Vagueness, Partial Belief, and Logic

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In his recent work on vagueness and indeterminacy, and in particular in Chapter 5 of *The Things We Mean*,[1] Stephen Schiffer advances two novel theses:

1. Vagueness (and indeterminacy more generally) is a psychological phenomenon;

2. It is indeterminate whether classical logic applies in situations where vagueness matters.

He also puts forward a third thesis:

3. The "uncertainty" we have about whether to believe that Harry is bald when we take

him to be a borderline case is very different from ordinary uncertainty; so different

as to not really deserve the term 'uncertainty'.

This last thesis is opposed to the views of "epistemic theorists" such as Timothy Williamson[2], but the thesis as I've stated it is probably one that most theorists of vagueness would accept. Even so, Schiffer develops it in a highly novel way, according to which there is a special kind of partial belief heretofore unrecognized.

After an introductory section reviewing some background and locating his approach with respect to some others, I will turn to the thesis about the indeterminacy of logic, move on to his version of the thesis about two kinds of "uncertainty", then to the thesis that vagueness is a psychological phenomenon, and then back to the thesis about the indeterminacy of logic.

I should say that Schiffer's views and the arguments that support them are extremely

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[1] Oxford University Press 2003. All page references will be to this book.

[2] *Vagueness*. Routledge 1994.

interesting and important, and I have probably learned more from him on this subject than I have from anyone else—from many conversations over the years, as well as from his writings. By most standards, our views are quite close; so in order to give him his fair share of abuse I will have to stress some issues that those with more radically different views may regard as relatively minor.

1. Background on epistemicism and supervaluationism. Schiffer begins his chapter by considering a number of arguments related to the Sorites paradox, one of which is essentially as follows:[3]

> I   A person with $50,000,000 is rich
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> II   A person with 37¢ is not rich
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> So   III   There is an amount of money such that a person with that amount of money is rich but a person with 1¢ less isn't.

I'll call this *the Least Number Sorites*. (Of course, we're indulging in the fiction that richness is simply a matter of net assets; and since 'rich' is doubtless context-relative, we're imagining the context fixed.) Presumably the premises of this argument are true, and the argument is classically valid. (The conclusion is essentially a disjunction of about five billion conjunctions of form *Rich(x) and not-Rich(x −0.01)*; it follows from the premises by about that many applications of excluded middle together with the distributive law.) But the conclusion of the argument is *prima facie* implausible: it seems to assert a *sharp cut-off* between the rich and the non-rich, and this

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[3] In Schiffer's version the conclusion is of form $\neg\forall x[A{\rightarrow}B]$ instead of $\exists x[A{\wedge}\neg B]$, but since he stipulates that the conditional is material and shows no tendency to dispute normal quantifier interchange rules or double-negation elimination, this difference is presumably not significant.

seems contrary to the evident vagueness of the term.

Not only is the conclusion *prima facie* implausible, it also raises some embarrassing questions, as Schiffer points out. For instance, it seems fairly clear that no one knows where this sharp cutoff is, and that sociologists could never find out the answer to that question no matter how heavily their studies were funded. And (the embarrassing part) it doesn't seem as if this inevitable ignorance can be explained in *anything like* the ways that one can explain other inevitable ignorance (e.g. about the very remote past or the fine details of what's happening inside a given black hole). Rather, the natural explanation of our inability to know the location of the cutoff is that the issue "isn't fully factual". But *that* explanation doesn't seem to fit with III.

Admittedly, it is very difficult to make clear sense of the claim that the location of the cutoff "isn't fully factual". The difficulty has led many to reluctantly conclude that *the epistemic theory* is right. According to it, there must be a sharp cutoff, even though we can never know where it is; vagueness is just a kind of inevitable ignorance. The epistemic view has it that the location of the boundary between rich and non-rich is a straightforward matter of fact; an omniscient being (one who knew all the facts) would know where the cutoff point is. *Our* inability to know the cutoff point is due to human limitations; and there is no conceptual bar to aliens from outer space knowing this fact about which we are ignorant.

Let's be clear: on the epistemic view, it isn't simply that the aliens might have a precise concept instead of our concept of richness, and know where the cutoff of that precise concept is. No, it's that they might know where the cutoff of *our* concept of richness is. (And it isn't merely that they might have a more precise concept of cutoff than we have, call it cutoff*, and know the

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cutoff* of our concept of richness. No, it's that they might know where the cutoff of our concept

of richness is, *in our sense of cutoff*.) If only we were endowed with a special organ for detecting

richness, we'd be as well off as they are. This position appears to be an almost inevitable result of

accepting the validity of the Least Number Sorites (and hence, presumably, accepting its

conclusion III). Schiffer is highly skeptical of the position, and I think rightly so.

He is also properly skeptical of attempts to accept the validity of the argument but avoid

the epistemic theory. The most influential such attempt is called *supervaluationism*. According

to it, III does not assert a sharp cut-off between the rich and the non-rich. In one formulation of

supervaluationism, the claim that there is a sharp cutoff is rather

> III_T There is a number of dollars x_0 (a multiple of 0.01) such that *it is true that*
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> a person with x_0 dollars is rich and *it is true that* a person with $x_0 - 0.01$
>
> dollars isn't.

Obviously this would be equivalent to III if *it is true that p* were assumed equivalent to p (in a

sense of equivalence that allows for substitutivity of equivalents in extensional contexts), but this

form of supervaluationism takes *it is true that p* to be strictly stronger than p. According to this

version of supervaluationism, III is perfectly acceptable, and so is

> *It is true that* there is a number of dollars x_0 such that a person with x_0 dollars is
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> rich and a person with $x_0 - 0.01$ dollars isn't.

But III_T, with the 'it is true that' distributed to the inside, is not: sentences of form 'a person with

x dollars is rich' for which x is near to the critical x_0 are neither true nor false, i.e. neither they nor

their negations are true.

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In making *it is true that p* stronger than *p*, this form of supervaluationism seems to do violence to the ordinary notion of truth; but a variant form of supervaluationism avoids that, by explicating the notion of sharp cut-off not by III_T but by

III_D There is a number of dollars x_0 such that *it is determinately true that* a person with x_0 dollars is rich and *it is determinately true that* a person with x_0-0.01 dollars isn't.

Supervaluationism of this form mimics that of the other form: again, III is deemed acceptable, as is

It is determinately true that there is a number of dollars x_0 such that a person with x_0 dollars is rich and a person with x_0-0.01 dollars isn't;

but III_D isn't, and this is supposed to capture the idea that there is no sharp cutoff.

I think these two forms of supervaluations differ only verbally. Schiffer disagrees with that assessment, for reasons I don't fully understand (195-6); but it probably doesn't matter much, because he is skeptical of both forms, and his discussion suggests that neither form succeeds in its attempt to be significantly different from epistemicism.

For in the first place, it's obscure what 'determinately true' means in the III_D version: as Schiffer notes, the usual supervaluationist attempts to explain it are blatantly circular. It seems to me equally obscure what 'true' means in the III_T version: the usual explanation of 'true' in terms of the equivalence between 'it is true that *p*' and '*p*' obviously is inapplicable to 'true' as used in III_T, and the point about the circularity of a supervaluationist explanation carry over.

In the second place and more important, it is unclear why anyone should care if in this

special sense of determinate truth (or of truth), claims such as 'a person with x_0 dollars is rich' and 'a person with $x_0 - 0.01$ dollars isn't rich' aren't determinately true (or aren't true). *For III alone, without III$_T$ or III$_D$, suffices for a commitment to the existence of the critical value x_0.* If there is such a critical value, why can't we know it? It wouldn't be much of an explanation to say that knowledge of p requires that p be determinately true, or be true in the special sense assumed in III$_T$: for the only uncontentious requirement here is that one can't know that p unless p. If one really thinks there is a critical point x_0 satisfying III, why couldn't there be a god or an alien who had the means to reliably ascertain what this critical value is?

It's worth emphasizing that Schiffer does not object to the claim that it is indeterminate where the cutoff between rich and non-rich is: indeed, such a claim is an important part of his own theory. But (i) we need some explanation of indeterminacy that explains why this claim is of interest; and (ii) we need to drop our commitment to III, since that commitment undermines any interest the claim might have. I think this is exactly the right approach to take.

It's also worth emphasizing that Schiffer doesn't assert that the cutoff between the rich and the non-rich *isn't* sharp—that is, he does not assert the negation of III. Indeed, he notes (quite correctly) that though accepting the conclusion III would commit one to epistemicism or something in that ballpark, accepting the negation of III would appear to be as bad or worse.

One reason for this (the one Schiffer gives) is that unless normal quantifier and negation rules are questioned (a radical step that Schiffer shows no inclination to take), the negation of III is equivalent to

N-III For any amount of money, either a person with that amount of money is not rich or

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a person with 1¢ less is rich.

But that gives rise to another form of Sorites argument: N-III together with I classically imply the negation of II, and yet I is obviously true while the negation of II is obviously not. Moreover, the classical inference involved here seems intuitively to be at least as strong as the one in the Least Number Sorites: it involves only universal instantiation and "disjunctive modus ponens" (the classically valid rule of inferring *q* from *not p or q* together with *p*). Universal instantiation is hard to question (and can be avoided, in an alternative version of the argument); so accepting the negation of III would seem to require giving up disjunctive modus ponens (i.e. regarding it as invalid or at least not determinately valid), and that seems a high cost.

Actually this reason against accepting N-III isn't in the end one that Schiffer is entitled to, for he eventually argues (224) that (disjunctive) modus ponens *isn't* determinately valid. (I think this is a mistake, as I'll argue in the next section, but it is his view.) Because of this, it's worth mentioning another argument he might have given. The argument is that on the natural assumption that adding money doesn't destroy richness (together with the assumption, implicit in the whole discussion, that richness is just a matter of net assets) N-III implies

> For any amount of money, either a person with that amount of money is not rich or
> a person with that amount of money is rich.

But that conclusion (generalized excluded middle) can itself be argued to lead to epistemicism or its supervaluationist variant, by a route very much like the one above: it seems to posit unknowable facts, as to whether people with *x* dollars in net assets are rich (even for values of *x*

that intuitively correspond to the "border region").[4]

So we have strong reason to reject III and strong reason to reject its negation; and we can accommodate both these reasons, by rejecting the instance of excluded middle *III or not-III*. Rejecting here doesn't mean accepting its negation; what it means, roughly, is adopting a policy that precludes accepting it. The claim that we should reject this instance of excluded middle will turn out to be pretty much equivalent to the claim that III should be regarded as neither determinately true nor determinately false, on Schiffer's eventual explication of determinateness.

So we can summarize his attitude toward the Least Number Sorites by saying that it has determinately true premises, but that it's indeterminate whether it's conclusion is true. And I think this is exactly the line one ought to take.

2. Schiffer on the indeterminacy of logic. Since Schiffer takes the premises of the Least Number Sorites to be true (indeed, determinately true), but doesn't take the conclusion to be true (and indeed, assumes it not to be determinately true), it would seem natural for him to conclude that the Least Number Sorites isn't really valid, despite the fact that classical logic says it is. That

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[4] If it seems puzzling that III and its negation both lead to essentially the same problem, it may help to reflect on the negation of an instance of excluded middle: *not-(p or not-p)*. This certainly implies *not-p* (the negation of a disjunction certainly implies the negation of the first disjunct), and that in turn certainly implies *p or not-p* (a disjunction is certainly implied by its second disjunct). So the negation of an instance of excluded middle implies that instance, by reasoning that would be very hard to contest; and so if accepting an instance of excluded middle leads to problems, accepting its negation leads to the very same problems.

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is the conclusion offered by most proponents of a non-classical "logic of vagueness". As noted, the question of the validity of the Least Number Sorites turns on the law of excluded middle—about fifty billion applications of it, in fact, many of which are intuitively to "borderline propositions". So a consequence of this non-classical view is that the law of excluded middle is not unrestrictedly valid.[5]

But Schiffer insists that the proper conclusion is *not* that the Least Number Sorites or the law of excluded middle is invalid. (He strongly suggests that that would constitute what he calls a "Happy-Faced Solution" to the Sorites paradox, since it would offer a definite place where the Sorites argument goes wrong. He is quite insistent that we need an "Unhappy-faced Solution".) He says, rather, that the question of whether the argument is valid is itself indeterminate: the argument is neither determinately valid nor determinately invalid.[6] Or rather, he says that that's so

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[5] If validity is a purely formal notion, the 'unrestrictedly' is unnecessary: the view is that *no* sentence of form *p or not-p* is valid, though of course those in which the disjuncts display no relevant vagueness are true and are appropriate to use as non-logical premises in arguments. But some people may prefer to speak of those instances of excluded middle in which the disjuncts display no relevant vagueness as valid, and I have formulated the claim in the text so as to accommodate them. I doubt that there is more than a verbal issue here.

[6] One sometimes hears people advocate another means of avoiding the conclusion that some classically valid arguments are invalid: they say that classical logic is not properly applied in situations where vagueness may matter. This way of "saving" classical logic seems wholly uninteresting—one can always "save" a logic from revision by declaring that cases that seem problematic for it aren't really in its scope—and Schiffer doesn't display any tendency to go in that

on the common sense notion of validity; and I believe he thinks that any notion on which the argument is determinately invalid is a merely technical notion lacking philosophical interest. In the ordinary sense of 'valid', calling an argument valid requires not that it necessarily preserve *determinate* truth, but that it necessarily preserve *truth* (in the ordinary sense of truth in which asserting the truth of p is equivalent to asserting p). So because there is no possible case where we are in a position to say that the premises are true and that the conclusion isn't, we can't assert that the argument is invalid, we can only say that it isn't determinately valid. His conclusion would seem to be that the epistemic theory (or the disjunction of it and supervaluationism, if that is taken as a distinct theory) is not determinately false, despite his criticisms of it;[7] we shouldn't believe it, but we shouldn't disbelieve it either.

I have two points to make about this (for now—there will be a further discussion in the final section). The less interesting is that I don't think it at all clear that "the ordinary sense of the term 'valid'" is what Schiffer takes it to be. In fact, I'm not at all clear that there is a unique "ordinary sense": there are a bunch of different ways in which one might explicate 'valid' which happen to coincide under the assumption of classical logic (or if you like, of its determinate validity) but can come apart elsewhere, and I doubt that one of these senses is privileged as what

direction.

[7] 'Determinate' here is used not in accordance with the supervaluationist theory, but in a sense that Schiffer explains and that I will get to shortly; for now it suffices to say that to call a theory not determinately true does not imply that it isn't true, but does imply that it can't rationally be believed with certainty under ideal conditions, and that one can't rationally have even a positive *standard* degree of belief in it under ideal conditions.

is "really meant" by people unaware of the distinction.[8] And even if there is a unique "ordinary sense" of 'valid', I don't see why we should glorify it: if there is another sense of 'valid' that is extensionally equivalent to the first under the assumption of classical logic but which may no longer be when that assumption is removed, shouldn't the important question be which of the two is more theoretically fruitful?

My second and I think more interesting point is that even if we stick to Schiffer's preferred sense of 'valid'—necessary truth preservation—it is not at all clear that the inference from I and II to III isn't determinately invalid. For what sort of necessity is involved here? Not "metaphysical necessity", whatever exactly that is: for the inference from 'There is water in the sink' to 'There is H_2O in the sink' preserves truth by metaphysical necessity, but is not valid. The notion of necessity that is involved, rather, is *logical* necessity. And it could certainly be argued that the fact that premises I and II are determinately true and III isn't is enough to show that *logic* doesn't require that if I and II are true then so is III, so that it's enough to show that *it is not logically necessary that* the inference preserves truth.

I don't say that it is *clear* that on Schiffer's "ordinary sense" of validity, the inference in question is determinately invalid. What the above paragraph seems to me to show, rather, is that

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[8] If one of the senses *is* "the one really meant" by the ordinary folk, I think ascertaining which one it is that has this status would require an exquisite faculty of distinguishing meaning from deep theoretical commitment or presupposition; a faculty not all that far removed from the faculty of detecting the cutoff between the rich and non-rich, whose absence the epistemic theorist bemoans.

even this "ordinary sense" is non-univocal: its application may be clear in the (determinately) classical case, but once we give up our allegiance to classical assumptions there is room to interpret the necessity claim in different ways, and the difference matters to whether we regard question of necessary truth-preservation as determinate.

It may seem that it doesn't much matter to broader issues whether the inferences are declared invalid or merely not determinately valid. Perhaps not—though from Schiffer's insistence on the latter choice over the former (e.g. 229), it is clear that *he* takes it to matter.

And one place that it *seems* to matter is to Schiffer's frequent hand-wringing about how hard it is to wear a happy face while theorizing about vagueness. (Pp. 196-8, 224-31.) Since he takes it to be indeterminate whether the Least Number Sorites is invalid (and because he says more generally that it is indeterminate whether *any* classical inferences are invalid, a matter I'll come back to in the final section of the paper), then he presumably also think that despite his criticisms of epistemicism and supervaluationalism, they are not determinately incorrect.[9] And he presumably thinks that standard theories of vagueness involving non-classical logics, which take epistemicism and supervaluationism to be determinately incorrect, do not offer acceptable theories of vagueness: they are too "happy-faced". I can't say with complete confidence that Schiffer accepts these conclusions, but they are ones that a reader would naturally draw from his

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[9] I don't think this exhausts Schiffer's rationale for hand-wringing: that seems to be partly based also on his view (i) that there is a reasonably clear contrast between analyzing a concept and reforming it, (ii) that the requirements for analyzing it are fairly stringent, and (iii) that a philosopher should only be happy if he can analyze it.

discussion, and he doesn't disavow them.

But however we decide on these matters, there is a vastly more important issue in the vicinity. It concerns Schiffer's *rationale* for declaring that these inferences are not determinately invalid. That rationale (that validity is necessity of truth preservation) not only undermines claims of invalidity, it undermines claims of validity as well. For instance, as Schiffer himself notes (224), if we interpret 'valid' in this way then modus ponens comes out not determinately valid. Consider the argument from *A* and *If A then B* to *B*, where the "if ... then" is taken as the material conditional: when *A* is indeterminate and *B* is clearly false, we're in no position to conclude that the argument even *actually* preserves truth, let alone does so *necessarily*. To me it seems odd not to declare modus ponens valid in Schiffer's theory, given that he would agree that once we've established the premises of a modus ponens we've established its conclusion.

If it were merely modus ponens that we couldn't declare valid, we could learn to live with that. But the point about the determinateness of validity claims may go far deeper than Schiffer acknowledges: it may indicate that on Schiffer's "ordinary definition" of validity, *no* argument comes out determinately valid.

The reason is that talk of necessary truth preservation involves a conditional: necessarily, *if* the premises are true *then* so is the conclusion. The question then arises how the conditional in the truth-preservation claim is to be understood. Schiffer doesn't give the answer in this chapter, but of course it depends on the question of what kind of embeddable conditional connectives are

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available once you give up commitment to excluded middle.[10] And it is not obvious that any embeddable conditionals are available that would allow Schiffer to declare any inferences necessarily truth preserving, and hence valid in his ordinary sense.

For instance, one possible view[11] is that the only embeddable conditional connective that makes sense is the material conditional $p \supset q$ (equivalent to *not-p or q*). (That would not preclude our making sense of conditional sentences that don't behave like material conditionals, as when I deny that if I run for President I'll win while also denying that I'll run for President: for we can say that my low "degree of belief" that if I run for President I'll win doesn't consist in my attaching a low value to a claim that involves an embeddable conditional connective, it is simply a matter of having a low conditional degree of belief in winning given the assumption of running for President.) Now, the material conditional has the property that when its antecedent and consequent are indeterminate, it too is indeterminate[12] (where 'indeterminate' means 'neither determinately true nor determinately false'). But a claim of necessary truth-preservingness has the form "Necessarily, if the premises are true then so is the conclusion"; and whatever the premises

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[10] I say *embeddable* conditionals because an explication of 'valid' in terms of a conditional must explain its use not only in atomic sentences but also in the scope of other operators.

[11] Which has recently been given a vigorous defense by Tim Maudlin: *Truth and Paradox*, Oxford University Press 2004.

[12] At least, this is so on Schiffer's conception of indeterminacy, on which the rules for determinate truth, determinate falsity and indeterminacy behave compositionally as in the strong Kleene truth tables.

and conclusion are, it is always logically possible for them to be indeterminate.[13] So necessary truth-preservation, *understood in terms of the material conditional*, is something that can *never* determinately hold. Not even the inference from a sentence to itself, or from a conjunction to its conjuncts, would come out determinately valid in Schiffer's sense if we took that to be defined in terms of the material conditional.[14] And this problem is unaffected by how we interpret 'necessarily' (as long as it implies 'actually'): the problem is the commitment to actual truth-preservation, as defined by the material conditional.

To what extent is this a problem for Schiffer? His own view of conditionals, given in a later chapter, is not the material conditional view (nor is it a version of the conditional degree of belief view which does without an embeddable conditional). Still, it seems to give rise to a similar problem. For on his view, a conditional *if p then q* is determinately true only if it "owes its determinate truth ... to the satisfaction of a non-conditional sufficient condition for its being determinately true" (298), and in the case where the antecedent and consequent aren't both determinately true the sufficient condition must be that the antecedent "determinately metaphysically or physically entails" the consequent. Let's apply this to a conditional of form

(C)   If the premises of argument X are true then so is the conclusion,

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[13] I'm assuming that the premises and conclusion are stated in a language whose only logical connectives are those under discussion in Schiffer's current chapter, namely the Kleene connectives.

[14] When I made the point in a seminar a couple of years ago, I restricted it to arguments in which the conclusion is not among the premises. I'm grateful to Cian Dorr for pointing out that that restriction is completely unnecessary.

where neither the premises of X nor the conclusion of X is determinately true. (Argument X can be of any form you like, say from the premise p to the conclusion p or q; in that example, focus on the possibility that p and q are both indeterminate.) Then the view seems to say that that conditional can only be determinately true if it "owes its determinate truth" to the determinate validity of argument X. But the validity of argument X, according to Schiffer, just *is* its necessary truth-preservingness, which can only hold if (C) is true and thus can only hold determinately if (C) is determinately true. Given this, it would seem that (C) can't be determinately true: if it were, it would have to "owe" that determinate truth to the determinate truth of itself, which seems absurd.[15]

I think, then, that on Schiffer's own view of the conditional, his views about the determinate validity of inferences require that *no inference be determinately valid*—a conclusion that strikes me as having an intolerably high cost.

But the argument that no inference comes out determinately valid depends not only on his view about validity, but also on his view about conditionals. I myself think that there is good

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[15] One might weaken Schiffer's claim a bit, by dropping the 'owe' rhetoric and simply saying that in cases where the antecedent and consequent of a conditional aren't both determinately true then the conditional is determinately true only if the antecedent "determinately metaphysically or physically entails" the consequent. That would be enough to undermine my argument that the inference from p to p or q comes out not determinately valid for Schiffer, but it would not be enough to settle that it was determinately valid on his theory. I guess you could say that that inference wouldn't be *determinately determinately* valid; no inference would.

reason to want another kind of conditional connective when we weaken classical logic, so perhaps the point of the previous three paragraphs is not worth making too big a deal of. Even so, it seems a somewhat unfortunate feature of his account of validity that its acceptability should turn on this technical issue about conditionals.

Moreover, if the only kinds of conditionals that evade the argument of the previous two paragraphs are deemed technical devices unknown to ordinary folk, then presumably they are unavailable for understanding Schiffer's "ordinary sense" of validity. We'd still be stuck with the conclusion that in "the ordinary sense" of validity, no inferences are determinately valid, a conclusion that not even Schiffer wants to endorse. So if one insists that only "the ordinary sense" is of philosophical interest, perhaps one should wear a *very* unhappy face!

3. Schiffer on partial belief. I now turn to Schiffer's view that there are two different kinds of partial belief (belief that may fall short of certainty, i.e. belief to a degree that may be less than 1).

What Schiffer calls *standard partial belief* (SPB) is *initially* explained as the kind of partial belief that under suitable idealizations of logical omniscience can be identified with subjective probability; the usual laws of subjective probability constrain it, in the sense that a rational agent would try to make her degrees of belief accord with those laws to the extent possible.

What Schiffer calls *vagueness-related partial belief* (VPB) is something one has (to a non-zero degree) only to propositions that one regards as perhaps indeterminate. Imagine a person who seems to you neither a clear case of being bald nor a clear case of not being bald.

Then you have a (non-zero) VPB in the claim that he is bald, and a (non-zero) VPB in the claim that he is not bald. These VPB's obey unusual laws. For instance, Schiffer thinks that if Harry is a "paradigm borderline case" of baldness and also a "paradigm borderline case" of thinness, then our VPB's in the following claims should all be ½:

Harry is bald

Harry is not bald

Harry is thin

Harry is not thin

Harry is bald and thin

Harry is bald and not thin

Harry is thin and not bald

Harry is not thin and not bald.

So though the last four claims are mutually exclusive in classical logic, Schiffer thinks that our vagueness-related degrees of belief in them will add to more than 1 (as Paul Horwich pointed out to me).[16] Just *why* he thinks this is something I defer for the moment; for now the point is only to illustrate how different VPB is from SPB on Schiffer's account.

The claim that there are two kinds of partial belief may seem mysterious. Is it that propositions divide into two classes, those that we have SPB's toward and those that we have

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[16] Schiffer does however think that for the classical partition {p, *not-p*}, the vagueness-related degrees of belief can never sum to more than 1 (and must sum to exactly 1 when p is a clearly borderline proposition).

VPB's toward? Even if we allowed such a division of propositions to depend on the agent and her situation, that would be a highly unpromising view, for many reasons. For instance, presumably we could conjoin (or disjoin) propositions in one class with those in the other, and if so, would our degree of belief in the conjunction (or disjunction) be an SPB or a VPB? And consider our attitude toward a person we meet who's wearing a hat; we have no idea if he's a clear case of baldness, a clear case of non-baldness, or a borderline case. In that situation, is our degree of belief that he is bald an SPB or a VPB?

Because of issues like these, Schiffer rejects the idea that there are for a given agent two classes of propositions, those she has SPB's toward and those she has VPB's toward. (That had been his view in earlier papers.) Instead he adopts the much more promising view, that toward any kind of proposition we can and typically do have both kinds of degrees of belief. In the situation with the person wearing a hat, suppose we think it 20% likely that he is clearly bald, 50% likely that he is clearly non-bald, and 30% likely that he is borderline; ignore higher order vagueness. In that case, the SPB that he is bald is .2 and the SPB that he is not bald is .5; the VPB's that he is bald and that he is not bald are underdetermined by the description, but they are both non-zero and add to .3. In general, SPB's and VPB's are governed by the following laws among others:

> (i) $0 \leq S(p) \leq 1$ and $0 \leq V(p) \leq 1$
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> (ii) $S(p) + S(\textit{not-p}) + V(p) + V(\textit{not-p}) = 1$
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> (iii) $V(p) > 0$ if and only if $V(\textit{not-p}) > 0$.

(This is discussed in Section 5.7 of his chapter.)

Given this, Schiffer's initial explanation of SPB's as involving the standard probability laws seems quite misleading: the fact is that in situations where vagueness may matter (i.e. where V(p) and hence V(*not-p*) are non-zero for some of the propositions that are in question), the usual laws of subjective probability may dramatically fail *for SPBs as well as for VPB's*, on Schiffer's view. In particular, the standard degrees of belief in p and its negation will add up to less than 1 in this situation. In cases where one is sure that p is a borderline case, V(p) and V(*not-p*) will sum to 1; so the standard degrees of belief in p and *not-p* will in those cases be 0.

In n. 38 (pp. 209-210) Schiffer criticizes a view of mine that uses only a single notion of degree of belief Q with the property that Q(p) and Q(*not-p*) can sum to less than 1, and are each 0 in cases where the agent is sure that the p is borderline (borderline true). Given that his own standard degree of belief function S has this property, I take it that the point of the criticism must be not that what I say about degree of belief is wrong but rather that it is incomplete: that it neglects another important kind of degree of belief. (Or rather, that it neglects something *that may or may not be a kind of degree of belief, but ought to be discussed while discussing degree of belief*: for Schiffer eventually concedes that VPB's may be partial beliefs only in a Pickwickian sense (232).)

Is there this other important kind of degree of belief (or degree of Pickwickian belief)? To some extent the plausibility of Schiffer's case that there is depends on his initial characterization of standard partial belief, which as we've seen he doesn't in the end believe, according to which SPB's obey the classical laws of probability (qualifications about lack of logical omniscience etc. aside). If SPB's really had to obey the standard laws, then the case for VPB's would be strong.

For suppose we are faced with Harry who we take to be a clear borderline case of baldness. If there were only SPB's *and they had to obey classical laws*, we would need to believe to some degree z that Harry is bald and to degree $1-z$ that he is not, and these degrees of belief would have to function very much like our other degrees of belief, e.g. in who will win the next presidential election. Don't worry about the precision involved in assigning a particular number to our degree of belief about Harry's baldness—that problem arises even for our degrees of belief about the election, and can presumably be handled in a refinement of the theory that allows for indeterminacy in degrees of belief. Worry instead about the fact that there is a formal similarity between the partial belief in Harry's baldness and the partial belief in the election result, even though these cases intuitively seem very different. As Schiffer puts it, standard degree of belief is usually thought of as a measure of *uncertainty* or *ignorance*; but intuitively, in the case of Harry's baldness *there is no fact about which we are ignorant*. Admittedly, that intuition is not easy to make sense of, as epistemic theorists are fond of pointing out. Still, almost everyone thinks there's an important sense in which the question of Harry's baldness isn't something that we are ignorant about in the sense in which we might be ignorant of Thales' maternal grandmother's day of birth; and Schiffer's claim that our attitudes are or should be VPB's rather than SPB's is supposed to capture the intuition reasonably well.

So given *Schiffer's initial characterization* of SPB's (according to which they must obey the standard laws of probability, qualifications about logical omniscience aside), it looks as if one needs VPB's to account for the fact that even though we refuse to fully believe that Harry is bald or fully believe that he is not bald, we don't think of ourselves as ignorant about anything. But given *Schiffer's actual theory*, one doesn't need VPB's to account for this. His actual theory is

that in a case of certainty that Harry is borderline, our standard degrees of belief that he is bald and also that he is not bald are both 0. I believe that on his theory, our standard degree of belief that *he is either bald or not bald* is also 0 (as is our standard degree of belief that *it is not the case that he is either bald or not bald*).[17] This seems enough by itself to undermine the sense that we are committed to belief in a fact about which we are ignorant, even without introducing VPB's. What Schiffer's introduction of VPB's adds is that in this situation, we *also* have non-zero VPB's in the claim that Harry is bald and that he is not bald, which add to 1. Whatever purpose the introduction of these VPB's may serve, it doesn't seem to be needed, or to help, merely for the purpose of undercutting the idea that our situation with regard to Harry's baldness is one of ignorance.

What then does Schiffer think VPB's *are* needed for? On the basis of his criticism mentioned several paragraphs back, it seems that he thinks it's needed to reflect *the ambivalent attitude* we have in borderline cases. Suppose Harry is a pretty clear borderline case of baldness; we do nonetheless have some tendency to call Harry bald and some tendency to call him non-bald; and as more hair is removed we may have more of a tendency to call him bald, even while still thinking of him as a borderline case. I believe that Schiffer takes the ratio of our VPB in Harry's baldness to our VPB in his non-baldness to reflect the ratio of our inclination to call him bald over

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[17] This pair of claims follows by (ii) from the claim that in clear borderline cases, $V[\textit{not-(p or not-p)}] = 1 - V[\textit{p or not-p}]$, which appears to be part of what's asserted in the negation law on 218. (This assumes that *p or not-p* counts as borderline when *p* does; but since $V[\textit{p or not-p}]$ is taken to be the maximum of $V[p]$ and $V[\textit{not-p}]$, and hence ½ in the paradigm case where $V[p]$ is ½, I have little doubt that this is what Schiffer intends.

our inclination to call him non-bald.

There is no doubt that we do have such conflicting inclinations in borderline cases. Whether we really need a theory of VPB's to explain or characterize this seems more doubtful. I'm inclined to think that our conflicting inclinations in such cases have a lot to do with contextual shifts in how we use the term 'bald': when we have more tendency to regard Harry as bald than not bald, even while regarding him as borderline, that's because we find it more natural to shift to a standard loose enough to count him as bald than to one strong enough to exclude him. One piece of evidence for this diagnosis is that if you alter the example to make it seem indeterminate in *all* contexts—e.g., "The minimum number of hairs required for someone to be bald is even"—then we no longer feel ambivalent, we simply reject both the claim and its negation. But I don't offer this diagnosis with total confidence; and especially given that Schiffer concedes that vagueness-related partial-belief may only be partial belief in a Pickwickian sense, there may be little to fight about here. Perhaps the only point to make is that Schiffer's assumption that there are neat compositional laws governing "vagueness-related degrees of belief" appears under-motivated if the only role of such "degrees of belief" is to characterize our conflicting inclinations in cases where our standard degrees of belief in a sentence and its negation add to less than 1.

Aside from this general worry about whether there are *any* neat laws concerning "vagueness-related degrees of belief" in Schiffer's sense, I think that the particular laws that he proposes are quite unmotivated. In particular, he proposes that the VPB of a disjunction is the maximum of the VPB's of the disjuncts, and that the VPB of a conjunction is the minimum. I grant that there's no obvious reason why the principles appropriate to SPB's should apply, but

don't find these rules either intuitively evident or motivated by the conception of VPB's given two paragraphs back. In the example above, where V(Harry is bald) and V(Harry is thin) are each ½ and there is no connection between them, why should V(Harry is bald and thin) also be ½? (E.g., a shift of context to make Harry come out *both* clearly bald and clearly thin would seem harder than one to make him come out *either* clearly not bald or clearly not thin.) I just don't see any intuitive pull to his proposal.

4. Vagueness as a psychological phenomenon. Despite my doubts about the need for a notion of "vagueness-related partial belief" and the details of Schiffer's account of it, I think that his view that standard partial belief does not obey classical laws is important to an understanding of vagueness: in particular, it is hard to see how one could have any real alternative to the epistemic view if standard degrees of belief were applicable to vague propositions and had to obey the classical laws. If the standard laws of partial belief are assumed, then (i) our degree of belief that Harry is bald and our degree of belief that he is not bald must add to 1, and (ii) our degree of belief that he is either bald or not bald must be 1. It is hard to see how accepting either of these two claims wouldn't involve a commitment to the straightforward factuality of the question of Harry's baldness; but by abandoning both, Schiffer makes room for a view on which we have no commitment to the question being factual.[18]

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[18] It is possible to give up (i) without giving up (ii): the resulting view has a bit of the flavor of supervaluationism. It is a slight improvement on supervaluationism, I think, in that it can be used to give non-circular content to claims about indeterminacy; and on this basis, I once advocated it: "Indeterminacy, Degree of Belief and Excluded Middle", *Nous* 34: 1-30 (2000). But I soon came to realize that the deepest problems with supervaluationism can't be avoided as long

One might respond that claims about partial belief can at best illuminate *our beliefs about* vagueness and indeterminacy; they can say nothing about what vagueness and indeterminacy *really are*. I'd be skeptical of such a response. Nonetheless, I am even more skeptical of Schiffer's particular means of combating the response, which is to explicate indeterminacy in terms of partial belief. More particularly, Schiffer defines a *VPB** in p as a belief state formed under ideal epistemic conditions in which the SPB in p and in *not-p* would both be zero, so that the VPB in p and in *not-p* would both be non-zero and would sum to 1. He then proposes that for it to be indeterminate[19] whether p *just is* for it to be the case that it is possible for someone to have a VPB* in p.[20]

It seems to me hard to take this proposed explication seriously:

(A) In one direction, let t be a certain time in the early history of the universe when it was

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as (ii) is maintained. (For my own view on this and other matters related to this paper, see my "No Fact of the Matter", *Australasian Journal of Philosophy* 81: 457-80 (2003); and "The Semantic Paradoxes and the Paradoxes of Vagueness", in JC Beall, ed., *Liars and Heaps* (Oxford University Press, 2004), pp. 262-311.)

[19] Schiffer regards vagueness as a species of indeterminacy, so for vagueness a further condition is required; I won't discuss this.

[20] Presumably when he says that it's possible for someone to have a VPB* in p, he means that it's possible for someone to do so with "the facts as regards p itself" being as they actually are. That is, it's no objection to what he intends that even if p is perfectly determinate, it might have been the case *both that p is indeterminate and that someone had a VPB* that reflected this indeterminacy*.

far too hot for life to be possible, and let r be a certain region; consider the claim p that in r at t there was a roundish configuration of particles, and suppose there actually was a configuration of particles in that region that was *a borderline case of* being roundish (and no configurations that were clear cases of being roundish). If one recognizes indeterminacy at all, p will clearly be an example of it; but since life was then impossible, there seems to be no serious sense in which it is possible for there to be a person in the ideal epistemic conditions required for a VPB* in this claim (given the interpretation of this required by note 20 above).

I suppose Schiffer could resist this by saying that a being who could survive the extraordinary high temperatures and observe the configuration is *logically* possible even if not physically possible, and that that's all that matters; but then, why isn't a being who has a magical baldness detector or roundishness detector also possible in the relevant sense, so that a being without one isn't ideal? It isn't open to Schiffer to answer this by saying that claims about exact shape are determinate but claims about roundishness and baldness are often not, and that the relevant kind of possibility is one that allows for physically impossible beings with access to the determinate facts but not to the indeterminate ones: to invoke that *in this context* would presuppose a notion of determinateness not based on the epistemic attitudes of possible agents.

(B) In the other direction, consider an intuitively determinate claim, say that Dick Cheney is wealthy. Schiffer's proposed explication requires that no one could have a VPB* that Cheney is wealthy: if someone were to have anything other than an SPB of 1 in Cheney's

wealthiness, that person wouldn't be in ideal epistemic circumstances. But couldn't there

be someone fully aware of Cheney's tremendous assets, who nonetheless is unsure

whether to call Cheney wealthy on the ground that he has quite a bit less than Bill Gates?

It is no good to respond to this by saying that such a person isn't in ideal conditions

because he has incorrect views about indeterminacy; that response presupposes that the

notion of indeterminacy is available independently of the ideality constraints on degrees of

belief, which doesn't fit with Schiffer's proposal of using ideality of degrees of belief in

explicating determinateness.

For an example with a different flavor, consider an even clearer case of a determinate

truth, that Napoleon was less than six feet tall. Schiffer's proposed explication requires

that no one could have a VPB* that he was less than six feet tall. But consider an

extraordinarily rational Dummett-like anti-realist who believes that contingent claims

about the past are all indeterminate, and in accordance with this, has only VPB's in such

claims. Again, the only obvious way to defend the claim that such a philosopher is not in

ideal conditions is to say that he has an incorrect theory of indeterminacy, and that defense

seems unavailable in the current dialectical context since it seems to require a sense of

indeterminacy not based on the epistemic attitudes of agents.[21]

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[21] Another worry is that the question of whether the conditions are ideal is a normative

question, which obviously can't be cashed out in reliabilist terms (the frequency with which beliefs

formed in these conditions are true) since we are dealing with beliefs that may be indeterminate.

In a later chapter Schiffer argues that normative claims of this sort are indeterminate. It would

seem, then, that he must hold that it's indeterminate whether the conditions under which the

In my view, Schiffer's attempt to explicate indeterminacy in terms of the psychology of ideal agents is overkill: the point that rational standard degrees of belief can depart from the laws of classical probability, in ways that allow for the standard degrees of belief in p and in *not-p* to sum to less than 1 and for the standard degree of belief in *p or not p* to also be less than 1, is enough to illuminate the nature of vagueness without any such explication. So why does Schiffer insist on an explication in terms of the agent's attitudes? He seems to argue (211) that if indeterminateness weren't explainable in terms of epistemic attitudes then a "third possibility" view of vagueness would have to be correct, where this is defined (191) as a view on which indeterminate claims are neither true nor false. I agree with Schiffer that "third possibility" views ought to be rejected: the claim that 'p' is true should be regarded as equivalent to p itself, and the claim that 'p' is false should be regarded as equivalent to *not-p*; so if it is indeterminate whether p, it is indeterminate whether 'p' is true, and also indeterminate whether it is false.[22] But why he thinks that rejecting the explicability of indeterminacy in terms of agents' attitudes should require the view that indeterminate sentences are neither true nor false is a complete mystery to me: the definability of indeterminacy in other terms seems to have little if any bearing on the question of the relation between indeterminacy on the one hand and truth and falsity on the other.

Dummett-like philosopher adopts a high VPB in Napoleon's being less than six feet tall are ideal; from which it would seem to follow that this (and all other) questions about the past aren't *determinately* determinate.

[22] Moreover, if Schiffer is right in claiming that the indeterminacy of p implies the indeterminacy of *p or not-p*, as I think he is, then it is indeterminate whether 'p' is either true or false.

5. More on validity and the status of classical logic. Schiffer's view that determinateness is a purely psychological concept does play a role for him: one of the reasons he gives against explaining validity in terms of necessary preservation of *determinate* truth is that this would make validity depend on psychological notions. This reason becomes suspect if the points made in the previous section are correct.

Putting that aside, let's return to the question of what validity should be taken to be. There is much to be said for the view that even in classical logic it should not be taken as a defined term, but as a primitive notion that governs our inferential or epistemic practices. (For instance, when we discover that the inference from p and q to r is valid, then we should ensure that our degree of belief in r is no lower than our degree of belief in the conjunction of p and q.) From this viewpoint (which is arguably implicit in natural deduction formulations of logic) we can explain why, in classical logic, we identify the valid inferences with those that, of logical necessity, preserve truth. For assuming "disquotational" rules for truth and the usual introduction and elimination rules for conjunction, the validity of the inference from $p_1,...,p_n$ to q is equivalent to the validity of the inference from *True($<p_1>$) and ... and True($<p_n>$)* to *True($<q>$)*. And now assuming the usual introduction and elimination rules for the conditional, this is equivalent to the validity of the conditional *If True($<p_1>$) and ... and True($<p_n>$) then True($<q>$)*. Validity of a single claim is presumably equivalent to the logical necessity of that claim, so the natural-deduction rules of classical logic plus the truth predicate *show* that valid inferences are those that of logical necessity preserve truth.

But note that this derivation will be inapplicable to any logic that doesn't contain both of

the classical rules for the conditional (the introduction and elimination rules). And the fact is that *none* of the usual non-classical logics of vagueness contain both of these rules (unless one takes attempts at treating vagueness using intuitionist logic seriously). Indeed, if we are wedded to keeping the general intersubstitutivity of *True(<p>)* with *p*, we can't possibly maintain both rules: as those familiar with Curry's Paradox know, we could then prove anything. If, as I think is the best policy in dealing with vagueness, we keep modus ponens (the elimination rule) and give up the unrestricted introduction rule for the conditional, we will accept the validity of some arguments without being able to assert that they preserve truth. It is still the case (i) that if we accept the validity of an argument then we shouldn't believe that all the premises are true without believing that the conclusion is true; and (ii) the existence of a case where all the premises are true and the conclusion isn't true would entail the argument's invalidity. But (i) and (ii) don't rule out there being cases of determinate validity and of determinate invalidity where an indeterminacy in one or more of the premises or the conclusion makes it indeterminate whether the inference preserves truth.[23]

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[23] If it still seems weird to call an inference valid and yet refrain from asserting that it is necessarily truth-preserving, consider the following. A professional magician claims to be able to "preserve evenness", in the following sense: you write down whatever you like on a card, in secret, and he'll then write down a single numeral, and he claims that he will have written the name of an even number if and only if you have written at least one term standing for an even natural number. You write down "the number of nanoseconds between the start of Ashcroft's life and the start of Falwell's", and he writes '0'. It seems indeterminate whether he's preserved evenness. But a natural response is that once you consider cases like this, you'll see that the statement of the goal was too strong: all he really should have claimed is that *in any totally clear case of your either referring to an even number*

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The viewpoint I've just sketched is my preferred view of the matter: it makes certain classical inferences (e.g. the Least Number Sorites) determinately invalid. And note that it does so without defining validity in terms of determinate truth, or semantic values, or anything like that; so that even if we were to agree that determinacy is a psychological notion and that the notion of validity is not to be defined in psychological terms, the approach just suggested would in no way be undermined.

I offer the above view of the matter as one possibility; I do not insist on it. Indeed I'm willing to grant Schiffer's preferred use of 'valid', on which the valid inferences are by definition those that preserve truth of logical necessity. But let me repeat two points from Section 2 before making some further ones:

1. It isn't obvious that classical inferences like the Least Number Sorites don't come out determinately invalid *even on Schiffer's definition*. (Similarly for the law of excluded middle). On a strong but natural reading of 'logical necessity', they definitely fail at the goal of preserving truth *by logical necessity*.

2. Unless Schiffer uses a non-material conditional, and one different from his own preferred conditional, to define 'truth-preserving', *no* inference will turn out determinately *valid* on Schiffer's definition. (This is so whichever way one reads the 'necessarily' in his definition.)

A further point to be made concerns Schiffer's suggestions (e.g. at several places on 229)

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or not doing so, he will write an even number iff you do.

that it is indeterminate whether there are *any* classical inferences that are invalid. Let us concede

to Schiffer (putting aside the doubt raised in point 1 above) that in Schiffer's sense of 'valid'

neither the law of excluded middle nor the Least Number Sorites are determinately invalid. Still,

aren't there other determinately invalid inferences? In most non-classical logics of vagueness

there is a conditional for which *if p then q* is not generally equivalent to *not-p or q* (though it is

equivalent *given* the assumption that excluded middle holds for antecedent and consequent); and

in most of these logics quite a few classically-valid inferences involving the conditional will lead

from determinate truth to determinate untruth, and hence will be clearly invalid in Schiffer's sense.

For instance, in most logics for vagueness, there can be sentences *p* for which *if p then not-p* and

if not-p then p are both determinately true;[24] but in classical logic these together imply that the

earth is made of Jello. Similarly, in classical logic *p if and only if (if p then q)* implies *p and q*, but

in most logics of vagueness the former can be determinately true and the latter determinately false.

I take this point to be fairly significant. The apparent "punch" to Schiffer's claim that

inferences like the Least Number Sorites aren't determinately invalid was that the epistemic view

can't be determinately false. But if what I've just argued is right, then even if you concede to

Schiffer that *those* classical inferences aren't determinately invalid in Schiffer's sense (a

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[24] E.g. in Lukasiewicz continuum valued logic, this will hold of any sentence with value ½.

More generally, in any logic designed to not only handle vagueness but also allow for a naive

truth theory in face of the paradoxes, it will hold of sentences that assert their own untruth: for if

L is such a sentence, the naive truth theory will imply that *True(<L>) if and only if not-*

True(<L>), so this classical contradiction will be taken to be clearly true.

concession which, I repeat, one need not make), still there are *other* classical inferences that *will* be determinately invalid, and that is enough to make the epistemic view (and supervaluationism) determinately false.

Schiffer might resist this conclusion, by insisting that the above "counterexamples" to classical logic involve a conditional that differs in meaning from any in classical logic: the connective of classical logic, he might say, is the material conditional (or perhaps the Schiffer conditional mentioned earlier), and none of the classical inferences involving *that* lead from determinate truth to determinate falsity. But to start with a small point, I see no obvious ground for insisting that the 'if ... then' of these logics of vagueness differs in meaning from the classical conditional, given that in these logics *if p then q* will completely coincide with *not-p or q* when restricted to determinate sentences; or, to put it another way, they are equivalent relative to the classical assumptions *p or not-p* and *q or not-q*. To insist that the classical conditional is not only equivalent to *not-p or q* relative to the classical assumption of excluded middle, but is synonymous with it (so that the equivalence persists even when the assumption of excluded middle is dropped), seems highly contentious, and I know of no plausible argument for it.

A more significant point is that were Schiffer to take this "change of meaning" line it would have an extremely high cost: it would commit him to the conclusion that no classical inference is determinately valid in the ordinary sense. For by point 2 above, none is truth-preserving as defined by the material conditional, or as defined by the Schiffer conditional either, and the line now under consideration has it that any sense in which inferences are truth-preserving simply changes the ordinary meaning of 'truth-preserving'. No inference (whether licensed by

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classical logic or not) would be determinately valid, in the ordinary sense; the ordinary sense of

validity would have become completely useless.