiar, difficulty in making sense of *second*-order quantification into modal contexts when the modality is logical. There are perhaps two main accounts of the quantifier '$\forall I$' that might reasonably be adopted in this case. One is substitutional and takes the variable 'I' to range over appropriate substituends (predicates or the like); the other is 'extensional' and takes 'I', in effect, to range over enumerations of objects of the domain.

Under the first of these accounts, it is hard to see why any domain should be extendible, for in the formalization $\forall I(\forall_I x \exists y(y = x) \supset\ \Diamond\exists x{\sim}\ \exists_I y(y = x))$ we may let I be the predicate of self-identity. The antecedent $\forall_I x\exists y(y = x)$ will then be true while the consequent $\Diamond\exists x{\sim}\exists_I y(y = x)$, which is equivalent to $\Diamond\exists x{\sim}\ \exists y(y = x)$, will be false.

The second of the two accounts does not suffer from this difficulty since the interpretation I will be confined to the objects that it enumerates. But it is now hard to see why any domain should be *in*extendible. For let $a_1, a_2, a_3, \ldots$ be an enumeration of all of the objects in the domain. Then it is logically possible that these are not all of the objects ($\Diamond\exists x{\sim}(x = a_1\ \vee x = a_2\ \vee x = a_3\ \vee \ldots)$), since there can be no logical guarantee that any particular objects are all of the objects that there are. This is especially clear if there are infinitely many objects $a_1, a_2, a_3, \ldots$. For if it were logically impossible that some object was not one of $a_1, a_2, a_3, \ldots$, then it would be logically impossible that some object was not one of $a_2, a_3, \ldots$, since the logical form of the existential proposition in the two cases is the same. But there *is* an object that is not one of $a_2, a_3, \ldots$, viz. a_1! Thus just as considerations of empirical vicissitude are irrelevant to the question of extendibility, so are considerations of logical form.

It should also be fairly clear that it will not be possible to define the relevant notion of necessity by somehow relativizing the notion of logical necessity. The question is whether we can find some condition φ such that the necessity of ψ in the relevant sense can be understood as the logical necessity of $\varphi\ \supset\ \psi$. But when, intuitively, a domain of quantification is inextendible, we will want φ to include the condition $\forall x(x = a_1\ \vee x = a_2\ \vee x = a_3 \vee \ldots)$, where $a_1, a_2, a_3, \ldots$ is an enumeration of all the objects in the domain; and when the domain is extendible, we will want φ to exclude any such condition. Thus we must already presuppose whether or not the domain is extendible in determining what the antecedent condition φ should be (and nor are things better with metaphysical necessity, since the condition may then hold of necessity whether we want it to or not).

[11] The issue is discussed in Fine (1990).

2.4 POSTULATIONAL POSSIBILITY

We have seen that the postulational modalities are not to be understood as, or in terms of, the metaphysical or logical modalities. How then are they to be understood? I doubt that one can provide an account of them in essentially different terms—and in this respect, of course, they may be no different from some of the other modalities.[12] However, a great deal can be said about how they are to be understood and in such a way, I believe, as to make clear both how the notion is intelligible and how it may reasonably be applied. Indeed, in this regard it may be much less problematic than the more familiar cases of the metaphysical and natural modalities.

It should be emphasized, in the first place, that it is not what one might call a 'circumstantial' modality. Circumstance could have been different; Bush might never have been President; or many unborn children might have been born. But all such variation in the circumstances is irrelevant to what is or is not postulationally possible. Indeed, suppose that D is a complete description of the world in basic terms. It might state, for example, that there are such and such elementary particles, arranged in such and such a way. Then it is plausible to suppose that any postulational possibility is compatible with D. That is:

$\Diamond A \supset \Diamond(A \,\&\, D)$.

Or, equivalently, D is a postulational necessity ($\Box$ D); there is not the relevant possibility of extending the domain of quantification so that D is false. Postulational possibilities, in this sense, are possibilities *for* the actual world, and not merely possible alternatives *to* the actual world.

Related considerations suggest that postulational necessity is not a genuine modality at all. For when a proposition is genuinely necessary there will be a broad intuitive sense in which the proposition *must be* the case. Thus epistemic necessity (or knowledge) is not a genuine modality since there is no reason, in general, to suppose that what is known must be the case. Similarly for postulational necessity. That there are swans, for example, is a postulational necessity but it is not something that, intuitively, must be the case. Thus it is entirely compatible with the current 'modal' approach that it is not merely considerations of metaphysical modality, but genuine considerations of modality in general, that are irrelevant to questions of extendibility.

The postulational modalities concern not a possible variation in circumstance but in interpretation. The possibility that there are more sets, for example, depends upon a reinterpretation in what it is for there to be a set. In this respect, postulational possibility is more akin to logical possibility, which may be taken to concern the possibility for reinterpreting the primitive non-logical notions. However, the kind of reinterpretation that is in question in the case of postulational possibility is much more circumscribed

[12] Metaphysical modality is often taken to be primitive and Field (1989, 32) has suggested that logical modality is primitive. In Fine (2002), I argued that there are three primitive forms of modality—the metaphysical, the natural, and the normative. Although postulational modality may also be primitive, it is not a genuine modality in the sense I had in mind in that paper.

than in the case of the logical modality, since it primarily concerns possible changes in the interpretation of the domain of quantification and is only concerned with other changes in interpretation in so far as they are dependent upon these.

But if postulational possibility is a form of interpretational possibility, then why does the postulational possibility of a proposition not simply consist in the existence of an interpretation for which the proposition is true? It is here that considerations of extendibility force our hand. For from among the interpretations that there are is one that is maximal. But it is a postulational possibility that there are objects which it does not pick out; and so this possibility cannot consist in there actually being an interpretation (broader than the maximal interpretation) for which there is such an object.[13]

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Nor can we plausibly take the postulational possibility of a proposition to consist in the metaphysical possibility of our specifying an interpretation under which the proposition is true. For one thing, there may be all sorts of metaphysical constraints on which interpretations it is possible for us to specify. More significantly, it is not metaphysically possible for a quantifier over pure sets, say, to range over more pure sets than there actually are, since pure sets exist of necessity. So this way of thinking will not give us the postulational possibility of there being more pure sets than there actually are.

The relationship between the relevant form of interpretational possibility and the existence of interpretations is more subtle than either of these proposals lead us to suppose. What we should say is that the existence of an interpretation of the appropriate sort *bears witness* or *realizes* the possibility in question.[14] Thus it is the existence of an interpretation, given by the Russell jump, that bears witness to the possibility that there are objects not picked out by the given interpretation. However, to say that a possibility may be realized by an interpretation is not to say that it consists in the existence of an interpretation or that it cannot obtain without our being able to specify the interpretation.

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But still it may be asked: what bearing do these possibilities have on the issue of unrestricted quantification? We have here a form of the 'bad company' objection. Some kinds of possibility—the metaphysical or the logical ones, for example—clearly have no bearing on the issue. So what makes this kind of possibility any better? Admittedly, it differs from the other kinds in various ways—it is interpretational rather than circumstantial and interpretational in a special way. But why think that these differences matter?[15]

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I do not know if it possible to answer this question in a principled way, i.e., on the basis of a clear and convincing criterion of relevance to which it can then be shown that the modality will conform. But all the same, it seems clear that there is a notion of the required sort, one which is such that the possible existence of a broader interpretation

[13] We have here a kind of *proof* of the impossibility of providing a possible worlds semantics for the relevant notion of interpretational possibility. Any semantics, to be genuinely adequate to the truth-conditions, would have to be homophonic.

[14] What is here in question is the legitimacy of the inference from φ^I to $\Diamond\varphi$, where φ^I is the result of relativizing all the quantifiers in φ to I. This might be compared to the inference from φ-is-true-in-w to $\Diamond\varphi$, with the world w realizing the possibility of φ.

[15] I am grateful to Timothy Williamson for pressing this question upon me.

is indeed sufficient to show that the given narrower interpretation is not absolutely unrestricted. For suppose someone proposes an interpretation of the quantifier and I then attempt to do a 'Russell' on him. Everyone can agree that if I succeed in coming up with a broader interpretation, then this shows the original interpretation not to have been absolutely unrestricted. Suppose now that no one in fact does do a Russell on him. Does that mean that his interpretation was unrestricted after all? Clearly not. All that matters is that the interpretation should be possible. But the relevant notion of possibility is then the one we were after; it bears directly on the issue of unrestricted quantification, without regard for the empirical vicissitudes of actual interpretation.

Of course, this still leaves open the question of what it is for such an interpretation to be possible. My opponent might think it consists in there existing an interpretation in a suitably abstract sense of term or in my being capable of specifying such an interpretation. But we have shown these proposals to be misguided. Thus the present proponent of the modal approach may be regarded as someone who starts out with a notion of possible interpretation that all may agree is relevant to the issue and who then finds good reason not to cash it out in other terms. In this case, the relevance of the notion he has in mind can hardly be doubted.

2.5 RESTRICTIONISM

To better understand the relevant notion of postulational possibility we must understand the notion of interpretation on which it is predicated. Postulational possibilities lie in the possibilities for reinterpreting the domain of quantification. But what is meant here by a reinterpretation, or change in interpretation, of the quantifier?

The only model we currently have of such a change is one in which the interpretation of the quantifier is given by something like a predicate or property which serves to restrict its range. To say that a proposition is postulationally necessary, on this model then, is to say that it is true no matter how the restriction on its quantifiers might be relaxed; to say that an interpretation of the quantifier is extendible is to say that the restriction by which it is defined can be relaxed; and to say that a quantifier is indefinitely extendible is to say that no matter how it might be restricted the restriction can always be relaxed.

Unfortunately, the model, attractive as it may be, is beset with difficulties. Consider the claim that possibly there are more sets than we currently take there to be ($\forall I[\forall x \exists_I y(y = x) \supset \Diamond \exists y \forall_I x(y \neq x)]$). In order for this to be true, the current quantifier '$\forall x$' over sets must not merely be restricted to sets but to sets of a certain sort, since otherwise there would not be the possibility of the set-quantifier '$\exists y$' having a broader range. But it is then difficult to see why the current interpretation of the quantifier '$\forall x$' should not simply be restricted to sets.

For surely we are in possession of an unrestricted concept of a set, not *set of such and such a sort* but *set simpliciter*. When we recognize the possibility, via the Russell jump, of a new set, we do not take ourselves to be forming new concepts of set and membership. The concepts of set and membership, of which we were already in possession, are seen to be applicable to the new object; and there is no question of these concepts embodying some further implicit restriction on the objects to which they might apply.

But given that we are in possession of an unrestricted concept of set, then why is it not legitimate simply to restrict the quantifier to sets so conceived? It might of course be argued that the quantifier should always be restricted to a relevant sort, that we cannot make sense of quantification over objects as such without some conception of which kind of objects are in question. But such considerations, whatever their merits might otherwise be, are irrelevant in the present context. For the quantifier is already restricted to a sort, viz. *set*, and so we have as good a conception as we might hope to have of which kind of objects are in question. To insist upon a further restriction of the quantifier is like thinking that we cannot properly quantify over swans but only over black swans, say, or English swans.

There is another difficulty with the model. Any satisfactory account must account for the act of reinterpretation that is involved in the Russell jump. In making the Russell jump, we go from one interpretation of the quantifier to another; and we need to provide a satisfactory account of how this is done. To simplify the discussion, let us suppose that no set belongs to itself. The Russell set over a given domain is then the same as the universal set; and so the question of the intelligibility of the Russell jump can be posed in terms of the universal rather than the Russell set. Let us now suppose that we have an initial understanding of the quantifier. represented by '$\forall x$' and '$\exists x$'. We then seem capable of achieving a new understanding of the quantifier—which we may represent by '$\forall^+ x$' and '$\exists^+ x$'—in which it also ranges over a universal set. Under this new understanding, it is correct to say that there is a universal set relative to the old understanding ($\exists^+ x \forall y(y \in x)$). The question on which I wish to focus is: how do we come to this new understanding of the quantifier on the basis of the initial understanding?

It is clear that the condition $\forall y(y \in x)$ plays a critical role; since it is by means of this condition that the new understanding is given? But how? The only answer the restrictionist can reasonably give is that the condition is used to relax the condition on the quantifier that is already in play. Thus suppose that the initial quantifier $\forall x$ is implicitly restricted to objects satisfying the condition $\theta(x)$, so that to say $\forall x\varphi(x)$ is tantamount to saying $\forall x[\theta(x): \varphi(x)]$ (every θ-object is a φ-object). The effect of considering the condition $\forall y(y \in x)$ is then to weaken the initial restrictive condition $\theta(x)$ to $\theta(x) \lor \forall y(y \in x)$, so that to say $\forall^+ x\varphi(x)$ is tantamount to saying $\forall x[\theta(x) \lor \forall y(y \in x): \varphi(x)]$.

Unfortunately, this proposal does not deliver the right results. Intuitively, we wanted the quantifier $\forall^+ x$ to include one new object in its domain, the set of all those objects that are in the range of $\forall y$. But the condition $\forall y(y \in x)$ picks out *all* those sets that have all of the objects in the range of $\forall y$ as members, and not just the set that consists solely of these objects. If we had an unrestricted quantifier Πx, then we could pick out the intended set by means of the condition $\Pi y(y \in x \equiv \exists z(z = y))$ but under the present proposal, of course, no such quantifier is at hand.[16]

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[16] One might think that the new object should be defined by the condition: $\exists I[\forall x \exists_I y(y = x)$ & $\Box \forall y(y \in x \equiv \exists_I z(z = y))]$. But since the condition is modal, it is of little help in understanding the

There is a further difficulty, which is a kind of combination of the other two. As we have seen, the required restriction on the quantifier is not just to sets but to sets of such and such a sort. But how are we to specify the supplementary non-sortal condition? It is clear that in general this will require the use of a complex predicates and not just the use of simple predicates, such as 'set'. But how are the complex predicates to be specified except by the use of lambda-expressions of the form $\lambda x\varphi(x)$? And how is the implicit restriction on the lambda-operator λx in such expressions to be specified except by means of further complex predicates? Thus it is hard to see how the specification of the relevant class of restrictions might get 'off the ground'.[17]

2.6 EXPANSIONISM

The two obvious ways of understanding the postulational modality—the circumstantial and the interpretational—have failed. What remains? I believe that our difficulties stem from adopting an unduly narrow conception of what might constitute an interpretation of the quantifier. To understand better what alternative conceptions there might be, we need to reconsider the Russell jump and how it might be capable of effecting a change in the interpretation of the quantifier.

As I have remarked, the change in the interpretation of the domain of quantification is somehow given by the condition $\forall y(y \in x)$. But rather than thinking of that condition as serving to define a new predicate by which the quantifier is to be restricted, we should think of it as serving to indicate how the range of the quantifier is to be extended. Associated with the condition $\forall y(y \in x)$ will be an instruction or 'procedural postulate', $!x\forall y(y \in x)$, requiring us to introduce an object x whose members are the objects y of the given domain. In itself, the notation $!x\forall y(y \in x)$ is perhaps neutral as to how the required extension is to be achieved. But the intent is that there is no more fundamental understanding of what the new domain should be except as the domain that might be reached from the given domain by adding an object in conformity with the condition. Thus $!x\forall y(y \in x)$ serves as a positive injunction on how the domain is to be extended rather than as a negative constraint on how it is to be restricted.

It might be wondered why the present account of how the domain is to be extended is not subject to a form of the objection that we previously posed against the restrictionist account. For what guarantees that we will obtain the desired extension? What is to prevent the new object from containing members besides those in the range of y?

relevant sense of $\Box$. Also, there would appear to be something viciously circular about specifying an interpretation in this way, since the application of $\Box$ within such conditions must be understood by reference to the very interpretations it is being used to specify. At the very least, it is hard to see how such interpretations could be legitimate unless their application could be grounded in interpretations of an ordinary, nonmodal kind.

[17] Another possibility, under this approach, is to distinguish between free and bound variables. Free variables are absolutely unrestricted, bound variables are not; and conditions with free variables can then be used to specify the relevant restrictions on bound variables. But, as with the schematic approach, it is hard to see what prevents the free variable from being bound.

The answer lies in the nature of the postulational method. For not every object can be postulated into existence. We cannot postulate, for example, that there is to be an object whom everyone admires (!x∀y(y admires x)). And likewise, we cannot postulate an object which stands in the membership relationship to pre-existing objects. But this means that, once a universal set for a given domain has been introduced, no further objects that might be introduced can be among its members. Thus the membership—and hence identity—of the set will be fixed 'for all time', once it has been introduced.

The present account of domain extension should be sharply distinguished from the restrictionist and universalist accounts.[18] Under the universalist account, the old and new domains are to be understood as restrictions; and these restrictions, in turn, are to be understood as restrictions on an absolutely unrestricted domain. Under the restrictionist account, the old and new domains are also to be understood as restrictions; but these restrictions are not themselves to be understood as restrictions of some broader domain. Under the expansionist account, by contrast, the new domain is not to be understood as a restriction at all but as an expansion. What we are provided with is not a new way of seeing how the given domain might have been restricted but with a way of seeing how it might be expanded. We might say that the new domain is understood from 'above' under the universalist and restrictionist accounts, in so far as it is understood as the restriction of a possibly broader domain, but that it is understood from 'below' under the expansionist account, in that it is understood as the expansion of a possibly narrower domain.

Another major difference between the accounts concerns the conditions and consequences of successful reinterpretation. Any attempt to reinterpret the quantifier by means of a restricting predicate will be successful under the universalist account; and it will also be successful under the restrictionist account as long as the predicate does not let in 'too many' objects. However, belief that there is a new object, that the domain has in fact been extended, is not automatically justified under either of these accounts. They do indeed provide us with a new way in which there might be a new object for, given the new understanding of $\exists^{+}$y, it may now be true that $\exists^{+}$ y∀x(x ∈ y) even though it was not before true that ∃y∀x(x ∈ y). But success in the act of reinterpretation does not in itself guarantee that there *is* such an object. Under the expansionist account, by contrast, success in the act of reinterpretation does guarantee that there is such an object. Thus if the attempt to reinterpret the quantifier $\exists^{+}$y by means of the injunction !x∀y(y ∈ x) is successful, then the inference to $\exists^{+}$y∀x(x ∈ y) will be secure.

However, successful reinterpretation, in this case, cannot simply be taken for granted. We do not need to show that there is an object of the required sort in order to be sure of success. Indeed, such a demand would be self-defeating since its satisfaction

[18] It should also be distinguished from a view that takes quantification to be relative to a conceptual scheme. One major difference is this. A procedural postulate presupposes a prior understanding of the quantifier and so it should be possible, under the postulationalist approach, to understand the quantifier in the absence of any postulates. However, it is not usually thought to be possible, under the conceptualist approach, to understand the quantifier apart from any conceptual scheme.

would require the very understanding of the quantifier that we are trying to attain. But in order successfully to postulate an object we do need to demonstrate the legitimacy of the postulate, i.e. the postulational possibility of there being an object of the prescribed sort. Given this possibility, we may then use the condition by which the object is given to secure an interpretation of the quantifier in which there *is* such an object.

It is a remarkable feature of the understanding we achieve through the Russellian jump that the very act of reinterpretation serves to secure the existence of the object in question. It is not as if we can think of ourselves as successfully reinterpreting the quantifier and then go on to ask whether, under this reinterpretation, there is indeed an object of the required sort. The one guarantees the other; and it is a key point in favor of the present approach that it is in conformity with what we take ourselves to be doing in such cases.

There is a third important difference. Both the restrictionist/universalist and the expansionist accounts allow the interpretation of the quantifier to be relative—relative to a restricting predicate in the one case and to a procedural postulate in the other. But the relativity can plausibly regarded as internal to the content in the first case. If I restrict the interpretation of the quantifier to the predicate θ, then what I am in effect saying when I say '$\exists x\varphi(x)$' is that some θ φ's. But the relativity cannot plausibly be regarded as internal to the content in the second case. If I expand the interpretation of the quantifier by means of the postulate α, then what I am in effect saying when I say '$\exists x\varphi(x)$' is simply that something φ's (but in the context of having postulated α), not that something φ's in the domain as enlarged by α. For to say that something φ's in the domain as enlarged by α is to say that something suitably related to α is a φ; and I cannot make proper sense of what this 'something' might be unless I have *already* enlarged the domain by α. We might say that the relativity in the interpretation of the quantifier is understood from the 'inside' under the universalist and restrictionist accounts but from the 'outside' under the expansionist account.[19]

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This feature of the postulationism might be thought to be at odds with our previous insistence that a postulate should serve to reinterpret the quantifier. For surely, if I reinterpret the quantifier, then what I say, before laying down a postulate, is different from what I say afterwards. Indeed, it might be thought that the postulationist, as I have characterized him, faces an intolerable dilemma. For a postulate may result in a statement changing its truth-value. But that can be so only because of a change in content or of a change in the circumstances (in what it is for the statement to be true or in what it is that renders the statement true or false). Yet, for different reasons, we have wanted to reject both of these alternatives.

I think that, in the face of this dilemma, we are forced to recognize a quite distinctive way in which a postulate may result in a change of interpretation—one that is

[19] These various differences are discussed in more detail in Fine (2005b); and other forms of external relativism are discussed in Fine (2005c) and Fine (2005d). I should note that there are some similarities between my views on domain expansion and Glanzberg's (this volume). Thus his notion of a 'background domain' corresponds to my notion of an unrestricted domain, as given by a postulational context; and his notion of an 'artifactual object' corresponds to my notion of an object of postulation.

intermediate, as it were, between a change in content and a change in circumstance, as these are normally conceived. We should bear in mind that, on the present view, there is no such thing as *the* ontology, one that is privileged as genuinely being the sum–total of what there is. There are merely many different ontologies, all of which have the same right (or perhaps we should say no right) to be regarded as the sum-total of what there is.[20] But this means that there is now a new way in which a statement may change its truth-value—not through a change in content or circumstance, but through a change in the ontology under consideration. There is another parameter in the picture and hence another possibility for determining how a statement may be true. Postulation then serves to fix the value of this parameter; rather than altering how things are within a given ontology or imposing a different demand on the ontology, it induces a shift in the ontology itself.[21]

The postulational conception of domain extension provide us with two distinct grounds upon which universalism might be challenged. It might be challenged on the ground that any interpretation of the quantifier must be restricted; and it might also be challenged on the ground that any interpretation of the quantifier is subject to expansion. It should be clear that these two grounds are independent of one another. Thus one might adopt a form of restrictionism that is either friendly or hostile to expansionism. In the first case, one will allow the expansion of the domain but the expansion must always be relative to an appropriately restricted domain (to *sets*, say, or *ordinals*); while in the second case, one will not allow an expansion in the domain and perhaps not even accept the intelligibility of the notion. Similarly, one might adopt a form of expansionism that is hostile to restrictionism. On this view, there is nothing to prevent the quantifier from being completely unrestricted; in saying '$\exists x\varphi(x)$', one is saying something φ's, period. However, this is not to rule out the possibility of expanding the unrestricted domain; the resulting quantifier is then unrestricted, but relative to a 'postulate'. Indeed, on this view it is *impossible* to regard expansion as a form of de-restriction, since there is no existing restriction on the quantifier to be relaxed.[22]

I have taken universalism to be the view that there is absolutely unrestricted quantification. Usually, the term 'absolutely' in the formulation of this view is taken to mean 'completely'; there is absolutely no restriction, i.e. no restriction whatever. But if I am right, the view is really a conjunction of two distinct positions, one signified by 'unrestricted' and the other by 'absolutely'. The first is the affirmation of unrestricted (i.e. completely unrestricted) quantification. The second is the rejection of any relativity in the interpretation of the quantifier beyond a restriction on its range; once the range of the quantifier has been specified by means of a suitable predicate, or even by the absence of a predicate, then there is nothing else upon which its interpretation might depend. It is because the view is essentially conjunctive in this way that we have been

[20] Of course, this is not how the postulationist should express himself. What he refuses to privilege is his current ontology as opposed to the various ontologies that might be realized through postulation.

[21] This new form of indexicality is further discussed and developed in Fine (2005c, 2005d).

[22] I might note that there are some intermediate positions. Thus one might suppose that there is an inexpandible domain, but one that can only itself be reached through expansion.

able to find two distinct grounds—restrictionism and expansionism—upon which it might be challenged.

I myself am tempted by the view that embraces expansionism but rejects restrictionism. I am a believer in what one might call 'relatively unrestricted' quantification. However, opposition to universalism—at least, when the issue of extendibility is at stake—has not usually been of this form. The critical question of how an extension in the domain might be achieved has rarely been broached and it has usually been supposed, if only tacitly, that the relevant interpretation of the quantifiers can only be given by means of a restriction, so that it is only through a change in the restriction that the desired change in the domain of quantification might be achieved.

We are therefore left with a radical form of restrictionism, one which requires not only a 'visible' restriction to a sort but also an 'invisible' restriction to some form of nonsortal condition (whose exact identity is never made clear). But, as I have argued, such a form of restrictionism is highly implausible, both in itself and as an account of extendibility. For the need for a non-sortal restriction lacks any independent motivation and a change in the non-sortal restriction is not, in any case, capable of accounting for the desired extension in the domain. The restrictionists have operated within an unduly limited model of how domain extension might be achieved; and I believe that it is only by embracing expansionism that a more adequate account of domain extension and a more viable form of opposition to universalism can be sustained.

2.7 EXPRESSIVITY

• Q2 I• wish, in conclusion, to consider one of the most familiar objections to the limitativist position. It is that it prevents us from saying things that clearly can be said. It seems evident, for example, that we can say that *absolutely everything* is self-identical. But how can such a thing be said, under the limitavist view, if the quantifier by which it is said is either restricted or subject to expansion? Or again, we may wish to assert that no donkey talks (cf. Williamson, 2003). Our intent, in making such a claim, is that it should concern absolutely all donkeys. But then what is to prevent it from being true simply because the domain has been limited—either through restriction or lack of expansion—to objects that are not talking donkeys?

These difficulties can be overcome by using the modal operator to strengthen the universal claims. Instead of saying everything is self-identical ($\forall x(x = x)$), we say necessarily, whatever might be postulated, everything is self identical ($\Box\forall x(x = x)$); and instead of saying no donkey talks ($\forall x(Dx \supset \sim Tx)$), we say necessarily no donkey talks ($\Box\forall x(Dx \supset \sim Tx)$). The claims, if true, will then exclude the possibility of counter-example under any extension of the domain.

If we were to read the 'absolutely' in 'absolutely all' as the postulational box, then we could even preserve some similarity in form between the natural language rendering of the claim and its formalization. However, in many cases we can rely on the unqualified non-modal claim and use suitable 'meaning postulates' to draw out the modal implications. Consider no donkey talks ($\forall x(Dx \supset \sim Tx)$), for example. It is plausibly part of the meaning of 'dog' that dogs cannot be introduced into the domain through postulation ($\exists I[\forall x \exists_I y(y = x) \& \Box \forall x(Dx \supset \exists_I y(y = x))]$) and it is plausibly part of the meaning of 'talk' that no non-talking object can be made to talk through postulation ($\forall x(\sim Tx \supset \Box \sim Tx)$).[23] But with the help of these meaning postulates, we can then derive the strengthened modal claim ($\Box \forall x(Dx \supset \sim Tx)$) from the nonmodal claim ($\forall x(Dx \supset \sim Tx)$. We therefore see that in these cases the unqualified nonmodal claims are themselves capable of having the required deductive import.

A similar device can be used, in general, to simulate the effect of absolutely unrestricted quantification. Suppose that Πx is the absolutely unrestricted quantifier of the universalist and that $\varphi(x)$ is a condition whose satisfaction is indifferent to postulational context. Then instead of saying $\Pi x \varphi(x)$, we may say $\Box \forall x \varphi(x)$, where $\forall x$ is the relatively unrestricted quantifier of the expansionist. In general, $\varphi(x)$ may be a condition whose satisfaction is sensitive to postulational context—as with the condition $\exists y(y = x)$ to the effect that x is in the current range of the quantifier. To take care of such cases, we must make use of some device to take us back to the current context (once we are within the scope of $\Box$). To this end, we can appeal to the current interpretation of the quantifier. Thus instead of saying $\Pi x \varphi(x)$, we may say $\exists I(\forall x \exists_I y(y = x) \& \Box \forall x \varphi(x)^I)$, where the embedded condition $\varphi(x)^I$ is the result of relativizing the quantifiers in $\varphi(x)$ to I.[24]

The locution Πx, as understood by the expansionist, behaves like a quantifier, it conforms to all of the right first-order principles; and the universalist can even conceive of it as having a quantificational semantics. But it is not a quantifier. Indeed, contradiction would ensue if the expansionist supposed that there were some genuine quantifier $\forall x$ for which $\Pi x \varphi(x)$ was equivalent to $\forall x \varphi(x)$, for he would then be in no position to perform a Russell jump on $\forall x$ and thereby assert the postulational possibility of some object not in the current domain ($\exists I[\forall x \exists_I y(y = x) \& \Diamond \exists x \sim \exists_I y(y = x)]$).

This curious hybrid status of the quasi-quantifier Πx is able to account for what it right and wrong about Schematism. The schematist takes us to be committed to the schematic truth of $\mathbf{x} = \mathbf{x}$; and he correctly perceives that this is not a matter of being committed to any particular universal truth, i.e. there is no understanding of the universal quantifier $\forall x$ for which the commitment to $\mathbf{x} = \mathbf{x}$ is equivalent to the commitment to $\forall x(x = x)$. But from this he incorrectly infers that to be committed to the schematic truth of $\mathbf{x} = \mathbf{x}$ is not to be committed to any particular truth (something that we previously saw to be implausible); for to be committed to $\mathbf{x} = \mathbf{x}$ is to be committed to $\Pi x(x = x)$ (or $\Box \forall x(x = x)$). Thus it is by appeal to the quasi-quantifier Πx that we may correctly represent the form of generality implicit in a schematic commitment.

The hybrid status of Πx can also be used to make sense of the obscure distinction between actual and potential infinity. It has been thought that some infinite domains

[23] I might note, incidentally, that it is unclear how such meaning postulates could have any plausibility under a radical form of restrictionism.

[24] Similar definitions of possibilist quantification in terms of actualist quantification have been proposed in connection with the metaphysical modalities (see Fine (2003) and the accompanying references). When $\varphi(x)$ contains only the unrestricted quantifiers of the universalist, the more complicated form of analysis is not required.

are definite or complete while others are 'always in the making'. But what does this mean? We can take quantification over an actually infinite domain to be represented by a genuine quantifier $\forall x$ and quantification over a potentially infinite domain to be represented by the quasi-quantifier Πx. The domain is then potential in that it is incapable of being exhausted by any actual domain ($\Box\forall I(\forall x \exists_I y(y = x) \supset \Sigma x \sim \exists_I y(y = x))$; and we can take the peculiar features of quantification over a potential domain, and its inability to sustain domain expansion, to rest upon its underlying modal form.[25]

We see, once the notion of postulational necessity is on the table, that the charge of expressive inadequacy is without merit. The expansionist can, in his own way, say everything that the universalist says. The difficulty over expressive inadequacy lies, if anywhere, in the other direction. For the expansionist can make claims about what is or is not postulationally possible or necessary. But how is the universalist to express these claims? Presumably, for a proposition to be postulationally necessary is for it to be true in all relevant domains. Not all domains whatever, though, since any of the domains should be capable of expansion. But then which ones? It seems to me that, in response to this question, the universalist must either make a substantive assumption about the domains in question, such as that they are all of 'smaller size' than the universe as a whole, or he must work with a primitive notion of the relevant domains. They are ones that in some unexplained sense are 'definite' or 'complete'.

Of course, the universalist will not be happy with the way the expansionist expresses absolutely unrestricted generality. This notion, he wants to say, is quantificational, not modal. But likewise, the expansionist will not be happy with the way the universalist expresses postulational necessity. This notion, he wants to say, is modal in form, not quantificational. It therefore appears as if there is some kind of stale-mate, with neither side enjoying a decided advantage over the other.

I believe, however, that there are some general theoretical considerations that strongly favor the expansionist point of view. For the idea behind expansionism can be used as the basis for a new approach to the philosophy of mathematics and to the philosophy of abstract objects in general. This approach is able to provide answers to some of the most challenging questions concerning the identity of these objects, our understanding of the language by which they are described, and our knowledge of their existence and behavior. Its ability to answer these questions and to throw light over such a wide terrain may well be regarded as a decisive point in favor of the expansionist position.[26]

REFERENCES

Cartwright R. (1994)] 'Speaking of Everything', *Nous* 28, v. 28, no. 1, pp. 1–20.
Dummett M. (1981) *Frege: Philosophy of Language*, Duckworth: London.

[25] Dummett's ambivalence between rejecting absolutely unrestricted quantification (1981, pp. 529, 533) and allowing it within the setting of intuitionistic logic (1981, 529–30) can perhaps also be explained in similar terms.

[26] Some details of the expansionist approach are to be found in Fine (2005b) and I hope to give a fuller account elsewhere.

Dummett M. (1991) *Frege: Philosophy of Mathematics*, Cambridge, Massachusetts: Harvard University Press.

Field H. (1989) *Realism, Mathematics and Modality*, Oxford: Blackwell.

Fine K. (1990) 'Quine on Quantifying In', in *Proceedings of the Conference on Propositional Attitudes*, ed. Anderson, Owens, CSLI, reprinted in Fine (2005e).

____ (2002) 'The Varieties of Necessity', in *Conceivability and Possibility*, eds. T. S. Gendler and J. Hawthorne, Oxford: Clarendon Press, 253–82, reprinted in Fine (2005e).

____ (2003) 'The Problem of Possibilia', in *The Oxford Handbook of Metaphysics*, eds. D. Zimmerman and M. Loux, Oxford: Clarendon Press, 161–179; reprinted in Fine (2005e).

____ (2005a) 'Response to Papers on "The Limits of Abstraction" ', *Philosophical Studies*.

____ (2005b) 'Our Knowledge of Mathematical Objects', to appear in a volume on epistemology, Oxford: Clarendon Press, 2005, eds. J. Hawthorne *et al*.

____ (2005c) 'The Reality of Tense', to appear in *Synthese*, 2005, in a volume dedicated to Arthur Prior.

____ (2005d) 'Tense and Reality', in Fine (2005d).

____ (2005e) *Modality and Tense: Philosophical Papers*, Oxford: Clarendon Press.

Glanzberg M. (2001) 'The Liar in Context', *Philosophical Studies* 103, 217–51.

____ (2004a) 'A Contextual-Hierachical Approach to Truth and the Liar Paradox', *Journal of Philosophical Logic* 33, 27–88.

____ (2004b) 'Quantification and Realism', *Philosophical and Phenomenological Research* 69, 541–72.

Lewis D. (1991) *Parts of Classes*, Oxford: Blackwell.

McGee, V. (2000) 'Everything', in *Between Logic and Intuition* (eds. G. Sher and T. Tieszen), Cambridge: Cambridge University Press, 54–78.

Parsons C. (1974) 'Sets and Classes', *Nous* 8, 1–12.

Parsons T. (1980) *Non-existent Objects*, New Haven: Yale University Press.

Putnam H. (2000) 'Paradox Revisited II: Sets—A Case of All or None', in *Between Logic and Intuition: Essays in Honor of Charles Parsons*, eds. G. Sher and R. Tieszen, Cambridge: Cambridge University Press, 16–26.

Rosen G. (1995) 'Armstrong on Classes as States of Affairs', *Australasian Journal of Philosophy* 73 (4), 613–625.

Shapiro S. (2003) 'All Sets Great and Small; and I do mean ALL', *Philosophical Perspectives* 17, 467–90.

Williamson T. (2003) 'Everything', *Philosophical Perspectives* 17, 415–65, to be reprinted in *The Philosopher's Annual, vol. 26*, ed. P. Grim *et al*., available online at www.philosophersannual.org.

Queries in Chapter 2

Q1. Please provide closing quote

Q2. We have changed section no. ‘2.6’ to ‘2.7’. If this is fine. Please clarify.