DEMONSTRATION BY *REDUCTIO AD IMPOSSIBILE* IN *POSTERIOR ANALYTICS* 1. 26

MARKO MALINK

1. Aristotle's thesis in *Posterior Analytics* 1. 26

At the beginning of the *Analytics*, Aristotle states that the subject of the treatise is demonstration (ἀπόδειξις). A demonstration, for Aristotle, is a kind of deductive argument. It is a deduction through which, when we possess it, we have scientific knowledge (ἐπιστήμη). While the *Prior Analytics* deals with deduction in general, the nature of demonstration is studied in the *Posterior Analytics*.

In *Posterior Analytics* 1. 24–6, Aristotle sets out to compare different kinds of demonstration. He begins by explaining why, in his view, universal demonstrations are better than particular ones, and positive demonstrations are better than negative ones (1. 24–5). He goes on, in chapter 1. 26, to argue that direct demonstrations are better than those that proceed by *reductio ad impossibile*. In the opening sentence of the chapter, Aristotle writes:

ἐπεὶ δ' ἡ κατηγορικὴ τῆς στερητικῆς βελτίων, δῆλον ὅτι καὶ τῆς εἰς τὸ ἀδύνατον ἀγούσης.[1] (*Post. An.* 1. 26, $87^{a}1$–2)

Since positive demonstration is better than privative demonstration, clearly it is also better than that which leads to the impossible.

Aristotle is here referring to the main result of the preceding chapter, that direct positive demonstrations are better than direct negative (or privative) ones. Based on this, he seeks to establish in 1. 26 that direct positive demonstrations are better than those by *reductio ad impossibile*. He does so by arguing that direct negative demonstrations are better than those by *reductio*.[2]

[1] For the Greek text of Aristotle's *Analytics*, I follow the edition of T. Waitz, *Aristotelis Organon Graece* [*Organon*], 2 vols. (Leipzig, 1844–6). All translations are my own unless otherwise noted.

[2] See Philop. *In An. Post.* 290. 32–291. 5 Wallies; J. Zabarella, *Opera logica* [*Opera logica*] (Frankfurt a.M., 1608), 972 E–F.

Marko Malink, *Demonstration by* reductio ad impossibile *in* Posterior Analytics *1. 26*. In: Oxford Studies in Ancient Philosophy Volume LVIII. Edited by: Victor Caston, Oxford University Press (2020).

DOI: 10.1093/oso/9780198858997.003.0004

To show that direct negative demonstrations are better than those by *reductio*, Aristotle argues that the former are superior in explanatory power to the latter in that they proceed from premisses which are prior to the conclusion. In the final section of the chapter, he states this thesis as follows:

> εἰ οὖν ἡ ἐκ γνωριμωτέρων καὶ προτέρων κρείττων, εἰσὶ δ᾽ ἀμφότεραι ἐκ τοῦ μὴ εἶναί τι πισταί, ἀλλ᾽ ἡ μὲν ἐκ προτέρου ἡ δ᾽ ἐξ ὑστέρου, βελτίων ἁπλῶς ἂν εἴη τῆς εἰς τὸ ἀδύνατον ἡ στερητικὴ ἀπόδειξις, ὥστε καὶ ἡ ταύτης βελτίων ἡ κατηγορικὴ δῆλον ὅτι καὶ τῆς εἰς τὸ ἀδύνατόν ἐστι βελτίων. (*Post. An.* 1. 26, 87^a25–30)

> Thus, if a demonstration which proceeds from what is more known and prior is superior, and if in both kinds of demonstration conviction proceeds from something's not holding, but in the one from something prior and in the other from something posterior, then privative demonstration will be better without qualification than demonstration leading to the impossible. Consequently, it is also clear that positive demonstration, which is better than privative demonstration, is also better than that which leads to the impossible.

In this passage, Aristotle compares direct negative demonstrations and those by *reductio*. Both kinds of demonstration make use of negative propositions, that is, propositions asserting that 'something does not hold'.[3] They differ from each other with respect to the priority relations that obtain between the premisses and the conclusion. While direct negative demonstrations proceed from premisses that are more known than and prior to the conclusion, demonstrations by *reductio* proceed from premisses that are posterior to the conclusion.

In *Posterior Analytics* 1. 2, Aristotle distinguishes between priority 'in nature' and priority 'to us'. Likewise, he distinguishes between being more known 'in nature' and being more known 'to us'. He regards the latter distinction as equivalent to the former, using the phrases 'prior' and 'more known' interchangeably in this context.[4] In chapter 1. 26, Aristotle makes it clear that the sense in which the premisses of a direct negative demonstration are prior to

[3] Philop. *In An. Post.* 298. 28–299. 4; M. Mignucci, *L'argomentazione dimostrativa in Aristotele: commento agli* Analitici secondi *I* [*L'argomentazione dimostrativa*] (Padua, 1975), 566. The phrase τὸ μὴ εἶναί τι at 87^a26 is used by Aristotle to designate negative as opposed to affirmative propositions (similarly, *Post. An.* 1. 2, 72^a20; 1. 23, 84^b30–1; 1. 25, 86^b8–9).

[4] *Post. An.* 1. 2, 71^b33–72^a5; see R. D. McKirahan, *Principles and Proofs: Aristotle's Theory of Demonstrative Science* [*Principles*] (Princeton, 1992), 30–1;

the conclusion is priority in nature (φύσει, 87ᵃ17). Accordingly, when he refers to these premisses as 'more known', he does not mean that they are necessarily more known to us, but that they are more known in nature.[5]

By contrast, the premisses of a demonstration by *reductio* are not prior but posterior in nature to the conclusion. Aristotle does not specify whether this claim holds for all demonstrations by *reductio* or whether it is a claim that admits of exceptions. The answer to this question will emerge from his argument in chapter 1. 26. For now, what is important is that Aristotle draws a clear contrast between direct negative demonstrations and those by *reductio*: the former proceed from premisses that are prior in nature to the conclusion, the latter from premisses that are posterior in nature to the conclusion.[6]

In Aristotle's view, premisses that are not prior in nature to the conclusion fail to reveal the cause (αἰτία) of the *demonstrandum*, and hence are not explanatory of the conclusion (αἴτια τοῦ συμπεράσματος, 1. 2, 71ᵇ22).[7] Thus, the premisses of demonstrations by *reductio* are not explanatory of the conclusion. At the same time, Aristotle holds that, in order to have scientific knowledge of a thing, one needs to grasp the cause, or explanation, of that thing (71ᵇ9–12). Hence, demonstrations by *reductio* in which the premisses are posterior in nature to the conclusion are not capable of producing scientific knowledge of the conclusion. This does not mean that Aristotle takes these demonstrations to be invalid. There is no doubt that he regards them as valid deductive arguments in which the conclusion follows necessarily from the premisses.[8] Accordingly, he takes them to be capable of producing conviction (πίστις) of the

J. Barnes, *Aristotle's* Posterior Analytics, 2nd edn. [*Posterior Analytics 2nd edn.*] (Oxford, 1993), 95–7.

[5] J. H. von Kirchmann, *Erläuterungen zu den zweiten Analytiken des Aristoteles* [*Erläuterungen*] (Leipzig, 1878), 116.

[6] See R. Grosseteste, *Commentarius in Posteriorum Analyticorum libros* [*Commentarius*], ed. P. Rossi (Florence, 1981), 253–4.

[7] Aristotle maintains that, in order for the premisses of a demonstration to be explanatory (αἴτια) of the conclusion, they must be prior in nature to the conclusion (1. 2, 71ᵇ31; 2. 15, 98ᵇ17); see Zabarella, *Opera logica*, 660 C–D; D. Bronstein, *Aristotle on Knowledge and Learning: The* Posterior Analytics [*Knowledge and Learning*] (Oxford, 2016), 128. For the premisses to be explanatory of the conclusion is for them to reveal the cause (αἰτία) of the *demonstrandum* (1. 2, 71ᵇ30–1 and 71ᵇ9–12); cf. Bronstein, *Knowledge and Learning*, 35–8.

[8] See *Pr. An.* 2. 11, 62ᵃ11–17.

conclusion.[9] He denies, however, that they can produce scientific knowledge (ἐπιστήμη).

In *Posterior Analytics* 1. 2, Aristotle defines a demonstration as a deduction that is capable of producing scientific knowledge (71^{b}18–19). In order to have this capacity, he argues, the premisses of a demonstration must be more known in nature than the conclusion, prior in nature to the conclusion, and explanatory of the conclusion (71^{b}19–25).[10] Insofar as demonstrations by *reductio* fail to meet these conditions, they are not genuine demonstrations as defined in chapter 1. 2. Instead, they are demonstrations in a broader sense. Aristotle countenances such a broader sense, for example, in *Posterior Analytics* 1. 13, when he distinguishes between deductions 'of the why' (τοῦ διότι) and those 'of the fact' (τοῦ ὅτι). Although deductions 'of the fact' are not explanatory and do not reveal the cause of the *demonstrandum*, Aristotle is willing to refer to them as 'demonstrations' (78^{a}30, 78^{b}14). Similarly, when he speaks of 'demonstration' by *reductio ad impossibile* in 1. 26, it may be a demonstration 'of the fact' but not a genuine demonstration.[11] By contrast, direct negative demonstrations proceed from premisses that are prior in nature to the conclusion. Thus, provided that they satisfy the other conditions laid down in Aristotle's characterization of demonstration in chapter 1. 2, they are genuine demonstrations in which the premisses are explanatory of the conclusion. As such, they are capable of producing scientific knowledge.

Aristotle's view that demonstrations by *reductio* are not explanatory proved influential. For example, the view is endorsed by Proclus in his commentary on the first book of Euclid's *Elements*:

ὅταν μὲν οὖν ὁ συλλογισμὸς ᾖ δι' ἀδυνάτου τοῖς γεωμέτραις, ἀγαπῶσι τὸ σύμπτωμα μόνον εὑρεῖν, ὅταν δὲ διὰ προηγουμένης ἀποδείξεως, τότε πάλιν, εἰ μὲν ἐπὶ μέρους αἱ ἀποδείξεις γίγνοιντο, οὔπω δῆλον τὸ αἴτιον, εἰ δὲ καθ' ὅλον καὶ ἐπὶ πάντων τῶν ὁμοίων, εὐθὺς καὶ τὸ διὰ τί γίγνεται καταφανές. (Proclus *In Eucl. I* 202. 19–25 Friedlein)

[9] This is clear from the fact that Aristotle regards demonstrations by *reductio*, just like direct negative ones, as 'convincing' (πισταί, 87^{a}26); see Philop. *In An. Post.* 298. 28–299. 2.

[10] See Philop. *In An. Post.* 29. 1–14; Zabarella, *Opera logica*, 653 F–654 B; McKirahan, *Principles*, 31; Bronstein, *Knowledge and Learning*, 127–8.

[11] Avicenna *al-Shifā': al-Burhān* 1. 8, 90. 15 Afifi. Thanks to Riccardo Strobino for sharing parts of his translation of Avicenna's *al-Burhān*.

When geometers reason through the impossible, they are content merely to discover the property [of a given subject]. But when their reasoning proceeds through a principal demonstration, then, if the demonstrations are partial, the cause is not yet clear, whereas if it is universal and applies to all like things, the 'why' at once becomes evident.[12]

According to Proclus, demonstrations by *reductio ad impossibile* serve to establish the fact that a given subject has a certain property. They do not, however, reveal the cause, or the 'why', of that fact. Instead, the cause can be revealed by means of a 'principal' demonstration, that is, a direct one.[13] Provided that the direct demonstration exhibits the appropriate level of generality, it succeeds in revealing the cause of the *demonstrandum*. Thus, like Aristotle, Proclus holds that direct demonstrations possess an explanatory power that those by *reductio* lack. Now, Proclus was thoroughly familiar with Aristotle's logical works and wrote a commentary on the *Posterior Analytics*.[14] His commentary on the *Elements* contains numerous references to the *Posterior Analytics*, including an exposition of Aristotle's account of exactness in chapter 1. 27.[15] Thus, it seems clear that, in his remarks on demonstration by *reductio*, Proclus is following Aristotle's treatment in chapter 1. 26.

The same view of demonstration by *reductio* was held by a number of thinkers in the early modern period.[16] For example, in his 1615 essay *De mathematicarum natura dissertatio*, Giuseppe Biancani

[12] The translation follows the one given by T. L. Heath, *The Thirteen Books of Euclid's* Elements, vol. i: Introduction and Books I, II [*Elements*] (Cambridge, 1908), 150 n. 1, with some modifications.

[13] As Heath (*Elements*, 150 n. 1) notes, the phrase 'principal demonstration' (*προηγουμένη ἀπόδειξις*) at 202. 21–2 is used by Proclus to refer to direct demonstrations. *Pace* G. R. Morrow, *Proclus: A Commentary on the First Book of Euclid's* Elements, 2nd edn. [*Proclus*] (Princeton, 1992), 158–9.

[14] As a student, Proclus learned Aristotle's logical works by heart (Marinus *Vita Procli* 9. 33–6 Saffrey and Segonds). For Proclus' (lost) commentary on the *Posterior Analytics*, see C. Helmig, 'Proclus' Criticism of Aristotle's Theory of Abstraction and Concept Formation in *Analytica Posteriora* II 19', in F. A. J. de Haas, M. Leunissen, and M. Martijn (eds.), *Interpreting Aristotle's* Posterior Analytics *in Late Antiquity and Beyond* (Leiden, 2010), 27–54 at 27–9.

[15] For Proclus' exposition of *Post. An.* 1. 27, see *In Eucl. I* 59. 10–60. 1; cf. Morrow, *Proclus*, 47.

[16] See P. Mancosu, 'On the Status of Proofs by Contradiction in the Seventeenth Century', *Synthese*, 88 (1991), 15–41 at 15–36; id., *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* [*Mathematics*] (Oxford, 1996), 24–8, 58–64, 83–4, and 100–15; id., 'On Mathematical Explanation', in E. Grosholz and H. Berger (eds.), *The Growth of Mathematical Knowledge* (Dordrecht, 2000), 103–19 at 108–16.

denied that demonstrations by *reductio* proceed 'from a cause', citing the passage from Proclus' commentary just quoted.[17] In his *Lectiones mathematicae* from the 1660s, Isaac Barrow asserts that, as for reasoning by *reductio ad impossibile*, 'Aristotle teaches, and everyone grants, that reasoning of this sort does not at all furnish knowledge that is very perspicuous and pleasing to the mind'.[18] The point is made more explicit by Arnauld and Nicole in the *Port-Royal Logic*:

> Those demonstrations which show that a thing is such, not by its principles, but by some absurdity which would follow if it were not so, are very common in Euclid. It is clear, however, that while they may convince the mind, they do not enlighten it, which ought to be the main result of science. For our mind is not satisfied unless it knows not only that a thing is, but why it is, which cannot be learned from a demonstration by reduction to the impossible.[19]

Arnauld and Nicole criticize Euclid and other geometers for giving demonstrations by *reductio* in cases when a direct demonstration is available. These geometers, they argue, 'have not sufficiently observed that, in order to have perfect knowledge of a truth, it is not enough to be convinced that it is true, if beyond this we do not penetrate into the reasons, derived from the nature of the thing itself, why it is true'.[20]

[17] G. Biancani, *De mathematicarum natura dissertatio* (Bologna, 1615), 10, 12, and 20.

[18] 'Certe docet philosophus, et omnes fatentur, ejusmodi ratiociniis haud ita perspicuam animoque blandientem comparari scientiam', I. Barrow, *The Mathematical Works of Isaac Barrow*, ed. W. Whewell (Cambridge, 1860), 377. Accordingly, Barrow maintains that 'this mode of demonstration, as all acknowledge, is rather obscure and ignoble' ('qui demonstrandi modus, ut omnes agnoscunt, obscurior est et ignobilior', 357).

[19] 'Ces sortes de démonstrations qui montrent qu'une chose est telle, non par ses principes, mais par quelque absurdité qui s'ensuivrait si elle était autrement, sont très ordinaires dans Euclide. Cependant il est visible qu'elles peuvent convaincre l'esprit, mais qu'elles ne l'éclairent point, ce qui doit être le principal fruit de la science. Car notre esprit n'est point satisfait, s'il ne sait non seulement que la chose est, mais pourquoi elle est; ce qui ne s'apprend point par une démonstration qui réduit à l'impossible.' A. Arnauld and P. Nicole, *La Logique ou l'art de penser* [*Logique*], ed. P. Clair and F. Girbal, 2nd edn. (Paris, 1993), 4. 9. 3, 328.

[20] '. . . il semble qu'ils n'ont pas assez pris garde qu'il ne suffit pas pour avoir une parfaite science de quelque vérité, d'être convaincu que cela est vrai, si de plus on ne pénètre par des raisons prises de la nature de la chose même pourquoi cela est vrai.' Arnauld and Nicole, *Logique*, 4. 9. 1, 326.

A similar view is expressed by Kant in the *Critique of Pure Reason*. In the course of specifying the methods of proof admissible in the discipline of pure reason, Kant excludes proof by *reductio ad impossibile*, or 'apagogic' proof, on the following grounds:

> The third special rule of pure reason, if it is subjected to a discipline in regard to transcendental proofs, is that its proofs must never be apagogic but always ostensive. Direct or ostensive proof, in all kinds of cognition, is that which combines with the conviction of truth insight into the sources of the truth; apagogic proof, by contrast, can produce certainty, but cannot enable us to comprehend the truth in its connection with the grounds of its possibility. Hence the latter is more of an emergency aid than a procedure which satisfies all the aims of reason.[21]

This view of 'apagogic' proof was common among theorists working in the Kantian tradition in the nineteenth century.[22] In his *Theory of Science*, Bolzano argues that proofs by *reductio*, while they can produce conviction, cannot exhibit 'the objective ground' of the *demonstrandum*.[23] Similarly, Trendelenburg states in his *Logical Investigations* that 'indirect proof, as Aristotle has already shown, possesses less scientific value than direct proof. . . . Indirect proof does not provide any insight into the inner grounds of the thing.'[24] Among more recent authors, Lipton suggests that proofs

[21] 'Die dritte eigenthümliche Regel der reinen Vernunft, wenn sie in Ansehung transscendentaler Beweise einer Disciplin unterworfen wird, ist: daß ihre Beweise niemals apagogisch, sondern jederzeit ostensiv sein müssen. Der directe oder ostensive Beweis ist in aller Art der Erkenntniß derjenige, welcher mit der Überzeugung von der Wahrheit zugleich Einsicht in die Quellen derselben verbindet; der apagogische dagegen kann zwar Gewißheit, aber nicht Begreiflichkeit der Wahrheit in Ansehung des Zusammenhanges mit den Gründen ihrer Möglichkeit hervorbringen. Daher sind die letzteren mehr eine Nothhülfe, als ein Verfahren, welches allen Absichten der Vernunft ein Genüge thut.' I. Kant, *Kritik der reinen Vernunft*, in *Kant's gesammelte Schriften I*, Königlich Preußische Akademie der Wissenschaften (Berlin, 1911), vol. iii, A 789/B 817.

[22] For example, L. H. Jakob, *Grundriß der allgemeinen Logik und kritische Anfangsgründe der allgemeinen Metaphysik*, 4th edn. (Halle/Saale, 1800), 130; W. T. Krug, *System der theoretischen Philosophie,* 1. Theil: Denklehre oder Logik (Königsberg, 1806), 595–7; J. G. C. C. Kiesewetter, *Grundriß einer allgemeinen Logik nach Kantischen Grundsätzen,* 1. Theil: Reine allgemeine Logik, 4th edn. (Leipzig, 1824), i. 149 and ii. 493; E. Reinhold, *Die Logik oder die allgemeine Denkformenlehre* (Jena, 1827), 409–10.

[23] B. Bolzano, *Wissenschaftslehre: Versuch einer ausführlichen und größtentheils neuen Darstellung der Logik mit steter Rücksicht auf deren bisherige Bearbeiter* [*Wissenschaftslehre*], 4 vols. (Sulzbach, 1837), iv. 270–1 and 278 n. 2 (§530).

[24] 'Der indirekte Beweis hat, wie schon Aristoteles zeigt, geringeren wissenschaftlichen Werth, als der direkte. . . . Der indirekte Beweis öffnet daher keine

by *reductio* 'work by showing necessity but without providing an explanation', and that they fail to be explanatory 'because they do not show what "makes" the theorem true'.[25] Similarly, Poston holds that '*reductio* proofs provide conclusive grounds *that* a claim is true without removing the mystery as to *why* the claim is true'.[26] Of course, this is not to say that all these theorists agree with Aristotle, or with each other, on why it is that proofs by *reductio* fail to be explanatory. Far from it. Nonetheless, they share a commitment to the same general thesis, which derives from Aristotle's discussion in *Posterior Analytics* 1. 26.

While Aristotle's thesis in *Posterior Analytics* 1. 26 exerted a long-lasting influence, its precise import has remained somewhat obscure. There is no consensus in the literature on why exactly demonstrations by *reductio* fail to be explanatory in Aristotle's view. This is mainly because the argument Aristotle gives in support of his thesis is compressed and raises a number of interpretive questions. In fact, Aristotle's argument in chapter 1. 26 has been regarded as problematic since antiquity. Philoponus reports that 'all commentators together have attacked Aristotle on the exposition of these things, saying that he gives an incorrect account of deduction through the impossible'.[27] Accordingly, Zabarella notes that 'I have considered this passage for a very long time and have not found anything in other commentators in which I could quite

Einsicht in die inneren Gründe der Sache', F. A. Trendelenburg, *Logische Untersuchungen,* 2. Band, 3rd edn. (Leipzig, 1870), 440. For Trendelenburg's discussion of *Post. An.* 1. 26, see id., *Elementa logices Aristoteleae*, 5th edn. (Berlin, 1862), 127–9. By contrast, Lotze argues that some indirect proofs are explanatory in that they derive the conclusion from its grounds; H. Lotze, *System der Philosophie,* 1. Theil: Drei Bücher der Logik [*Logik*] (Leipzig, 1874), 263–76.

[25] P. Lipton, 'Understanding without Explanation', in H. W. de Regt, S. Leonelli, and K. Eigner (eds.), *Scientific Understanding: Philosophical Perspectives* (Pittsburgh, 2009), 43–63 at 48–9; id., 'Mathematical Understanding', in J. Polkinghorne (ed.), *Meaning in Mathematics* (Oxford, 2011), 49–54 at 50.

[26] T. Poston, *Reason and Explanation: A Defense of Explanatory Coherentism* (New York, 2014), 79. Against this, others argue that some proofs by *reductio* are explanatory, e.g. M. Colyvan, *An Introduction to the Philosophy of Mathematics* (Cambridge, 2012), 80; M. Lange, *Because Without Cause: Non-Causal Explanations in Science and Mathematics* (Oxford, 2017), 234 and 445 n. 9; G. Hanna, 'Reflections on Proof as Explanation', in A. J. Stylianides and G. Harel (eds.), *Advances in Mathematics Education Research on Proof and Proving* (Cham, 2018), 3–18 at 13–14.

[27] Philop. *In An. Post*. 291. 10–12: *ἐν αὐτῇ τῇ ἐκθέσει τούτων ἁπαξάπαντες οἱ ἐξηγηταὶ ἐπελάβοντο τοῦ Ἀριστοτέλους ὡς κακῶς τὸν διὰ τοῦ ἀδυνάτου ἐκτιθεμένου συλλογισμόν.*

acquiesce'.[28] Modern readers are in no better a position than Zabarella. Thus, Mignucci concludes that 'it is difficult to make sense of the confused argument that Aristotle advances' in *Posterior Analytics* 1. 26.[29] Smith holds that the argument is 'very unsatisfactory' and that 'we cannot actually make c. 26 into a coherent account of *per impossibile* proof'.[30] Likewise, Detel regards the argument as 'muddled' and, 'as a matter of fact, unsuccessful'.[31]

The plan of this paper is as follows. I begin by considering the preliminary part of Aristotle's argument in chapter 1. 26, in which he explains the difference between direct negative demonstrations and those by *reductio* (Section 2). Next, I turn to the core part of the argument, which appeals to the relation of priority in nature between scientific propositions (Section 3). I argue that this priority relation is determined by the order of terms in acyclic chains of immediate universal affirmations (Sections 4 and 5). Given this characterization of priority in nature, Aristotle's argument in *Posterior Analytics* 1. 26 turns out to be coherent and successful (Section 6). Finally, I discuss how Aristotle answers an objection to his argument by emphasizing the mereological structure of direct demonstrations (Section 7).

2. Direct negative demonstration versus demonstration by *reductio*

In the *Analytics*, demonstrations take the form of deductions governed by the three syllogistic figures. Aristotle focuses on deductions that consist of four kinds of categorical proposition:

AaB	A belongs to all B	(universal affirmative)
AeB	A belongs to no B	(universal negative)
AiB	A belongs to some B	(particular affirmative)
AoB	A does not belong to some B	(particular negative)

[28] 'locum hunc diutissime consideravi & apud alios nihil inveni, in quo plane possem acquiescere', Zabarella, *Opera logica*, 976 A.

[29] 'È difficile dare un senso alla confusa argomentazione che Aristotele avanza . . .', Mignucci, *L'argomentazione dimostrativa*, 559; similarly, id., *Aristotele*: Analitici secondi (Rome, 2007), 233.

[30] R. Smith, 'The Syllogism in *Posterior Analytics* I' ['Syllogism'], *Archiv für Geschichte der Philosophie*, 64 (1982), 113–35 at 133–4.

[31] 'verkorkst' and 'faktisch erfolglos', W. Detel, *Aristoteles:* Analytica posteriora, 2 vols. [*Analytica posteriora*] (Berlin, 1993), ii. 450.

A demonstration is positive if its conclusion is an a- or i-proposition; it is negative (or privative) if its conclusion is an e- or o-proposition.

In *Posterior Analytics* 1. 25, Aristotle has argued that direct positive demonstrations are better than direct negative ones. His aim in chapter 1. 26 is to show that direct negative demonstrations are better than those by *reductio*. He begins by explaining the difference between these two kinds of demonstration, as follows:

δεῖ δ' εἰδέναι τίς ἡ διαφορὰ αὐτῶν. ἔστω δὴ τὸ A μηδενὶ ὑπάρχον τῷ B, τῷ δὲ Γ τὸ B παντί· ἀνάγκη δὴ τῷ Γ μηδενὶ ὑπάρχειν τὸ A. οὕτω μὲν οὖν ληφθέντων δεικτικὴ ἡ στερητικὴ ἂν εἴη ἀπόδειξις ὅτι τὸ A τῷ Γ οὐχ ὑπάρχει. ἡ δ' εἰς τὸ ἀδύνατον ὧδ' ἔχει. εἰ δέοι δεῖξαι ὅτι τὸ A τῷ B οὐχ ὑπάρχει, ληπτέον ὑπάρχειν, καὶ τὸ B τῷ Γ, ὥστε συμβαίνει τὸ A τῷ Γ ὑπάρχειν. τοῦτο δ' ἔστω γνώριμον καὶ ὁμολογούμενον ὅτι ἀδύνατον. οὐκ ἄρα οἷόν τε τὸ A τῷ B ὑπάρχειν. εἰ οὖν τὸ B τῷ Γ ὁμολογεῖται ὑπάρχειν, τὸ A τῷ B ἀδύνατον ὑπάρχειν. (*Post. An.* 1. 26, 87^a2–12)

We must understand what the difference between them is [i.e. between direct negative demonstration and demonstration by *reductio ad impossibile*]. Let A belong to no B and B to all C; it follows necessarily that A belongs to no C. If these premisses are assumed, the privative demonstration that A does not belong to C will be ostensive. Demonstration leading to the impossible, on the other hand, proceeds as follows. If it is required to prove that A does not belong to B, we must assume that it does belong, and that B belongs to C; hence it follows that A belongs to C. But let it be known and agreed that this is impossible. Therefore, A cannot belong to B. If, then, it is agreed that B belongs to C, it is impossible for A to belong to B.

In the first part of this passage, Aristotle considers a negative demonstration that is direct (or 'ostensive').[32] This demonstration takes the form of the syllogistic mood Celarent in the first figure:

[32] In the *Prior Analytics*, the term 'ostensive' (δεικτικός) is used to designate direct deductions as opposed to deductions by *reductio ad impossibile* and other deductions 'from a hypothesis' (ἐξ ὑποθέσεως); see *Pr. An.* 1. 7, 29^a31–3; 1. 23, 40^b25–9, 41^a21–37; 1. 29, 45^a23–$^b15$; 2. 14 *passim*. In the *Posterior Analytics*, δεικτικός is sometimes used to designate positive as opposed to negative demonstrations (1. 23, 85^a2; 1. 25, 86^a32, 86^b30–9); see M. Frede, 'Stoic vs. Aristotelian Syllogistic', *Archiv für Geschichte der Philosophie*, 56 (1974), 1–32 at 29. At 1. 26, 87^a5, however, the term designates direct deductions; see Philop. *In An. Post.* 296. 3; G. R. G. Mure, *Aristotle's* Analytica posteriora [*Posteriora*], in *The Works of Aristotle Translated into English*, ed. W. D. Ross and J. A. Smith, vol. i (Oxford, 1928), ad loc.; P. Pellegrin, *Aristote:* Seconds Analytiques [*Seconds Analytiques*] (Paris, 2005), 388 n. 3; J. Tricot, *Aristote:* Seconds Analytiques [*Seconds Analytiques*] (Paris, 2012), 145; *pace* Mignucci, *L'argomentazione dimostrativa*, 560.

AeB, BaC, therefore AeC

In the second part of the passage, Aristotle goes on to consider a demonstration by *reductio*. One might have expected this demonstration to derive the same conclusion as the preceding direct demonstration, AeC.[33] Aristotle would then be able to compare the two demonstrations with respect to their explanatory value regarding this conclusion. As reasonable as this strategy may seem, it is not the one adopted by Aristotle. Instead, he chooses to consider a demonstration by *reductio* that derives the conclusion 'A does not belong to B'. Accordingly, the assumption for *reductio* is the proposition 'A belongs to B'. Since Aristotle's formulation of these propositions does not contain any quantifying expressions such as 'no' or 'all', there is some ambiguity as to their quantity. The conclusion derived in the demonstration by *reductio* may be either an e- or an o-proposition, while the assumption for *reductio* may be either an a- or an i-proposition. Now, Aristotle takes the assumption for *reductio* to be the major premiss of a deduction in the first figure, the minor premiss being BaC. Since there are no (valid) first-figure deductions with a major i-premiss, it is clear that the assumption for *reductio* is not an i- but an a-proposition.[34] Thus, the subordinate deduction initiated by the assumption for *reductio* takes the form of the first-figure mood Barbara:

AaB, BaC, therefore AaC

Aristotle takes it that, in the demonstration by *reductio*, the proposition AaC is known and agreed to be 'impossible'. This allows him

The label 'direct' is not used by Aristotle, but derives from later authors who referred to direct deductions by the phrases *ἐξ εὐθείας* or *ἐπ' εὐθείας*. For *ἐξ εὐθείας*, see Galen *De sophismatis* 593. 17 Kühn; cf. R. B. Edlow, *Galen on Language and Ambiguity* (Leiden, 1977), 79 and 104. For *ἐπ' εὐθείας*, see Philop. *In An. Pr.* 248. 3, 300. 27 Wallies, *In An. Post.* 273. 27–8, 290. 32–291. 22, 296. 3 Wallies; ps.-Themist. *In An. Pr.* 107. 32, 108. 6 Wallies; ps.-Ammon. *In An. Pr.* 66. 33–6 Wallies.

[33] According to Philoponus, some ancient commentators criticized Aristotle for not considering a demonstration by *reductio* that derives the same conclusion as the direct demonstration (Philop. *In An. Post.* 294. 12–14). A similar criticism is expressed by Detel, *Analytica posteriora*, ii. 456–7.

[34] J. Pacius, *Aristotelis Stagiritae Peripateticorum principis Organum*, 2nd edn. [*Organum*] (Frankfurt a.M., 1597), 488. In the *Prior Analytics*, Aristotle considers 'indeterminate' propositions, which lack any quantifying expressions (*Pr. An.* 1. 1, 24^{a}16–22; see Ammon. *In An. Pr.* 18. 15–38 Wallies). However, since there are no (valid) first-figure deductions with an indeterminate major premiss (*Pr. An.* 1. 4, 26^{a}30–9 and 26^{b}21–5), the assumption for *reductio* cannot be indeterminate.

to conclude the *reductio* by inferring the conclusion of the demonstration. The conclusion thereby obtained is the contradictory opposite of the assumption for *reductio*. Given that this assumption is the universal affirmative AaB, the conclusion inferred in the demonstration by *reductio* is the particular negative AoB.

Against this, it is often thought that the conclusion of the demonstration by *reductio* is not AoB but the universal negative AeB.[35] In adopting this view, commentators are presumably guided by the idea that the demonstration by *reductio* is intended to derive the major premiss of the direct demonstration.[36] At the same time, they acknowledge that the assumption for *reductio* is the a-proposition AaB. Thus Aristotle would seem to commit the fallacy of concluding the *reductio* by inferring not the contradictory but the contrary opposite of the assumption for *reductio*. Yet, as Philoponus points out, 'it is not likely that Aristotle, the first and only one to have provided the logical methods, would commit such a grave error'.[37] In *Prior Analytics* 2. 11, Aristotle emphasizes that the conclusion of a deduction by *reductio* must be the contradictory opposite, not the contrary opposite, of the assumption for *reductio*.[38] He regards this as a constraint on *reductio* which is generally accepted (ἔνδοξον).[39]

[35] Philop. *In An. Post*. 294. 2–4 Wallies; Grosseteste, *Commentarius*, 253; R. Kilwardby, *Notule libri Posteriorum* [*Notule*], ed. D. Cannone (Cassino, 2003–4), 281; Albertus Magnus, *Opera omnia*, vol. ii, ed. A. Borgnet (Paris, 1890), 136; D. Soto, *In libros Posteriorum Aristotelis sive de demonstratione absolutissima commentaria* (Venice, 1574), 367–8; Zabarella, *Opera logica*, 973 A–B; Mignucci, *L'argomentazione dimostrativa*, 560–2; G. Striker, 'Review of J. Barnes, *Aristotle's* Posterior Analytics' ['Review of Barnes'], *Zeitschrift für philosophische Forschung*, 31 (1977), 316–20 at 317 n. 1; Smith, 'Syllogism', 132; H. Seidl, *Aristoteles:* Zweite Analytiken, 2nd edn. [*Zweite Analytiken*] (Würzburg, 1987), 265–6; Barnes, *Posterior Analytics 2nd edn*., 188; Detel, *Analytica posteriora*, ii. 451; Tricot, *Seconds Analytiques*, 145–6 nn. 2–3.

[36] Thus, Crivelli takes there to be an 'intuition that the conclusion of the demonstration through the impossible is the same as one of the premisses of the negative ostensive demonstration', P. Crivelli, 'Aristotle on Syllogisms from a Hypothesis' ['Hypothesis'], in A. Longo, *Argument from Hypothesis in Ancient Philosophy* (Naples, 2011), 95–184 at 165.

[37] Philop. *In An. Post*. 295. 21–3: οὐ γὰρ ἦν εἰκὸς τὸν Ἀριστοτέλη πρῶτον καὶ μόνον τὰς μεθόδους τὰς λογικὰς παραδεδωκότα τηλικοῦτον ἁμαρτεῖν ἁμάρτημα. Transl. by O. Goldin and M. Martijn, *Philoponus:* On Aristotle, *Posterior Analytics* 1. 19–34 (London, 2012), 98. A similar point is made by Smith, 'Syllogism', 121.

[38] *Pr. An*. 2. 11, 62ᵃ11–19; see also 2. 12, 62ᵃ28–32.

[39] *Pr. An*. 2. 11, 62ᵃ12–13 and 62ᵃ15–19; see R. Smith, *Aristotle:* Prior Analytics [*Prior Analytics*] (Indianapolis, 1989), 200.

Some commentators try to alleviate this problem by arguing that in the *Posterior Analytics* Aristotle adopts a framework in which a- and e-propositions are treated as exhaustive alternatives, so that the falsehood of an a-proposition entails the truth of the corresponding e-proposition.[40] This proposal, however, is problematic. Not only does Aristotle not give any indication of adopting such a framework in the *Posterior Analytics*; doing so would commit him to the implausible view that the e-proposition *No animal is human* is true, given that the a-proposition *Every animal is human* is false.[41] Zabarella suggests that in *Posterior Analytics* 1. 26 Aristotle treats a- and e-propositions as exhaustive alternatives because he ignores particular propositions and deals exclusively with universal propositions.[42] Again, this is dubious. While it is true that Aristotle tends to focus on universal propositions in the *Posterior Analytics*, he countenances particular propositions as well. For example, he considers third-figure deductions in *Posterior Analytics* 1. 14, noting that they do not derive a universal but a particular conclusion (79^a27–8). Moreover, he discusses demonstrations of the form Bocardo and Baroco in chapters 1. 21 and 1. 23.[43] Given this, why

[40] Zabarella, *Opera logica*, 973 A–B; Smith, 'Syllogism', 118–21 and 131–3; Barnes, *Posterior Analytics 2nd edn.*, 146 and 188.

[41] Barnes claims that, in addition to chapter 1. 26, there are three more passages in the *Posterior Analytics* in which Aristotle takes the falsehood of an a-proposition to entail the truth of the corresponding e-proposition: 1. 11, 77^a10–15; 1. 13, 78^b13–28; 1. 15, 79^a36–$^b4$ (Barnes, *Posterior Analytics 2nd edn.*, 146, 158, and 163). These passages, however, are far from conclusive since all of them can be interpreted without attributing to Aristotle this problematic assumption.

[42] Zabarella, *Opera logica*, 973 A–B. More generally, Solmsen suggests that particular propositions are absent from Aristotle's theory of demonstration in the *Posterior Analytics*; F. Solmsen, *Die Entwicklung der Aristotelischen Logik und Rhetorik* [*Entwicklung*] (Berlin, 1929), 143. Similarly, R. Smith, 'The Relationship of Aristotle's Two *Analytics*' ['Relationship'], *Classical Quarterly*, NS, 32 (1982), 327–35 at 327; Barnes, *Posterior Analytics 2nd edn.*, 114, 158, and 188.

[43] *Post. An.* 1. 21, 82^b21–8; 1. 23, 85^a7–12. Some commentators excise the reference to Baroco in 1. 23 by deleting the phrase ἢ μὴ παντί at 85^a9; e.g. W. D. Ross, *Aristotle's* Prior *and* Posterior Analytics*: A Revised Text with Introduction and Commentary* [*Analytics*] (Oxford, 1949), 587; H. Tredennick, *Aristotle:* Posterior Analytics [*Posterior Analytics*] (Cambridge, MA, 1960), 134; Barnes, *Posterior Analytics 2nd edn.*, 183; Detel, *Analytica posteriora*, ii. 411. In addition, Barnes (*Posterior Analytics 2nd edn.*, 173) excises the reference to Bocardo at 1. 21, 82^b21–8. These excisions, however, are not convincing and it is preferable to retain the text of the manuscripts in both passages; see A. D. Crager, *Meta-Logic in Aristotle's Epistemology* (diss. Princeton, 2015), 104–20; M. Malink, 'Aristotle on Principles as Elements' ['Elements'], *Oxford Studies in Ancient Philosophy*, 53 (2017), 163–213 at 184–5 nn. 63–4.

should particular propositions be excluded from consideration in I. 26? In any case, even if particular propositions are less prominent in the *Posterior Analytics* than universal ones, this does not justify inferring the truth of an e-proposition from the falsehood of the corresponding a-proposition.

It is preferable, then, to take the conclusion of the demonstration by *reductio* to be the particular negative AoB rather than the universal negative AeB. After all, Aristotle never asserts that the demonstration by *reductio* is intended to derive the universal negative premiss of the direct demonstration. It may at first glance seem natural to suppose that it is intended to derive this premiss; but, as we have already seen, we cannot presume that Aristotle's argument in I. 26 conforms to our antecedent expectations.

Having specified the two demonstrations, Aristotle points out a difference that obtains between them concerning the epistemic status of their negative premiss:

οἱ μὲν οὖν ὅροι ὁμοίως τάττονται, διαφέρει δὲ τὸ ὁποτέρα ἂν ᾖ γνωριμωτέρα ἡ πρότασις ἡ στερητική, πότερον ὅτι τὸ Α τῷ Β οὐχ ὑπάρχει ἢ ὅτι τὸ Α τῷ Γ. ὅταν μὲν οὖν ᾖ τὸ συμπέρασμα γνωριμώτερον ὅτι οὐκ ἔστιν, ἡ εἰς τὸ ἀδύνατον γίνεται ἀπόδειξις, ὅταν δ' ἡ ἐν τῷ συλλογισμῷ, ἡ ἀποδεικτική. (*Post. An.* I. 26, 87[a]12–17)

Thus the terms are similarly arranged [in both demonstrations], but there is a difference as to which of the two privative propositions is more known, the one that A does not belong to B or the one that A does not belong to C. When the conclusion that it does not hold is more known, the demonstration leading to the impossible comes about; and when the proposition in the deduction is more known, the demonstrative proof comes about.

Aristotle first notes that the two demonstrations employ terms that are 'similarly arranged'. Thus, he takes the two demonstrations to be related in that they are based on the same underlying structure of terms. In particular, they are based on a structure in which BaC is true and in which negative propositions concerning AB and AC are true.[44] There is, however, a difference as to 'which of the two' (*ὁποτέρα*) negative propositions is more known. One of the two

[44] Kirchmann, *Erläuterungen*, 114–15; Ross, *Analytics*, 595; Pellegrin, *Seconds Analytiques*, 388 n. 5. By contrast, Zabarella (*Opera logica*, 976 A–977 B) denies that the two demonstrations discussed at 87[a]3–17 are based on the same underlying structure of terms, arguing instead that they are unrelated and that the schematic letters 'A', 'B', and 'C' are meant to denote different terms in them. This interpretation, however, is not plausible since it conflicts with Aristotle's statement that 'the terms are similarly arranged' in the two demonstrations.

negative propositions in question is identified by Aristotle as 'the conclusion that it does not hold'.[45] This is the conclusion of the direct demonstration, AeC. The other negative proposition is identified as the one 'in the deduction'. This is the major premiss of the direct demonstration, AeB.

Aristotle maintains that in some cases AeB is more known than AeC, while in other cases the latter proposition is more known than the former. Since the relation of being more known in nature is not variable in this way, he presumably has in mind the varying degree to which each of these propositions may be known to the demonstrator.[46] Thus, given the truth of AeB, BaC, and AeC, Aristotle is describing the conditions under which a demonstrator will choose to employ either the direct demonstration or the one by *reductio*. If AeB is more known to the demonstrator than AeC, they will choose the direct demonstration, establishing the latter proposition on the basis of the former. If, on the other hand, AeC is more known to the demonstrator than AeB, they will choose the demonstration by *reductio*. In this case, the demonstrator cannot derive the universal proposition AeB, but at least is able to establish its particular counterpart, AoB.

[45] 87ᵃ15: *τὸ συμπέρασμα . . . ὅτι οὐκ ἔστιν*. In this phrase, *τὸ συμπέρασμα* refers to the conclusion of the direct demonstration, AeC, while the qualification *ὅτι οὐκ ἔστιν* indicates that this conclusion is a negative proposition; see Philop. *In An. Post.* 297. 1–13; Tredennick, *Posterior Analytics*, 151; Mignucci, *L'argomentazione dimostrativa*, 563; Barnes, *Posterior Analytics 2nd edn.*, 41; Detel, *Analytica posteriora*, i. 53; Pellegrin, *Seconds Analytiques*, 209 and 388 n. 6; Tricot, *Seconds Analytiques*, 146. By contrast, some commentators take *τὸ συμπέρασμα* at 87ᵃ15 to refer to the conclusion AaC of the subordinate deduction, rendering 87ᵃ14–15 as follows: 'when it is more known that the conclusion AaC is not (i.e. is false)'; Pacius, *Organum*, 489; Zabarella, *Opera logica*, 978 D; H. Maier, *Die Syllogistik des Aristoteles,* ii/1: Formenlehre und Technik des Syllogismus (Tübingen, 1900), 232–3 n. 1; Mure, *Posteriora*, ad loc.; Ross, *Analytics*, 594; Crivelli, 'Hypothesis', 165. However, this reading fits less well with the context than the former reading. All three occurrences of *συμπέρασμα* at 87ᵃ18–20 refer to the conclusion of the direct demonstration, AeC. Moreover, since *συλλογισμός* at 87ᵃ16 refers to the direct demonstration, it is natural to take *συμπέρασμα* at 87ᵃ15 to refer to the conclusion of this demonstration rather than to the conclusion of some other deduction. Finally, the two readings differ in the interpretation of *οὐκ ἔστιν* at 87ᵃ15. On the former reading, this phrase serves to demarcate negative from affirmative propositions. On the latter reading, the phrase is used to indicate that a proposition does not hold (or is false), whether this proposition is affirmative or negative (see e.g. *Pr. An.* 2. 2, 53ᵇ12–13; 2. 4, 57ᵇ1–2). While the former use of *οὐκ ἔστιν* appears at *Post. An.* 1. 26, 87ᵃ26 (see n. 3 above), the latter use does not appear elsewhere in chapter 1. 26.

[46] Kilwardby, *Notule*, 282; Kirchmann, *Erläuterungen*, 114–16; Ross, *Analytics*, 595; Crivelli, 'Hypothesis', 165–7.

In the demonstration by *reductio*, the proposition AaC is 'known and agreed' to be impossible ($87^{a}9$–10). This is because its contrary opposite, AeC, is known and accepted by the demonstrator and their interlocutors.[47] Thus, the demonstrator accepts both AeC and BaC, and uses these two propositions to establish AoB by *reductio*, as follows:

1.	AeC	(premiss)
2.	BaC	(premiss)
3.	| AaB	(assumption for *reductio*)
4.	| BaC	(iterated from 2)
5.	| AaC	(from 3, 4, by Barbara)
6.	AoB	(*reductio*: 1, 3–5)

In this derivation, the first two lines contain the premisses accepted by the demonstrator, and line 6 contains the conclusion of the demonstration, AoB.[48] The inference from the two premisses to the conclusion takes the form of the third-figure mood Felapton:

AeC, BaC, therefore AoB

A similar proof by *reductio* in which the premisses and the conclusion constitute an instance of Felapton is given by Aristotle in *Posterior Analytics* 1. 16. Having posited the minor premiss BaC ($80^{a}28$–9), Aristotle reasons as follows:

πάλιν ὃ τῷ Γ μηδενὶ ὑπάρχει, οὐδὲ τῷ Β παντὶ ὑπάρξει· εἰ γὰρ τῷ Β, καὶ τῷ Γ· ἀλλ' οὐχ ὑπῆρχεν.[49] (*Post. An.* 1. 16, $80^{b}2$–4)

That which belongs to no C will not belong to all B either; for if it belongs to B, it also belongs to C, but it was assumed that it does not belong.

In this passage, Aristotle makes it clear that the conclusion of the proof by *reductio* is the o-proposition that A does not belong to all B (*οὐδὲ τῷ Β παντὶ ὑπάρξει*). If I am correct, he presents a proof of

[47] Seidl, *Zweite Analytiken*, 265–6.

[48] The application of *reductio* in the last line relies on the fact that the conclusion of the subordinate deduction, AaC, is the contrary opposite of an accepted premiss, AeC. For similar proofs by *reductio* in which the conclusion of the subordinate deduction is the contrary rather than the contradictory opposite of an accepted premiss, see *Pr. An.* 1. 2, $25^{a}17$–19; 1. 7, $29^{a}36$–9; cf. P. Thom, *The Syllogism* [*Syllogism*] (Munich, 1981), 39–41; Crivelli, 'Hypothesis', 156–8.

[49] For continuity, I use the schematic letters 'A', 'B', '*Γ*' in place of Aristotle's '*Γ*', 'A', 'B' at $80^{a}28$–$^{b}4$.

the same form, giving rise to an instance of Felapton, in his discussion of demonstration by *reductio* in chapter 1. 26.

3. Aristotle's argument from priority in nature

So far, Aristotle has compared the two demonstrations with respect to what is more known to the demonstrator. He now goes on, in what is the core argument of the chapter, to compare them with respect to what is prior in nature:

φύσει δὲ προτέρα ἡ ὅτι τὸ Α τῷ Β ἢ ὅτι τὸ Α τῷ Γ. πρότερα γάρ ἐστι τοῦ συμπεράσματος ἐξ ὧν τὸ συμπέρασμα. ἔστι δὲ τὸ μὲν Α τῷ Γ μὴ ὑπάρχειν συμπέρασμα, τὸ δὲ Α τῷ Β ἐξ οὗ τὸ συμπέρασμα. (*Post. An.* 1. 26, 87ᵃ17–20)

By nature, however, the proposition that A does not belong to B is prior to the proposition that A does not belong to C. For the things from which a conclusion derives are prior to the conclusion; and that A does not belong to C is a conclusion, whereas that A does not belong to B is that from which the conclusion derives.

Aristotle claims that the proposition AeB is prior in nature to AeC. He takes this claim to help establish the main thesis of the chapter, that direct negative demonstration is better than that by *reductio* because it proceeds from premisses that are prior in nature to the conclusion (87ᵃ25–8).[50] In the direct negative demonstration discussed by Aristotle, the premiss AeB is prior in nature to the conclusion, AeC. In the demonstration by *reductio*, by contrast, the premiss AeC is not prior in nature to the conclusion, AoB. Instead, Aristotle seems to regard AeC as posterior in nature to AoB. At least, this is suggested by his remark that demonstration by *reductio* proceeds from premisses that are posterior in nature (87ᵃ27). Thus, there is a clear sense in which the direct negative demonstration considered by Aristotle in 1. 26 is better than the demonstration by *reductio*.

This argument from priority in nature raises a number of questions. One of them, pointed out by Zabarella, concerns Aristotle's discussion of *reductio ad impossibile* in *Prior Analytics* 2. 14.[51] In this chapter, Aristotle argues that 'whatever is proved through the

[50] See Philop. *In An. Post.* 297. 14–22 Wallies.
[51] Zabarella, *Opera logica*, 975 F–976 A.

impossible can also be concluded ostensively'.[52] He shows that every conclusion deducible from given premisses by *reductio ad impossibile* can also be deduced from these premisses without *reductio* by means of a direct deduction.[53] For example, consider the following derivation by *reductio* (63^{a}14–16):

1.	CaA	(premiss)
2.	CoB	(premiss)
3.	AaB	(assumption for *reductio*)
4.	CaA	(iterated from 1)
5.	CaB	(from 3, 4, by Barbara)
6.	AoB	(*reductio*: 2, 3–5)

Aristotle points out that, in this derivation, the conclusion in line 6 can be deduced directly from the premisses in lines 1 and 2 without the use of *reductio* by applying the second-figure mood Baroco (2. 14, 63^{a}7–16).[54] Thus, the derivation can be turned into a direct deduction simply by omitting the steps in lines 3–5. In the same way, the conclusion of the demonstration by *reductio* discussed by Aristotle in *Posterior Analytics* 1. 26 can be deduced from the two premisses directly without *reductio* by applying the third-figure mood Felapton. In the resulting direct negative demonstration, the premiss AeC fails to be prior in nature to the conclusion, AoB. Hence, there is a direct negative demonstration in which the premisses are not prior in nature to the conclusion.

Conversely, Aristotle asserts in *Prior Analytics* 2. 14 that whenever a conclusion is deducible from given premisses by means of a direct deduction, it can also be deduced from them by *reductio ad impossibile* (63^{b}14–18).[55] In particular, the conclusion of Aristotle's direct demonstration in Celarent can be deduced from the same premisses by *reductio*.[56] The result is a demonstration by *reductio* in which the premisses, AeB and BaC, are prior in nature to the conclusion, AeC. Thus, given the interchangeability of direct deduction and *reductio ad impossibile* stated by Aristotle in *Prior Analytics* 2. 14, it is not clear how he can maintain that direct negative

[52] *Pr. An.* 2. 14, 62^{b}38–40: ἅπαν δὲ τὸ δεικτικῶς περαινόμενον καὶ διὰ τοῦ ἀδυνάτου δειχθήσεται, καὶ τὸ διὰ τοῦ ἀδυνάτου δεικτικῶς.

[53] *Pr. An.* 2. 14, 62^{b}38–63^{b}13; see also 1. 29, 45^{a}23–^{b}8.

[54] See Ross, *Analytics*, 455.

[55] See also *Pr. An.* 1. 29, 45^{a}26–7.

[56] See *Pr. An.* 2. 14, 63^{a}32–5; cf. Ross, *Analytics*, 456.

demonstrations differ from those by *reductio* in that they proceed from premisses that are prior in nature to the conclusion.

In view of this problem, Gisela Striker argues that Aristotle's argument in *Posterior Analytics* 1. 26 does not turn on the priority relations that obtain, or fail to obtain, between the premisses and the conclusion in the two kinds of demonstration. Instead, she suggests, the argument turns on the way the conclusion is inferred from the premisses in each case:

> As Aristotle himself shows in *An. Pr.* B 14, one can form a 'genuine' premiss pair for the *demonstrandum* from the true premiss of the syllogism [i.e. the true premiss of the direct deduction initiated by the assumption for *reductio*] and the negation of the impossible conclusion. However, this further syllogism is not regarded by Aristotle as a part of the *reductio*, but as a different proof of the same *demonstrandum*—in the *reductio* the proposition to be proved is inferred 'from a hypothesis'. . . . Aristotle's argument for the superiority of 'categorical' proofs presumably relies on the fact that *reductio* proofs are not purely syllogistic.[57] (Striker, 'Review of Barnes', 318)

Striker is drawing attention to the fact that Aristotle regards arguments by *reductio ad impossibile* as deductions 'from a hypothesis' (ἐξ ὑποθέσεως, *Pr. An.* 1. 23, 40^{b}23–9). According to Aristotle, every argument by *reductio* contains a part that can be analysed as a direct deduction in the three syllogistic figures. This part of the argument appears within the subordinate deduction initiated by the assumption for *reductio*.[58] In Aristotle's view, no such syllogistic analysis is available for the final step of the argument, in which the assumption for *reductio* is discharged and the desired conclusion inferred. Aristotle describes this latter inference as being 'from a hypothesis' and acknowledges that it cannot be justified within his

[57] 'Man kann ja, wie Aristoteles selbst in *An. Pr.* B 14 zeigt, aus der wahren Prämisse des Syllogismus und der Negation der unmöglichen Conclusio ein "echtes" Prämissenpaar für das Demonstrandum bilden. Dieser zweite Syllogismus wird aber von Aristoteles nicht als ein Teil der *reductio* angesehen, sondern als ein anderer Beweis für dasselbe Demonstrandum—in der *reductio* erschließt man den zu beweisenden Satz "aus der Hypothese". . . . Aristoteles' Argument für die Überlegenheit der "kategorischen" Beweise stützt sich vermutlich darauf, daß die *reductio*-Beweise nicht rein syllogistisch sind.'

[58] See *Pr. An.* 1. 23, 41^{a}32–4; 1. 44, 50^{a}29–32; cf. Alex. Aphr. *In An. Pr.* 259. 9–11, 261. 24–8, 262. 3–4 Wallies; ps.-Ammon. *In An. Pr.* 66. 39–67. 4 Wallies; J. Lear, *Aristotle and Logical Theory* [*Logical Theory*] (Cambridge, 1980), 34; Crivelli, 'Hypothesis', 125.

theory of syllogistic moods.[59] According to Striker, this is what underlies Aristotle's argument in *Posterior Analytics* 1. 26.[60]

It may well be that Aristotle accepted the considerations put forward by Striker, taking them to show that direct demonstrations are superior to those by *reductio*. Nonetheless, there is little evidence to suggest that Aristotle in fact appeals to these considerations in *Posterior Analytics* 1. 26. In the core argument of the chapter (87^a17–20 and 25–8), Aristotle does not address the way in which the conclusion is derived from the premisses in a demonstration by *reductio*, nor does he allude to the fact that this derivation is 'from a hypothesis'. Instead, he focuses on the relations of priority in nature that obtain between the premisses and the conclusion of the respective demonstrations. This is in tension with the interpretation proposed by Striker. For relations of priority in nature obtain, or fail to obtain, between the premisses and the conclusion of a demonstration regardless of how the latter is derived from the former. If the same conclusion can be derived from the same premisses either directly or by *reductio*, the choice of derivation does not affect the priority relations obtaining between them. Consequently, Barnes rejects Striker's interpretation of 1. 26, noting that 'Aristotle's argument does not turn on the nature of *reductio* as such'.[61]

Moreover, Striker's interpretation does not sit well with the elaborate exposition of the two kinds of demonstration given by Aristotle in the first half of the chapter (87^a2–17). For, if Aristotle had in mind the point attributed to him by Striker, he could have provided simpler examples of the two kinds of demonstration than he does. He could have chosen examples in which the same conclusion is derived from the same premisses—or he could have omitted the examples altogether, since Striker's point can easily be stated in a general manner without appealing to any particular examples.

Finally, Striker's interpretation makes it difficult to see why the argument in 1. 26 appeals to priority in nature at all. For, on this interpretation, the argument turns on general features of *reductio* described by Aristotle in the *Prior Analytics*, and it is not clear how

[59] *Pr. An.* 1. 23, 41^a34; 1. 44, 50^a29–32.

[60] Similar suggestions have been made by Themist. *In An. Post.* 37. 3–7 Wallies; Avicenna *al-Shifā': al-Burhān* 1. 8, 90. 15–17; 3. 7, 244. 14–245. 14 Afifi; Zabarella, *Opera logica*, 976 E–977 D; and Crivelli, 'Hypothesis', 170 n. 157.

[61] Barnes, *Posterior Analytics 2nd edn.*, 188.

these features bear on considerations concerning priority in nature.[62] No doubt Aristotle's discussion in the *Prior Analytics* implies that demonstrations by *reductio* are inferior to direct ones in a number of respects. For example, it implies that demonstrations by *reductio*, unlike direct ones, involve false propositions.[63] Moreover, when Aristotle argues in *Prior Analytics* 2. 14 that every deduction by *reductio* corresponds to a direct deduction deriving the same conclusion from the same premisses, the direct deduction is significantly shorter than the one by *reductio*. Thus, direct demonstration is better than that by *reductio* for reasons of economy, since, as Aristotle notes in *Posterior Analytics* 1. 25, 'other things being equal, a demonstration through fewer items is better'.[64] Crucially, however, Aristotle does not put forward such straightforward arguments in 1. 26. Instead, he appeals to considerations of priority in nature, which do not follow from his general characterization of *reductio* in the *Prior Analytics*.

Thus, we are left with Zabarella's problem of how to reconcile Aristotle's argument in 1. 26 with his treatment of *reductio* in *Prior Analytics* 2. 14. To address this problem, it is helpful to take a closer look at the deductive frameworks employed in these two chapters. In *Prior Analytics* 2. 14, Aristotle takes for granted the fourteen syllogistic moods in the three figures established in *Prior Analytics* 1. 4–6:

First figure:	Barbara, Celarent, Darii, Ferio
Second figure:	Cesare, Camestres, Festino, Baroco
Third figure:	Darapti, Disamis, Datisi, Felapton, Ferison, Bocardo

In *Prior Analytics* 2. 14, Aristotle takes each of these moods to license a direct deduction from the two premisses to the conclusion. With respect to this system, he argues in 2. 14 that any conclusion deducible from given premisses by *reductio ad impossibile*

[62] Thus, when Crivelli ('Hypothesis', 159–73) interprets 1. 26 along the lines suggested by Striker, he neglects to explain Aristotle's appeal to priority in nature in the chapter.

[63] The fact that proofs by *reductio* involve false propositions has been taken to show that they are inferior to direct ones, e.g. by G. Galilei, *Tractatus de praecognitionibus et praecognitis and Tractatio de demonstratione*, ed. W. F. Edwards (Padua, 1988), 97; and L. Löwenheim, 'On Making Indirect Proofs Direct', *Scripta Mathematica*, 12 (1946), 125–39 at 126.

[64] *Post. An.* 1. 25, 86^{b}5–7: *καὶ ἡ διὰ τῶν ἐλαττόνων ἄρα ἀπόδειξις βελτίων τῶν ἄλλων τῶν αὐτῶν ὑπαρχόντων.*

can also be deduced from these premisses by means of a direct deduction using the fourteen moods.[65] Thus, the rule of *reductio ad impossibile* is redundant in Aristotle's full syllogistic system in which all fourteen moods are taken to license direct deductions.

This does not mean, however, that the rule of *reductio* is redundant in Aristotle's theory of the assertoric syllogism as a whole. In *Prior Analytics* 1. 1–7, Aristotle presents two deductive systems in which the rule of *reductio* plays an indispensable role. The first of these systems, expounded in *Prior Analytics* 1. 2 and 1. 4–6, includes among its principles the four first-figure moods which Aristotle regards as 'perfect' (*Pr. An.* 1. 4). In addition, the system includes three conversion rules (*Pr. An.* 1. 2), and a rule of *reductio ad impossibile*:[66]

1.	Perfect moods:	AaB, BaC, therefore AaC	(Barbara)
		AeB, BaC, therefore AeC	(Celarent)
		AaB, BiC, therefore AiC	(Darii)
		AeB, BiC, therefore AoC	(Ferio)
2.	Conversion:	AeB, therefore BeA	(e-conversion)
		AiB, therefore BiA	(i-conversion)
		AaB, therefore BiA	(a-conversion)
3.	Rule of *reductio ad impossibile*		

In *Prior Analytics* 1. 5–6, Aristotle employs these principles to establish the validity of syllogistic moods in the second and third figures. In most cases, he does so by means of direct deductions employing the perfect first-figure moods and conversion rules. There are, however, two valid moods that cannot be established in this way: Baroco and Bocardo. Aristotle establishes these moods by *reductio ad impossibile*, using the perfect mood Barbara.[67] Thus, the rule of *reductio* is not redundant in this deductive system, since Baroco and Bocardo cannot be established directly but only by *reductio*.[68]

[65] See e.g. Ross, *Analytics*, 454–6; N. Strobach, *Aristoteles:* Analytica priora, *Buch II* (Berlin, 2015), 348.

[66] See e.g. J. Corcoran, 'Completeness of an Ancient Logic', *Journal of Symbolic Logic*, 37 (1972), 696–702 at 697–8; id., 'Aristotle's Natural Deduction System', in id. (ed.), *Ancient Logic and its Modern Interpretations* (Dordrecht, 1974), 85–131 at 109–10; Smith, *Prior Analytics*, xix–xxi.

[67] *Pr. An.* 1. 5, 27^a36–27^b1; 1. 6, 28^b15–20.

[68] See Alex. Aphr. *In An. Pr.* 83. 12–25 Wallies; J. Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, 2nd edn. (Oxford, 1957),

In *Prior Analytics* 1. 7, Aristotle goes on to present a second deductive system, in which the rule of *reductio* plays an even more prominent role. In this system, Aristotle no longer includes the particular first-figure moods Darii and Ferio in the list of principles. Instead, he establishes these moods by *reductio ad impossibile*, using the first-figure mood Celarent.[69] In addition, Aristotle derives the rules of i- and a-conversion by *reductio* from e-conversion.[70] Thus, the deductive system presented by Aristotle in *Prior Analytics* 1. 7 can be taken to rest on the following principles:

1.	Perfect moods:	AaB, BaC, therefore AaC	(Barbara)
		AeB, BaC, therefore AeC	(Celarent)
2.	Conversion:	AeB, therefore BeA	(e-conversion)
3.	Rule of *reductio ad impossibile*		

Although this deductive system contains fewer principles than the previous one, it has the same deductive power. In particular, the system is strong enough to derive all fourteen moods of Aristotle's assertoric syllogistic. There are, however, only four moods that can be established by means of a direct deduction in this system, namely, the four universal moods Barbara, Celarent, Cesare, and Camestres. The remaining ten moods, including Felapton, cannot be established directly but only by *reductio*. Thus, in this system, a significant portion of the assertoric syllogistic turns out to depend on the rule of *reductio ad impossibile*. At the same time, Aristotle takes this system to be significant because it allows him to reduce all syllogistic moods to the universal first-figure moods Barbara and Celarent

54. In the case of Bocardo, Aristotle mentions an alternative proof using the method of ecthesis instead of *reductio* (*Pr. An.* 1. 6, 28^{b}20–1). He does not mention a proof by ecthesis for Baroco and it is not clear whether such a proof is available. For example, the formulation of the rule of ecthesis given by Parsons allows for a proof of Bocardo but not of Baroco; T. Parsons, *Articulating Medieval Logic* (Oxford, 2014), 36–7; similarly, T. Ebert and U. Nortmann, *Aristoteles:* Analytica priora*, Buch I* [*Analytica priora*] (Berlin, 2007), 333–7. Without appealing to ecthesis, the only way for Aristotle to establish Baroco and Bocardo in his deductive system is by *reductio*.

[69] *Pr. An.* 1. 7, 29^{b}1–19. See Alex. Aphr. *In An. Pr.* 113. 5–114. 30 Wallies; H. Weidemann, 'Aristotle on the Reducibility of All Valid Syllogistic Moods to the Two Universal Moods of the First Figure (*APr* A7, 29^{b}1–25)', *History and Philosophy of Logic*, 25 (2004), 73–8; J. Barnes, *Truth, etc.: Six Lectures on Ancient Logic* (Oxford, 2007), 364–6.

[70] *Pr. An.* 1. 2, 25^{a}17–22. For Aristotle's proof by *reductio* of a-conversion, see n. 48 above.

(1. 7, 29^b1–2 and 29^b24–5). Thus, when Aristotle summarizes the results of *Prior Analytics* 1. 1–7 in 1. 23, he makes special mention of the system introduced in 1. 7, emphasizing that he has shown how 'every deduction is completed through the first figure and is reduced to the universal deductions in this figure'.[71]

All told, then, we must distinguish three different syllogistic frameworks employed by Aristotle in the *Prior Analytics*. While of equal deductive power, these frameworks differ from each other in the extent to which they rely on *reductio ad impossibile*. In the full system of the assertoric syllogistic, *reductio* is redundant and is not needed to establish any moods. In the deductive system of chapters 1. 4–6, *reductio* is not redundant but is needed to establish Baroco and Bocardo. Finally, in the streamlined system of chapter 1. 7, *reductio* is needed to establish most syllogistic moods, with the exception only of the four universal moods.

In the *Posterior Analytics*, Aristotle does not specify the syllogistic framework that is meant to underlie his theory of demonstration. It is clear that, in chapters 1. 14–26, he relies on elements of the syllogistic theory developed in the *Prior Analytics*, but he does not describe them in any detail. For the most part, it is not necessary for Aristotle to do so in the *Posterior Analytics*. His account of demonstration is to a large extent compatible with different syllogistic frameworks, and entering into a discussion of these frameworks would be extraneous to his main aims in the *Posterior Analytics*. In chapter 1. 26, however, it is important for Aristotle to demarcate the moods that rely on *reductio ad impossibile* from those that do not. This demarcation depends on exactly which syllogistic framework is adopted.

As we have seen, the full system of the assertoric syllogistic, in which all fourteen moods are taken to license direct deductions, does not fit well with Aristotle's argument in *Posterior Analytics* 1. 26. The same is true for the deductive system presented in *Prior Analytics* 1. 4–6. For, in this system, the mood Felapton does not rely on *reductio*, but can be established by means of a direct deduction (*Pr. An.* 1. 6, 28^a26–9). Hence, in this system, Aristotle's

[71] *Pr. An.* 1. 23, 41^b3–5: *ἅπας τε συλλογισμὸς ἐπιτελεῖται διὰ τοῦ πρώτου σχήματος καὶ ἀνάγεται εἰς τοὺς ἐν τούτῳ καθόλου συλλογισμούς*. Similarly, 1. 23, 40^b17–19. See G. Striker, 'Perfection and Reduction in Aristotle's *Prior Analytics*', in M. Frede and G. Striker (eds.), *Rationality in Greek Thought* (Oxford, 1996), 203–19 at 205 n. 5; ead., *Aristotle's* Prior Analytics, *Book 1* [*Prior Analytics*] (Oxford, 2009), 108 and 170–1; Ebert and Nortmann, *Analytica priora*, 740 and 747.

demonstration by *reductio* in *Posterior Analytics* 1. 26 can be replaced by a direct deduction deriving the same conclusion from the same premisses, thus giving rise to Zabarella's problem. By contrast, this problem does not arise in the streamlined system from *Prior Analytics* 1. 7. For, in this system, Felapton cannot be established by means of a direct deduction but only by *reductio*. Hence, among the three systems, the one from chapter 1. 7 fits best with Aristotle's argument in *Posterior Analytics* 1. 26.

More generally, Aristotle's argument in 1. 26 fits with any variant of the system from *Prior Analytics* 1. 7 that yields the same demarcation between moods that rely on *reductio* and those that do not. For example, Aristotle might be taken to employ a variant of this system in which the second-figure moods Cesare and Camestres are not reduced by e-conversion to Celarent, but are posited as additional principles along with Barbara and Celarent. This would be in accordance with the fact that, in the first book of the *Posterior Analytics*, Aristotle does not mention any conversion rules and does not undertake to reduce second- and third-figure moods to those in the first figure.[72] In any case, all that is important for our purposes is that in *Posterior Analytics* 1. 26, Aristotle employs a deductive system in which the only moods that can be established by means of a direct deduction are the purely universal moods, with all other moods relying on *reductio*.[73]

Given such a deductive system, the only direct negative demonstrations in the three figures are those of the form Celarent, Cesare, and Camestres. Aristotle does not explicitly state in 1. 26 that direct affirmative demonstrations proceed from premisses that are prior in nature to the conclusion, but it seems clear that he is committed to this view.[74] In the present deductive system, the only

[72] See Smith, 'Syllogism', 115–17 and 121–2; id., 'Relationship', 331–3.

[73] *Pace* Lear (*Logical Theory*, 53), who maintains that in *Posterior Analytics* 1. 26, every demonstration by *reductio* can be replaced by a direct demonstration deriving the same conclusion from the same premisses. Lear suggests that, 'when Aristotle comes to criticize proof *per impossibile*, in *Posterior Analytics* A26, all he can say is that the premisses which are prior in nature—those from which the conclusion can be proved directly—are not sufficiently familiar to us' (Lear, *Logical Theory*, 53). This, however, is not correct. Aristotle's point at 1. 26, 87^a17–20 and 87^a25–30, is not that the premisses of demonstrations by *reductio* fail to be familiar to us, but that they fail to be prior in nature to the conclusion.

[74] Having argued that direct negative demonstrations are better than those by *reductio* in that they proceed from premisses which are prior in nature to the conclusion (87^a25–8), Aristotle goes on to infer: 'consequently, it is also clear that positive demonstration, which is better than privative demonstration, is also better than that

direct affirmative demonstrations in the three figures are those of the form Barbara.

Thus, Aristotle's thesis in 1. 26 amounts to the claim that all demonstrations of the form Barbara, Celarent, Cesare, and Camestres proceed from premisses that are prior in nature to the conclusion, whereas this is not the case in general for demonstrations that employ the rule of *reductio ad impossibile*. In order to be able to verify this claim, we need an account of what it is for one proposition to be prior in nature to another. In what follows, I provide such an account for both affirmative and negative propositions.

4. Priority in nature for a-propositions

As we have seen, when Aristotle speaks of 'demonstration' by *reductio ad impossibile* in 1. 26, he uses the term 'demonstration' in a broader sense than the one defined in *Posterior Analytics* 1. 2. Otherwise his claim in 1. 26 that demonstrations by *reductio* do not proceed from premisses that are prior in nature to the conclusion would be contradictory. Conversely, his claim that direct demonstrations do proceed from such premisses would be trivially true, simply in virtue of the definition of demonstration. What, then, is the intended reference of the term 'demonstration' in these claims?

Minimally, a demonstration is a deduction (συλλογισμός). Yet it is unlikely that Aristotle uses 'demonstration' in chapter 1. 26 to refer indiscriminately to all deductions. For he is clear that not every direct deduction has premisses that are prior in nature to the conclusion. For example, as he points out in *Posterior Analytics* 1. 6, a true conclusion may be deduced from false premisses, which are surely not prior in nature to the true conclusion.[75] Perhaps Aristotle uses 'demonstration' to denote all deductions in which the premisses are true? Again, such a use of the term would seem too broad. For, in Aristotle's view, demonstration is closely tied to scientific knowledge (ἐπιστήμη), and there are many true propositions that

which leads to the impossible' (ὥστε καὶ ἡ ταύτης βελτίων ἡ κατηγορικὴ δῆλον ὅτι καὶ τῆς εἰς τὸ ἀδύνατόν ἐστι βελτίων, 87ᵃ28–30). This would be odd if Aristotle did not think that direct affirmative demonstrations are better than those by *reductio* in the same respect as direct negative ones.

[75] *Post. An.* 1. 6, 75ᵃ3–4; see also *Pr. An.* 2. 2–4.

fall outside the purview of scientific knowledge. In particular, as he explains in *Posterior Analytics* 1. 30, truths about contingent chance events do not admit of demonstration and hence cannot be objects of scientific knowledge.[76] One would, therefore, not expect Aristotle in 1. 26 to refer to deductions that involve such truths as 'demonstrations'.

Instead, Aristotle may be taken to use 'demonstration' in 1. 26 to denote those deductions in which the premisses are scientific propositions. Scientific propositions are all indemonstrable premisses of a given science and the theorems derivable from them by means of demonstrations in the strict sense defined in *Posterior Analytics* 1. 2. Whenever the premisses of a deduction are such scientific propositions, the deduction counts as a 'demonstration' in the broader sense of the term employed in 1. 26. Thus, given a class of indemonstrable premisses, the strict demonstrations defined in 1. 2 determine the class of scientific propositions of the science under consideration. A 'demonstration' in the broader sense is then taken to be any deduction in which all premisses are scientific propositions, whether or not it satisfies the definition of demonstration given in 1. 2. Although Aristotle does not explicitly introduce this sense of 'demonstration', it is a natural way for him to use the term. It guarantees that 'demonstrations' are restricted to truths that fall under the purview of demonstrative science, while at the same time being broad enough to allow for a coherent reading of Aristotle's thesis in *Posterior Analytics* 1. 26.

Given this sense of 'demonstration', Aristotle's thesis in 1. 26 amounts to the following claim: in every deduction of the form Barbara, Celarent, Cesare, and Camestres in which the premisses are scientific propositions, the premisses are prior in nature to the conclusion; by contrast, there are deductions by *reductio* in which the premisses are scientific propositions but not prior in nature to the conclusion. While this is a substantive claim, it is not incoherent but stands a chance of being verified. As we will see, the claim can in fact be verified provided there is a suitable characterization of priority in nature.

In *Posterior Analytics* 1. 2–3, Aristotle emphasizes that the premisses of genuine demonstrations are prior in nature to the

76 *Post. An.* 1. 30, 87^b19–27. See also *Pr. An.* 1. 13, 32^b18–22.

conclusion.[77] He does not, in these chapters, provide an account of what it is for one proposition to be prior in nature to another. Aristotle does, however, offer some guidance on this question in *Posterior Analytics* 1. 15–25. There he considers chains of terms connected by universal affirmations, such as the following:

ἔστω δὴ τὸ Γ τοιοῦτον, ὃ αὐτὸ μὲν μηκέτι ὑπάρχει ἄλλῳ, τούτῳ δὲ τὸ Β πρώτῳ, καὶ οὐκ ἔστιν ἄλλο μεταξύ. καὶ πάλιν τὸ Ε τῷ Ζ ὡσαύτως, καὶ τοῦτο τῷ Β. (*Post. An.* 1. 19, 81^{b}30–2)

Let C be such that it itself does not further belong to any other thing, and that B belongs to it primitively, with nothing else between them. Again, let E belong to F in the same way, and F to B.

If universal affirmation is indicated by arrows pointing from the predicate to the subject term, the chain of universal affirmations described in this passage can be represented as follows:[78]

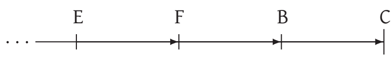

FIG. 1

In this diagram, each of the terms B, F, E, . . . belongs to its successor 'primitively' (*πρώτῳ*), that is, in such a way that there is no other term between them. Thus, each of these universal affirmations is immediate (*ἄμεσος*).[79]

Aristotle maintains that if a scientific proposition is immediate, there is no proposition that is prior in nature to it.[80] As such, immediate propositions are indemonstrable principles of a science.[81] For example, given that the a-proposition BaC is immediate, no proposition is prior in nature to it and there is no middle term by means of which it can be demonstrated. The indemonstrable a-propositions

[77] *Post. An.* 1. 2, 71^{b}19–72^{a}5; 1. 3, 72^{b}25–32; cf. n. 10 above.

[78] It is clear from the context (81^{b}10–18) that the verb *ὑπάρχειν* at 81^{b}30–2 is meant to indicate universal affirmations.

[79] Aristotle holds that A belongs to B primitively (*πρώτῳ* or *πρώτως*) just in case the a-proposition AaB is immediate (*ἄμεσος*); cf. 1. 16, 79^{b}25, with 1. 17, 81^{a}36; see also Philop. *In An. Post.* 194. 6–8, 186. 5–8, 186. 30–187. 6 Wallies. Accordingly, it is widely agreed that the primitive universal affirmations described by Aristotle at 1. 19, 81^{b}30–2 are immediate; Philop. *In An. Post.* 220. 18–20; Pacius, *Organum*, 464; Zabarella, *Opera logica*, 895 F; O. F. Owen, *The* Organon*, or Logical Treatises, of Aristotle*, vol. i [*Organon*] (London, 1889), 287; Mure, *Posteriora*, ad 81^{b}30; Mignucci, *L'argomentazione dimostrativa*, 404.

[80] *Post. An.* 1. 2, 72^{a}7–8; cf. R. Smith, 'Immediate Propositions and Aristotle's Proof Theory' ['Immediate Propositions'], *Ancient Philosophy*, 6 (1986), 47–68 at 48–55.

[81] *Post. An.* 1. 2, 72^{a}7–8; 1. 3, 72^{b}18–22.

of a science determine chains of terms connected by immediate universal affirmations. Aristotle refers to these chains as 'series' (*συστοιχίαι*).[82] In what follows, I refer to them as 'a-paths'. In Aristotle's example, there is an a-path from E to C and an a-path from F to B, the latter being a proper part of the former.

In Aristotle's syllogistic theory, the only way to deduce a universal affirmative conclusion AaB is by means of a-premisses forming a chain of universal affirmations leading from A to B.[83] If these premisses are immediate scientific propositions, they constitute an a-path from A to B. Aristotle holds that any such a-path gives rise to a demonstration of AaB.[84] For example, if there is an a-path from A to B through a single middle term C, there is a demonstration of AaB from the immediate premisses AaC and CaB. Hence each of these premisses is prior in nature to AaB.

More generally, if there is an a-path from A to B, every a-proposition that corresponds to a proper part of this a-path is prior in nature to AaB. Thus, for example, in *Posterior Analytics* 1. 25, Aristotle considers a structure of a-paths in which the proposition AaD is prior in nature to AaE:

ἔστω ἡ μὲν διὰ μέσων ἀπόδειξις τῶν Β Γ Δ ὅτι τὸ Α τῷ Ε ὑπάρχει, ἡ δὲ διὰ τῶν Ζ Η ὅτι τὸ Α τῷ Ε. ὁμοίως δὴ ἔχει τὸ ὅτι τὸ Α τῷ Δ ὑπάρχει καὶ τὸ Α τῷ Ε. τὸ δ' ὅτι τὸ Α τῷ Δ πρότερον καὶ γνωριμώτερον ἢ ὅτι τὸ Α τῷ Ε· διὰ γὰρ τούτου ἐκεῖνο ἀποδείκνυται. (*Post. An.* 1. 25, 86^{a}39–86^{b}5)

Let one demonstration show that A belongs to E through the middle terms B, C, and D, and let the other show that A belongs to E through F and G. Thus the proposition that A belongs to D and the one that A belongs to

[82] *Post. An.* 1. 15, 79^{b}7–11; 1. 17, 80^{b}27, 81^{a}21; 1. 29, 87^{b}5–14; see Smith, 'Syllogism', 122–6. See also Philop. *In An. Post.* 189. 11–13; Zabarella, *Opera logica*, 862 D; Mignucci, *L'argomentazione dimostrativa*, 341; Barnes, *Posterior Analytics 2nd edn.*, 163.

[83] *Pr. An.* 1. 26, 42^{b}32–3; see T. J. Smiley, 'What is a Syllogism?', *Journal of Philosophical Logic*, 2 (1973), 136–54 at 139–45; id., 'Aristotle's Completeness Proof', *Ancient Philosophy*, 14 (1994), 25–38 at 27; Thom, *Syllogism*, 181–3.

[84] See Smith, 'Syllogism', 126; McKirahan, *Principles*, 210–12 and 217. With respect to a-paths of immediate universal affirmations, Aristotle holds that 'when A belongs to B, then, if there is some middle term, it is possible to prove [i.e. demonstrate] that A belongs to B' (*ὅταν τὸ Α τῷ Β ὑπάρχῃ, εἰ μὲν ἔστι τι μέσον, ἔστι δεῖξαι ὅτι τὸ Α τῷ Β ὑπάρχει*, *Post. An.* 1. 23, 84^{b}19–20). Similarly, he writes: '. . . terms so related to a subject that there are other terms prior to them predicated of the subject are demonstrable [of the subject]' (*. . . ὧν πρότερα ἄττα κατηγορεῖται, ἔστι τούτων ἀπόδειξις*, 1. 22, 83^{b}33–4; transl. Mure, *Posteriora*, modified).

E are on a par. But that A belongs to D is prior to and more known than that A belongs to E; for the latter is demonstrated through the former.

In this passage, the terms A and E are connected by two distinct a-paths of immediate universal affirmations:[85]

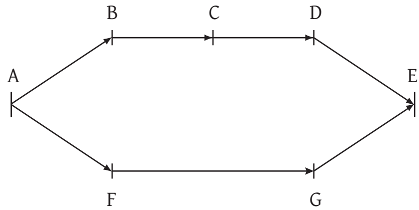

FIG. 2

The propositions AaD and AaE are 'on a par' in that both of them are demonstrable through the same number of middle terms (the former through B and C, and the latter through F and G).[86] At the same time, Aristotle asserts that AaD is prior in nature to, and more known in nature than, AaE.[87] Specifically, AaD is prior in nature to AaE in virtue of the fact that the a-path from A to D is a proper part of the a-path from A to E. If there are multiple a-paths from A to D, each of them is a proper part of some a-path from A to E. For the same reason, the proposition CaE is prior in nature to AaE, given that any a-path from C to E is a proper part of some a-path from A to E.[88]

In this way, a-paths provide a characterization of priority in nature between scientific a-propositions: for any arbitrary A, B, C, and D, the a-proposition AaB is prior in nature to CaD just in case

[85] See Zabarella, *Opera logica*, 964 E–F.

[86] Philop. *In An. Post.* 287. 22–5 Wallies; Mignucci, *L'argomentazione dimostrativa*, 548; Barnes, *Posterior Analytics 2nd edn.*, 187.

[87] When Aristotle writes *πρότερον καὶ γνωριμώτερον* at 86^{b}3–4, he does not mean priority and being more known 'to us' but 'in nature'; see M. F. Burnyeat, 'Aristotle on Understanding Knowledge' ['Understanding Knowledge'], in E. Berti (ed.), *Aristotle on Science: The* Posterior Analytics, *Proceedings of the Eighth Symposium Aristotelicum* (Padua, 1981), 97–139 at 127–8.

[88] However, this does not mean that the proposition CaE appears in a demonstration of AaE. In *Posterior Analytics* 2. 18, Aristotle argues that when AaE is demonstrated through B, C, and D, the final step of the demonstration should employ the least universal middle term, D, inferring AaE from AaD and DaE (99^{b}9–14; see Detel, *Analytica posteriora*, ii. 823 and 827; Tricot, *Seconds Analytiques*, 237 n. 4). Likewise, AaD should be inferred from AaC and CaD. On this account, the proposition CaE, though prior in nature to AaE, does not appear in the demonstration of AaE.

(i) there is an a-path from A to B, and (ii) any a-path from A to B is a proper part of some a-path from C to D.

Now, as Aristotle points out in *Posterior Analytics* 1. 3, priority is an asymmetric relation: if one proposition is prior in nature to another, the latter is not prior in nature to the former.[89] Does the proposed characterization of priority in nature satisfy this requirement of asymmetry? As it turns out, it fails to do so if there are cycles of immediate universal affirmations such as:

AaB, BaC, CaA

If each of these propositions is immediate, AaB is prior in nature to AaC since every a-path from A to B is a proper part of some a-path from A to C. At the same time, AaC is prior in nature to AaB, since every a-path from A to C can be extended to B. For example, the shortest a-path from A to C is a proper part of the following a-path from A to B:

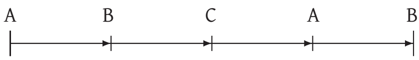

FIG. 3

Thus, the proposed characterization of priority in nature would fail to be asymmetric if there were cyclic a-paths. There is, however, evidence that Aristotle intends to exclude cyclic a-paths in the first book of the *Posterior Analytics*. In particular, he seems to exclude them in his discussion of reciprocal predication, or 'counterpredication', in chapter 1. 22. Based on the theory of predication developed in 1. 22, Aristotle argues against the possibility of counterpredication as follows:

ἔτι εἰ μή ἐστι τοῦτο τουδὶ ποιότης κἀκεῖνο τούτου, μηδὲ ποιότητος ποιότης, ἀδύνατον ἀντικατηγορεῖσθαι ἀλλήλων οὕτως, ἀλλ' ἀληθὲς μὲν ἐνδέχεται εἰπεῖν, ἀντικατηγορῆσαι δ' ἀληθῶς οὐκ ἐνδέχεται. ἢ γάρ τοι ὡς οὐσία κατηγορηθήσεται, οἷον ἢ γένος ὂν ἢ διαφορὰ τοῦ κατηγορουμένου . . . . ὡς μὲν δὴ γένη ἀλλήλων οὐκ ἀντικατηγορηθήσεται· ἔσται γὰρ αὐτὸ ὅπερ αὐτό τι. οὐδὲ μὴν τοῦ ποιοῦ ἢ τῶν ἄλλων οὐδέν, ἂν μὴ κατὰ συμβεβηκὸς κατηγορηθῇ· πάντα γὰρ ταῦτα συμβέβηκε καὶ κατὰ τῶν οὐσιῶν κατηγορεῖται. (*Post. An.* 1. 22, 83ᵃ36–ᵇ12)

If this is not a quality of that and that of this (a quality of a quality), it is impossible for one thing to be counterpredicated of another in this way. It is possible to make a true statement, but it is not possible to counterpredicate

[89] *Post. An.* 1. 3, 72ᵇ27–8; see also 2. 15, 98ᵇ16–21.

truly. For one alternative is that it is predicated as substance, i.e. being either the genus or the differentia of what is predicated. . . . Surely they will not be counterpredicated of one another as genera, for then something would be just what is some of itself. Nor will anything be counterpredicated[90] of a quality or of the other kinds of thing—unless it is predicated accidentally; for all these things are accidents, and they are predicated of substances.

While this passage presents many difficulties, it seems clear that its main focus is on counterpredication (*ἀντικατηγορεῖσθαι*).[91] By counterpredication, Aristotle means reciprocal predication—that is, cases in which A is predicated of B and B is predicated of A.[92] He considers various putative cases of counterpredication and argues that they are not admissible in the theory of predication developed in chapter 1. 22. He begins by noting that A and B cannot be counterpredicated of one another in such a way that one is a quality of the other and vice versa. Thus, for example, if *pale* and *musical* are

[90] It is natural to supply *ἀντικατηγορηθήσεται* from 83ᵇ9 as the main verb of this sentence; Waitz, *Organon*, ii. 357; Mure, *Posteriora*, ad loc.; Tredennick, *Posterior Analytics*, 123; J. Barnes, *Aristotle's* Posterior Analytics, 1st edn. [*Posterior Analytics 1st edn.*] (Oxford, 1975), 35; id. (ed.), *The Complete Works of Aristotle: The Revised Oxford Translation* [*Complete Works*], 2 vols. (Princeton, 1984), i 136; Seidl, *Zweite Analytiken*, 107; Tricot, *Seconds Analytiques*, 120. On the other hand, some authors take *κατηγορηθήσεται* to be the main verb of the sentence; Mignucci, *L'argomentazione dimostrativa*, 468–9; Barnes, *Posterior Analytics 2nd edn.*, 32; Pellegrin, *Seconds Analytiques*, 177 and 381 n. 14. Moreover, I take *τοῦ ποιοῦ ἢ τῶν ἄλλων* to be the genitive object of the verb of the sentence; Tredennick, *Posterior Analytics*, 123; Seidl, *Zweite Analytiken*, 107; Barnes, *Posterior Analytics 2nd edn.*, 32; Pellegrin, *Seconds Analytiques*, 177 (*pace* Barnes, *Posterior Analytics 1st edn.*, 35; id., *Complete Works*, i 136).

[91] Waitz, *Organon*, ii. 356–7; Mure, *Posteriora*, ad loc.; Ross, *Analytics*, 578–9; D. W. Hamlyn, 'Aristotle on Predication' ['Predication'], *Phronesis*, 6 (1961), 110–26 at 119–20; Barnes, *Posterior Analytics 1st edn.*, 169–70; J. Lear, 'Aristotle's Compactness Proof' ['Compactness'], *Journal of Philosophy*, 76 (1979), 198–215 at 214; id., *Logical Theory*, 31; Tricot, *Seconds Analytiques*, 118–20. *Pace* Mignucci, *L'argomentazione dimostrativa*, 460–9; Barnes, *Posterior Analytics 2nd edn.*, 177–8.

[92] Ross, *Analytics*, 578; Hamlyn, 'Predication', 119–20; Lear, 'Compactness', 214; id., *Logical Theory*, 31; Barnes, *Posterior Analytics 2nd edn.*, 178. This notion of counterpredication differs from the one employed by Aristotle in *Topics* 1. 5, according to which A is counterpredicated of B just in case A is true of every individual of which B is true and vice versa (*Top.* 1. 5, 102ᵃ18–30); see J. Brunschwig, *Aristote,* Topiques*, Livres I–IV* (Paris, 1967), 122. Thus, in the *Topics*, two terms are counterpredicated of one another if they are coextensive in the sense that they are true of the same class of individuals; see J. Barnes, 'Property in Aristotle's *Topics*', *Archiv für Geschichte der Philosophie*, 52 (1970), 136–55 at 137; O. Primavesi, *Die Aristotelische Topik: Ein Interpretationsmodell und seine Erprobung am Beispiel von* Topik *B* (München, 1996), 92. If A and B are coextensive, it does not follow that A is predicated of B or vice versa (see n. 103 below).

accidents in the category of quality, they are not counterpredicated of one another. Aristotle admits that if Callias is both pale and musical, there is a sense in which it is true to say that the pale thing is musical and the musical thing is pale (1. 22, 83^{a}7–12). He insists, however, that this is not an instance of true counterpredication.

In the last sentence of the passage, Aristotle strengthens this claim, asserting that nothing is counterpredicated of an accident that is in the category of quality or in any other non-substance category. Thus, if such an accident is predicated of a subject, the subject is not predicated of it. For example, if *large* is predicated of *log*, the latter is not predicated of the former. Again, Aristotle admits that there is a sense in which it is true to say that the large thing is a log (1. 22, 83^{a}1–3). In his view, however, such predications are merely accidental.[93] They are not proper predications and are not admissible in demonstrations (83^{a}14–21). For the same reason, the accident *musical* is not predicated of the accident *pale*. While it may be true to say that the pale thing is musical, Aristotle is clear that this is not a genuine predication and is not admissible in demonstrations (83^{a}10–21). More generally, he holds that no accident is the subject of a genuine predication (83^{b}21–2). Hence, if either A or B is an accident, they are not predicated of one another.

In the middle part of the passage, Aristotle discusses the possibility of counterpredication between items that are not accidents but are predicated 'as substance'. He states that two items cannot be counterpredicated of one another in such a way that one is a genus of the other and vice versa. It is not entirely clear whether he intends to exclude counterpredications between a definiendum and its definiens, such as *man* and *biped animal*. Philoponus argues that Aristotle intends to exclude them on the grounds that, in these cases, the counterpredicated items are the same whereas the predicate and the subject of genuine predications must be distinct.[94] If this is correct, Aristotle can be taken to exclude any counterpredications between items that are not accidents.

Accordingly, it is widely agreed that, in the passage just quoted, Aristotle excludes the possibility of any counterpredication.[95] This

[93] *Post. An.* 1. 22, 83^{a}1–18; see also 1. 19, 81^{b}23–9; *Pr. An.* 1. 27, 43^{a}33–6.

[94] Philop. *In An. Post.* 246. 14–24 Wallies.

[95] Waitz, *Organon*, ii. 356; Mure, *Posteriora*, ad loc.; Ross, *Analytics*, 578–9; Hamlyn, 'Predication', 120; Barnes, *Posterior Analytics 1st edn.*, 169–70; Lear, 'Compactness', 214; id., *Logical Theory*, 31; Tricot, *Seconds Analytiques*, 119–20.

is confirmed by his statement in chapter 1. 22 that all demonstrations in Barbara are based on chains of immediate, or primitive, predications. In all demonstrations by Barbara, Aristotle writes, 'it is necessary for there to be an item of which something is predicated primitively, and something else of this; and this must come to a stop, and there must be an item which is no longer predicated of anything prior and of which nothing else prior is predicated'.[96] At the same time, he denies that there are any such chains of primitive predications among counterpredicated items:

οὐ γὰρ ἔστιν ἐν τοῖς ἀντικατηγορουμένοις οὗ πρώτου κατηγορεῖται ἢ τελευταίου· πάντα γὰρ πρὸς πάντα ταύτῃ γε ὁμοίως ἔχει. (*Post. An.* 1. 19, 82ᵃ15–17)

Among counterpredicated items, there is none of which any is predicated primitively or of which it is predicated last; for in this respect at least every such item is related to every other in the same way.

In a class of counterpredicated items, every item is predicated of every other item. Thus, as far as predication is concerned, 'every such item is related to every other in the same way'. Aristotle infers from this that there is no basis for distinguishing immediate (or primitive) from mediate predications, and hence that there are no immediate predications among such items.[97] By the same reasoning, if an item is counterpredicated of something, it is not predicated immediately of anything and nothing is predicated immediately of it. Thus, in maintaining that all demonstrations in Barbara are based on chains of immediate predications, Aristotle in effect excludes counterpredication from the domain of demonstration.[98]

In *Posterior Analytics* 1. 19–22, Aristotle uses his account of predication to characterize the relation of universal affirmation in scientific propositions. In doing so, he assumes that if AaB is a scientific proposition, then A is predicated of B.[99] Hence, by excluding

[96] *Post. An.* 1. 22, 83ᵇ28–31: *ἀνάγκη ἄρα εἶναί τι οὗ πρῶτόν τι κατηγορεῖται καὶ τούτου ἄλλο, καὶ τοῦτο ἵστασθαι καὶ εἶναί τι ὃ οὐκέτι οὔτε κατ' ἄλλου προτέρου οὔτε κατ' ἐκείνου ἄλλο πρότερον κατηγορεῖται.* For *πρῶτον* at 83ᵇ29 indicating immediate predication, see Pacius, *Organum*, 474; Owen, *Organon*, 294; Barnes, *Posterior Analytics 2nd edn.*, 179.

[97] Waitz, *Organon*, ii. 350.

[98] Accordingly, Smith takes 82ᵃ15–17 to show that 'propositions involving convertible [i.e. counterpredicated] terms are somehow disqualified because of the lack of any priority' (Smith, 'Immediate Propositions', 62).

[99] *Post. An.* 1. 22, 83ᵃ18–21; see Zabarella, *Opera logica*, 909 F–910 A; Mignucci, *L'argomentazione dimostrativa*, 454–5. See also Lear, 'Compactness', 201–14; id.,

counterpredication, Aristotle excludes a-paths of the form AaB, BaA. More generally, he excludes any cyclic a-paths of the form A_1aA_2, $A_2aA_3, \ldots, A_naA_1$.[100] For, if there were such a-paths, both A_1aA_2 and A_2aA_1 would be scientific propositions, and hence A_1 and A_2 would be counterpredicated of one another.

Of course, Aristotle recognizes that sciences deal with terms that are coextensive in the sense that they are true of the same class of individuals.[101] For example, in the science of geometry, *triangle* is coextensive with *2R* (i.e. *having interior angles equal to two right angles*). Nevertheless, Aristotle denies in *Posterior Analytics* 1. 22 that coextensive terms such as *triangle* and *2R* are predicated of one another. As Philoponus notes in his commentary on 1. 22, Aristotle accepts that *2R* is predicated of *triangle* but denies that the latter is predicated of the former.[102] Consequently, while there is a demonstrable scientific a-proposition in which *2R* is the predicate and *triangle* the subject term, the converse a-proposition is not a scientific proposition. Thus, in Aristotle's theory of demonstration, the fact that A and B are coextensive does not imply that the proposition AaB is a demonstrable theorem or an indemonstrable principle of the science under consideration. This allows Aristotle to countenance coextensive terms while maintaining that there are no cyclic a-paths in a demonstrative science.[103]

Logical Theory, 18–31; R. Smith, 'Predication and Deduction in Aristotle: Aspirations to Completeness', *Topoi*, 10 (1991), 43–52 at 48–51.

[100] See Ross, *Analytics*, 578–9; Lear, 'Compactness', 214; id., *Logical Theory*, 31; Tricot, *Seconds Analytiques*, 118–19 n. 3. Smith argues that Aristotle's rejection of cyclic a-paths plays an important role in his argument for the finitude of demonstration in *Posterior Analytics* 1. 22; R. Smith, 'Aristotle as Proof Theorist', *Philosophia Naturalis*, 27 (1984), 590–97 at 593–4. See also Smith, 'Immediate Propositions', 62; id., 'Aristotle's Regress Argument' ['Regress'], in I. Angelelli and M. Cerezo (eds.), *Studies on the History of Logic: Proceedings of the III. Symposium on the History of Logic* (Berlin, 1996), 21–32 at 27.

[101] He refers to them as terms that 'follow one another' (ἀλλήλοις ἕπεσθαι, *Post. An.* 1. 3, 73ᵃ7) or terms that 'convert' (ἀντιστρέφειν, 1. 13, 78ᵃ27; 1. 19, 82ᵃ15; 2. 4, 91ᵃ16; 2. 16, 98ᵇ36).

[102] Philop. *In An. Post.* 246. 24–30 Wallies.

[103] In *Posterior Analytics* 1. 13, Aristotle states that the terms *near* and *non-twinkling* are counterpredicated of one another (ἀντικατηγορεῖσθαι, 78ᵃ28). Smith ('Immediate Propositions', 62) argues that this conflicts with Aristotle's rejection of counterpredication in *Posterior Analytics* 1. 19–22. However, the apparent conflict can be resolved. In chapter 1. 13, the term ἀντικατηγορεῖσθαι presumably does not express that two terms are predicated of one another in the sense of predication described in chapters 1. 19–22, for this sense of predication has not yet been introduced in 1. 13, but is only defined at 1. 19, 81ᵇ23–9 and 1. 22, 83ᵃ1–21. Instead,

Given that a-paths are acyclic, the proposed characterization of priority in nature satisfies Aristotle's requirement of asymmetry. To see this, recall that an a-proposition AaB is prior in nature to CaD just in case (i) there is an a-path from A to B, and (ii) any a-path from A to B is a proper part of some a-path from C to D. Hence, if AaB were prior in nature to CaD and vice versa, some a-path from A to B would be a proper part of some a-path from A to B, violating the condition of acyclicity. Thus, we have a well-defined, asymmetric relation of priority in nature between scientific a-propositions.

A-paths can naturally be represented by acyclic directed graphs such as the following:

FIG. 4

In this diagram, the demonstrable theorem AaB corresponds to an a-path containing the middle terms $C_1, \ldots, C_4$. Aristotle employs such diagrammatic representations in *Posterior Analytics* 1. 23, when he refers to immediate a-propositions as indivisible constituents of the theorems demonstrated from them ($84^{b}19$–31).[104] He states that every demonstrable a-proposition corresponds to an a-path containing one or more middle terms ($84^{b}31$–$85^{a}4$). Indemonstrable a-premisses, on the other hand, correspond to immediate a-paths. Thus, any scientific proposition AaB, demonstrable or otherwise, is underwritten by an a-path from A to B.

This allows us to verify part of Aristotle's claim in *Posterior Analytics* 1. 26 that the premisses of direct demonstrations are prior in nature to the conclusion. In particular, the claim can be verified for direct demonstrations of the form Barbara: whenever the premisses of a deduction in Barbara are scientific propositions, they are prior in nature to the conclusion. To see this, consider an instance of Barbara inferring AaC from AaB and BaC. Given that both premisses are scientific propositions, there is an a-path from A to B and an a-path

ἀντικατηγορεῖσθαι in 1. 13 can be understood in the weaker sense introduced in the *Topics*, in which it does not express that two terms are predicated of one another but only that they are coextensive (see n. 92). On this reading, Aristotle's acceptance of 'counterpredicated' terms in 1. 13 is consistent with his rejection of counterpredication in 1. 19–22.

[104] See Malink, 'Elements', 174–86.

from B to C. These two a-paths can be combined to form an a-path from A to C of which they are proper parts. More generally, any two a-paths from A to B and from B to C compose an a-path from A to C of which they are proper parts. Hence, both AaB and BaC are prior in nature to AaC.

Thus, Aristotle's claim holds for direct demonstrations of the form Barbara. To establish the claim for all direct demonstrations, it remains to verify it for those of the form Celarent, Cesare, and Camestres. This can be done in an analogous manner, by extending the present account to include not only a-paths but also e-paths.

5. Priority in nature for e-, i-, and o-propositions

In addition to immediate a-propositions, Aristotle countenances immediate e-propositions in the *Posterior Analytics*.[105] If an e-proposition is immediate, it is an indemonstrable principle of a science. It is atomic in the sense that there is no middle term through which it can be demonstrated:

ὥσπερ δὲ ὑπάρχειν τὸ Α τῷ Β ἐνεδέχετο ἀτόμως, οὕτω καὶ μὴ ὑπάρχειν ἐγχωρεῖ. λέγω δὲ τὸ ἀτόμως ὑπάρχειν ἢ μὴ ὑπάρχειν τὸ μὴ εἶναι αὐτῶν μέσον· οὕτω γὰρ οὐκέτι ἔσται κατ' ἄλλο τὸ ὑπάρχειν ἢ μὴ ὑπάρχειν. (*Post. An.* 1. 15, 79$^{\text{a}}$33–6)

Just as it is possible for A to belong to B atomically, so it is also possible for it atomically not to belong. By atomically belonging or not belonging I mean that there is no middle term between them; for, in this case, they no longer belong or do not belong by virtue of something else.

If AeB is immediate, B is 'first' (*πρῶτον*) among the terms to which A does not belong (1. 19, 82$^{\text{a}}$10–11). Accordingly, if both AeC and CaB are immediate, C is prior (*πρότερον*) to B among the terms to which A does not belong.[106] This is illustrated by the following diagram, in which the immediate e-proposition AeC is indicated by a zigzag line:

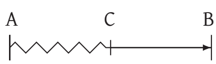

FIG. 5

This diagram represents an e-path from A to B, composed of an atomic e-path from A to C and an atomic a-path from C to B.

[105] *Post. An.* 1. 15, 79$^{\text{a}}$33–$^{\text{b}}$22; 1. 23, 84$^{\text{b}}$24–31.

[106] *Post. An.* 1. 19, 82$^{\text{a}}$9–13; 1. 21, 82$^{\text{b}}$5–11.

Similarly, if AeB is demonstrated by Camestres from immediate premisses CaA and CeB, the e-path from A to B is composed of an atomic a-path from C to A and an atomic e-path from C to B:

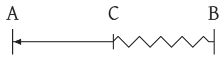

FIG. 6

In the *Posterior Analytics*, Aristotle considers complex negative demonstrations in which an e-conclusion is demonstrated by successive applications of Celarent, Cesare, Camestres, and Barbara.[107] In these demonstrations, the conclusion corresponds to compound e-paths such as the following:[108]

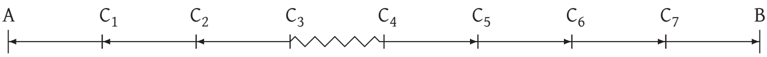

FIG. 7

In this diagram, the e-path from A to B is composed of an atomic e-path from C_3 to C_4 and two compound a-paths (one from C_3 to A and the other from C_4 to B).

Thus, for Aristotle, any given science determines a demonstrative structure which consists of a set of terms, T, equipped with the two relations of immediate universal affirmation and negation. In what follows, I will indicate these relations by '→' and '~', respectively. Thus, 'A→B' means that AaB is an immediate premiss of the science under consideration, and 'A~B' means that AeB is such a premiss. Both → and ~ are binary relations on T. As such, they can be represented as subsets of the Cartesian product $T \times T$. Moreover, we write '$\rightarrow^+$' to denote the transitive closure of → (i.e. the smallest transitive relation containing →). Thus, $A \rightarrow^+ B$ just in case there is an a-path from A to B (i.e. just in case there are $C_1, C_2, \ldots, C_n \in T$, $n \geq 1$, such that $A \rightarrow C_1$, $B = C_n$, and $C_i \rightarrow C_{i+1}$ for all $1 \leq i < n$).

Using this notation, the demonstrative structures employed by Aristotle in the *Posterior Analytics* can be characterized as follows:

> DEFINITION 1: A *demonstrative structure* is a triple $\langle T, \rightarrow, \sim \rangle$, in which T is a non-empty set of terms, and → and ~ are binary relations on T, such that for any A, B $\in T$:
>
> i. If $A \rightarrow B$, there is no $C \in T$ such that $A \rightarrow^+ C$ and $C \rightarrow^+ B$.

[107] *Post. An.* 1. 21, 82b4–21; 1. 23, 85a3–7; 1. 25, 86b10–27.

[108] Smith, 'Regress', 29; id., 'Aristotle's Theory of Demonstration', in G. Anagnostopoulos (ed.), *A Companion to Aristotle* (Oxford, 2009), 51–65 at 57–8.

ii. If A~B, then A≠B and B~A.
iii. If A~B, there is no C ∈ T such that $A\rightarrow^{+}C$ and either $B\rightarrow^{+}C$ or B=C.
iv. If A~B, there are no C, D ∈ T such that C~D and $C\rightarrow^{+}A$ and either $D\rightarrow^{+}B$ or D=B.

Condition (i) ensures that → is the relation of immediate as opposed to mediate universal affirmation. Moreover, it implies that → is acyclic (i.e. that there is no A ∈ T such that $A\rightarrow^{+}A$). Condition (ii) states that the relation of immediate universal negation, ~, is irreflexive and symmetric. Condition (iii) states that if A~B, then A and B are disjoint in the sense that there is no a-path from A to B or to a third term reachable from B. Finally, (iv) ensures that ~ is immediate as opposed to mediate universal negation.[109]

In a demonstrative structure, an a-path from A to B corresponds to a set of pairs $\langle A, C_1\rangle, \langle C_1, C_2\rangle, \ldots, \langle C_n, B\rangle$ such that $A\rightarrow C_1$, $C_i\rightarrow C_{i+1}$, and $C_n\rightarrow B$. Thus, a-paths can be defined as subsets of the relation →, as follows:

Definition 2: Let $\langle T, \rightarrow, \sim\rangle$ be a demonstrative structure. For any A, B ∈ T, an *a-path from A to B* is a set $\{\langle A, C_1\rangle, \langle C_1, C_2\rangle, \ldots, \langle C_{n-1}, C_n\rangle\} \subseteq \rightarrow$ ($n \geq 1$) such that C_n=B.

As we have seen, an e-path from A to B consists of an atomic e-path from C to D together with an a-path from C to A (unless C=A) and an a-path from D to B (unless D=B). Thus, if the atomic e-path is represented by the singleton $\{\langle C, D\rangle\}$ and the two possible a-paths by sets P and Q, e-paths can be defined as the union of these three sets:

Definition 3: Let $\langle T, \rightarrow, \sim\rangle$ be a demonstrative structure. For any A, B ∈ T, an *e-path from A to B* is a set $\{\langle C, D\rangle\}\cup P\cup Q$ such that:

i. C~D,
ii. either C=A and P=∅ or P is an a-path from C to A, and
iii. either D=B and Q=∅ or Q is an a-path from D to B.

[109] In addition to conditions (i)–(iv), the demonstrative structures considered by Aristotle in the *Posterior Analytics* satisfy a number of further conditions. For example, Aristotle holds that they do not contain any infinite chains of the form $A_1\rightarrow A_2$, $A_2\rightarrow A_3$, . . . (1. 22, 84ᵃ7–11 and 84ᵃ29–ᵇ2). For our present purposes, it is not necessary to make these additional conditions explicit.

Just like a-paths, e-paths are subsets of $T \times T$. Accordingly, parthood between these paths is given by the subset relation: an a- or e-path is a proper part of another a- or e-path just in case the former is a proper subset of the latter.[110]

Given this account of e-paths, the above characterization of priority in nature can be extended to cover e-propositions: AeB is prior in nature to CeD just in case (i) there is an e-path from A to B, and (ii) any e-path from A to B is a proper subset of some e-path from C to D. Likewise, AaB is prior in nature to CeD just in case there is an a-path from A to B and any such a-path is a proper subset of some e-path from C to D.

This allows us to verify Aristotle's claim in *Posterior Analytics* 1. 26 that the premisses of every direct negative demonstration are prior in nature to the conclusion. For any deduction of the form Celarent, Cesare, and Camestres, if the premisses are scientific propositions, they are prior in nature to the conclusion. To see this, consider an instance of Celarent in which AeC is deduced from AeB and BaC. Given that both premisses are scientific propositions, there is an e-path from A to B and an a-path from B to C. The union of these two paths is an e-path from A to C of which the first two paths are proper subsets. Likewise, any e-path from A to B and any a-path from B to C can be composed to form an e-path from A to C of which they are proper subsets. Hence, both AeB and BaC are prior in nature to AeC.

Similarly, consider an instance of Camestres in which the premisses, BaA and BeC, are scientific propositions. There is an a-path from B to A and an e-path from B to C. Their union is an e-path from A to C of which they are proper subsets. The same is the case for any a-path from B to A and any e-path from B to C. Hence, BaA and BeC are prior in nature to AeC. The same argument applies in the case of Cesare. Hence, given that the only direct deductions in the three syllogistic figures are those of the form Barbara, Celarent, Cesare, and Camestres, it follows that the premisses of every direct demonstration are prior in nature to the conclusion.

While a- and e-paths determine an order of priority in nature among scientific a- and e-propositions, they do not determine such an order for scientific i- and o-propositions. To this end, we need

[110] I am grateful to Kit Fine for suggesting to me this way of modelling parthood between paths.

to introduce i- and o-paths. Aristotle does not mention immediate i- or o-propositions in the *Posterior Analytics*, nor does he explain when an i- or o-proposition is prior in nature to another. His account of demonstration is therefore compatible with different ways of characterizing i- and o-paths. In what follows, I will adopt a simple characterization of these paths in terms of a- and e-paths. An i-path will be taken to consist of two a-paths with a common endpoint. For example, the following i-path from A to B consists of two a-paths from A and B to C_4:

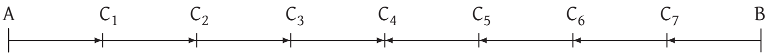

FIG. 8

Thus, i-paths can be defined as follows:

Definition 4: Let $\langle T, \rightarrow, \sim\rangle$ be a demonstrative structure. For any A, B $\in$ T, an *i-path from A to B* is a set P$\cup$Q such that for some C $\in$ T:

i. P is an a-path from A to C,
ii. Q is an a-path from B to C, and
iii. the sets P and Q are disjoint.

For example, if there are atomic a-paths from both *biped* and *animal* to *human*, they compose an i-path from *biped* to *animal*. It follows that the a-propositions *All humans are biped* and *All humans are animals* are prior in nature to the i-proposition *Some animals are biped*.

Condition (iii) in the definition excludes prolix i-paths that contain superfluous parts not needed to establish an i-proposition. For example, suppose that there is a unique a-path from *human* to *Greek*. Hence there are a-paths from both *biped* and *animal* to *Greek*, but these two a-paths are not disjoint since they both contain the a-path from *human* to *Greek*. Thus, condition (iii) ensures that they do not compose an i-path from *biped* to *animal*. This accounts for the intuition that *All Greeks are human* is not prior in nature to *Some animals are biped*.[111]

Similarly, an o-path can be taken to consist of an e-path and an a-path that have a common endpoint. For example, the following

[111] Condition (iii) may be strengthened, e.g. by requiring that no term other than C appear on both P and Q. For present purposes, however, it is not necessary to add this stronger condition.

o-path from A to B consists of an e-path from A to C_5 and an a-path from B to C_5:

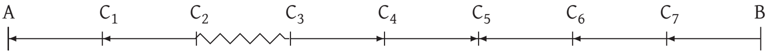

FIG. 9

Thus, o-paths can be defined as follows:

DEFINITION 5: Let $\langle T, \rightarrow, \sim\rangle$ be a demonstrative structure. For any A, B $\in$ T, an *o-path from A to B* is a set P$\cup$Q such that for some C $\in$ T:

i. P is an e-path from A to C,
ii. Q is an a-path from B to C, and
iii. the sets P and Q are disjoint.

For example, if there is an atomic e-path from *four-footed* to *human* and an a-path from *animal* to *human*, they compose an o-path from *four-footed* to *animal*. It follows that the propositions *No humans are four-footed* and *All humans are animals* are prior in nature to the o-proposition *Some animals are not four-footed*.

As before, condition (iii) excludes prolix o-paths that contain superfluous parts not needed to establish an o-proposition. Suppose, for example, that there is a unique e-path from *four-footed* to *Greek* and a unique a-path from *animal* to *Greek*. Since both of these paths contain the a-path from *human* to *Greek*, they are not disjoint and hence fail to compose an o-path from *four-footed* to *animal*. In this way, condition (iii) accounts for the intuition that *All Greeks are human* is not prior in nature to the o-proposition *Some animals are not four-footed*.

In a demonstrative structure, an i-proposition may be underwritten either by an i-path or by an a-path. Likewise, an o-proposition may be underwritten by an o- or e-path. If a path underwrites a proposition, the former is called a 'path for' the latter:

DEFINITION 6: Let $\langle T, \rightarrow, \sim\rangle$ be a demonstrative structure. For any A, B $\in$ T:

an a-path from A to B is a *path for* the propositions AaB, AiB, and BiA,
an e-path from A to B is a *path for* the propositions AeB and AoB,
an i-path from A to B is a *path for* the proposition AiB, and
an o-path from A to B is a *path for* the proposition AoB.

A proposition is satisfied in a demonstrative structure if there is a path for it in this structure:

> Definition 7: Let $\langle T, \rightarrow, \sim \rangle$ be a demonstrative structure with A, B $\in$ T. For any proposition AxB (where 'x' stands for 'a', 'e', 'i', or 'o'): AxB is *satisfied in* $\langle T, \rightarrow, \sim \rangle$ just in case there is a path for AxB in $\langle T, \rightarrow, \sim \rangle$.

Given this definition of satisfaction, all fourteen moods and conversion rules of Aristotle's assertoric syllogistic are sound with respect to the class of demonstrative structures. Whenever the premisses of any of these moods and conversion rules are satisfied in a demonstrative structure, the conclusion is satisfied in this structure.[112] Hence, assuming that all immediate premisses of a given science are satisfied in the underlying demonstrative structure, all theorems derivable from them by means of the deductive resources of Aristotle's assertoric syllogistic are satisfied in this structure. Thus, all scientific propositions—immediate premisses and demonstrable theorems alike—are satisfied in the demonstrative structure determined by the science under consideration.

It should be noted that there are cases in which neither AeB nor AiB is satisfied in a demonstrative structure.[113] Since Aristotle requires that one of these propositions be true, satisfaction in a demonstrative structure does not capture the truth-conditions of categorical propositions. Instead, it captures their status as scientific propositions: a proposition is satisfied in a demonstrative structure just in case it is either an immediate premiss or a demonstrable theorem of the science under consideration. Thus, if neither AeB nor AiB is satisfied in a demonstrative structure, one of them is true but neither is a scientific proposition of the science.

We are now in a position to provide a general characterization of priority in nature between scientific a-, e-, i-, and o-propositions:

> Definition 8: Let $\langle T, \rightarrow, \sim \rangle$ be a demonstrative structure with A, B, C, D $\in$ T. For any propositions AxB and CyD (where 'x'

[112] This can be verified by checking each mood and conversion rule. Consider, for example, the case of Baroco: BaA, BoC, therefore AoC. Given that both premisses are satisfied in a demonstrative structure, there is an a-path or from B to A and either an e- or o-path from B to C. It can be shown that the union of these two paths is an e-path or o-path from A to C, respectively. The other cases are similar.

[113] Similarly, there are cases in which neither AaB nor AoB is satisfied in a demonstrative structure.

and 'y' stand for 'a', 'e', 'i', or 'o'), AxB is *prior in nature to* CyD *in* $\langle T, \rightarrow, \sim\rangle$ just in case:

i. AxB is satisfied in $\langle T, \rightarrow, \sim\rangle$, and
ii. every path for AxB is a proper subset of some path for CyD.

Clearly, this relation of priority in nature is transitive. Moreover, given the acyclicity of $\rightarrow$, it is irreflexive, and hence asymmetric.[114] As such, it constitutes a well-defined relation of priority in nature between scientific propositions.

Of course, the present characterization of priority in nature is not stated by Aristotle in the *Posterior Analytics*. Nonetheless, it captures the way in which he took the relation of priority in nature between scientific propositions to be determined by a- and e-paths. Moreover, as we will see, this characterization allows us to verify Aristotle's thesis in *Posterior Analytics* 1. 26.

6. Accounting for Aristotle's thesis in *Posterior Analytics* 1. 26

The thesis of 1. 26 states that direct demonstrations proceed from premisses that are prior in nature to the conclusion, whereas demonstrations by *reductio* proceed from premisses that are posterior in nature to the conclusion. The first part of this thesis, regarding direct demonstrations, is captured by the following theorem:

> Theorem 1: Let $\langle T, \rightarrow, \sim\rangle$ be a demonstrative structure. For any deduction of the form Barbara, Celarent, Cesare, and Camestres, if both premisses are satisfied in $\langle T, \rightarrow, \sim\rangle$, then each premiss is prior in nature to the conclusion in $\langle T, \rightarrow, \sim\rangle$.

The theorem holds because, as we have seen, reasoning by Barbara from premisses satisfied in a demonstrative structure amounts to extending a-paths in this structure, and reasoning by Celarent, Cesare, and Camestres from such premisses amounts to extending e-paths.

Since all scientific propositions are satisfied in a demonstrative structure, every deduction of the form Barbara, Celarent, Cesare, and Camestres in which both premisses are scientific propositions

[114] If a proposition AxB were prior in nature to itself, every path for AxB would be a proper subset of some path for AxB. But, given that $\rightarrow$ is acyclic, no a- or e-path for AxB is a proper subset of another a- or e-path for AxB, and no i- or o-path for AxB is a proper subset of any path for AxB.

proceeds from premisses that are prior in nature to the conclusion. Given that a 'demonstration' is any deduction in which the premisses are scientific propositions, this claim covers all demonstrations instantiating the four universal moods. Hence, in a deductive system in which these are the only direct demonstrations in the three figures, every direct demonstration proceeds from premisses that are prior in nature to the conclusion.

By contrast, this is not the case for demonstrations that employ the rule of *reductio ad impossibile*. Consider, for example, Aristotle's demonstration by *reductio* in which the premisses and the conclusion constitute an instance of Felapton, deriving AoB from AeC and BaC. There are demonstrative structures in which both premisses of this demonstration are satisfied while neither premiss is prior in nature to the conclusion, but instead the conclusion is prior in nature to one of the premisses. Such a demonstrative structure can be specified using exactly the same arrangement of terms given by Aristotle in *Posterior Analytics* 1. 26 (87a3–12):

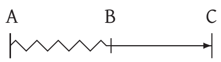

FIG. 10

The two premisses AeC and BaC are satisfied in this structure, since there is an e-path from A to C and an a-path from B to C. These two paths, however, are not disjoint and therefore cannot be combined to form an o-path from A to B.[115] Nor can they be combined to form any other path for AoB, since the only path for this proposition in the structure is the e-path from A to B. Thus, the two paths for AeC and BaC are not proper parts of any path for AoB. As a result, the premisses AeC and BaC are not prior in nature to the conclusion AoB. On the contrary, since the e-path from A to B is a proper part of the e-path from A to C, the conclusion AoB is prior in nature to the premiss AeC.

As we have seen, Aristotle takes AeC to be posterior in nature to AoB (1. 26, 87a25–8). Conversely, he takes AoB to be prior in nature to AeC. Of course, this does not mean that AoB can serve as a premiss in a demonstration of AeC, for no particular proposition

[115] The e-path from A to C and the a-path from B to C fail to constitute an o-path from A to B because condition (iii) in Definition 5 is violated (since the two paths overlap in the pair ⟨B, C⟩).

can serve as a premiss in deriving a universal conclusion.[116] Instead, Aristotle seems to regard AoB as prior in nature to AeC in virtue of the fact that the former asserts part of the content that is needed to demonstrate the latter. In the above demonstrative structure, this corresponds to the fact that every path for AoB is a path for AeB and part of a path for AeC. In other words, every path underwriting AoB is in fact a path underwriting AeB and is partly constitutive of a path underwriting AeC. This may help to explain why Aristotle does not use any quantifying expressions such as 'all', 'no', or 'some' in describing the demonstration by *reductio* in *Posterior Analytics* 1. 26 (87ᵃ6–12). For the relevant priority relation between AoB and AeC does not depend on the quantity of these two propositions, but only on the nature of the paths that underwrite them in the underlying demonstrative structure.

There are, then, instances of Felapton in which both premisses are satisfied in a demonstrative structure, but the conclusion is prior in nature to one of the premisses. In fact, there are such instances not only of Felapton but of most particular moods in the three figures:

> Theorem 2: There are deductions of the form Darii, Ferio, Festino, Darapti, Disamis, Datisi, Felapton, Ferison, and Bocardo such that:
>
> i. both premisses are satisfied in a demonstrative structure $\langle T, \rightarrow, \sim\rangle$, and
> ii. the conclusion is prior in nature to one of the premisses in $\langle T, \rightarrow, \sim\rangle$.

For the negative moods listed in this theorem, the claim can be established by means of the same demonstrative structure just described for the case of Felapton. For the affirmative moods listed in the theorem, it can be established by means of a similar demonstrative structure in which A is connected to B by an a-path instead of an e-path.[117] Thus, the demonstrative structure described by Aristotle in *Posterior Analytics* 1. 26 is not limited to the case of

[116] See *Pr. An.* 1. 24, 41ᵇ22–7.

[117] Specifically, consider a demonstrative structure in which A→B and B→C. The propositions AaC, BaC, AiC, CiB, and BiC are satisfied in this structure, but AiB is prior in nature to both AaC and AiC (since the only path for AiB, the a-path from A to B, is a proper part of the a-path from A to C).

Felapton (87^a3–12). Instead, Aristotle's discussion of this structure can be viewed as indicating a more general argument to establish Theorem 2 for negative particular moods and, with a simple modification, also for affirmative ones.

Theorem 2 covers all particular moods in Aristotle's three figures except the second-figure mood Baroco. The latter mood is not included because the theorem does not hold for it: whenever the premisses of an instance of Baroco are satisfied in a demonstrative structure, they are prior in nature to the conclusion. For all other particular moods in the three figures, there are instances in which the premisses are satisfied in a demonstrative structure while the conclusion is prior in nature to one of them. At the same time, there are also instances of these moods in which both premisses are prior in nature to the conclusion.[118] Thus, among demonstrations by *reductio*, there are some in which the premisses are prior in nature to the conclusion, as well as some in which the conclusion is prior in nature to one of the premisses.

These results suffice to justify Aristotle's thesis in *Posterior Analytics* 1. 26 that direct demonstration is better than demonstration by *reductio* on the grounds that the former proceeds from premisses that are prior in nature to the conclusion whereas the latter allows for the conclusion to be prior in nature to one of the premisses (87^a25–30). On the present account, this thesis should not be taken to mean that all direct demonstrations are better in this respect than all demonstrations by *reductio*. Rather, Aristotle's claim is that the method of direct demonstration is superior to the method of demonstration by *reductio* because the former guarantees that all premisses are prior in nature to the conclusion, whereas the latter does not come with such a guarantee but allows for cases in which the conclusion is prior in nature to one of the premisses. This differs from the interpretation of 1. 26 given by Gisela Striker and others, who take Aristotle in this chapter to state that all demonstrations by *reductio* are inferior to all direct ones.[119] If I am correct, such a universal thesis is neither intended by Aristotle nor supported by his argument in the chapter. Nonetheless, his argument

[118] Consider, for example, an instance of Darii, inferring AiC from AaB and BiC, when both AaB and BaC are satisfied in the underlying demonstrative structure.

[119] See n. 60 above.

succeeds in establishing a clear difference between the two methods of demonstration.

The function of a demonstration, for Aristotle, is to produce scientific knowledge:

. . . *καθ' ἣν μᾶλλον ἐπιστάμεθα ἀπόδειξιν βελτίων ἀπόδειξις· αὕτη γὰρ ἀρετὴ ἀποδείξεως* . . . (*Post. An.* 1. 24, 85^{a}21–2)

. . . the demonstration by which we have more scientific knowledge is the better demonstration; for this is the excellence of a demonstration . . .

In order to have scientific knowledge of a demonstrable theorem, one needs to derive it from premisses that are prior to it in nature. Hence, the method of direct demonstration is better than that of demonstration by *reductio* since it is more reliable at achieving 'the excellence of a demonstration'.[120] As Myles Burnyeat has pointed out, 'given Aristotle's belief that there is real priority and posteriority in nature', a demonstration should be 'not just a preferred ordering of humanly constructed knowledge, but a mapping of the structure of the real'.[121] Direct demonstration is better suited to this task than that by *reductio* because the former, but not the latter, always succeeds in mapping the relation of priority in nature encoded in structures of acyclic a-paths (*συστοιχίαι*).

Aristotle does not indicate whether those demonstrations by *reductio* in which the premisses are in fact prior in nature to the conclusion can have the status of genuine demonstrations producing scientific knowledge. His argument in *Posterior Analytics* 1. 26 does not preclude that some of them may have this status. On the present account, the only way to derive a scientific i- or o-proposition is by means of *reductio*. Thus, it would seem natural to accept that, for each of these propositions, there is a derivation by *reductio* which has the status of a genuine demonstration producing scientific knowledge of it. Aristotle may have independent reasons for regarding

[120] The former method is, so to speak, more demonstrative than the latter. Accordingly, when Aristotle distinguishes direct demonstration from demonstration by *reductio* in 1. 26, he refers to the former as 'demonstrative' (*ἀποδεικτική*, 87^{a}17; see Philop. *In An. Post.* 297. 9–12 Wallies). Similarly, he refers to it in 1. 24 as 'that which is said to demonstrate' (*ἡ ἀποδεικνύναι λεγομένη*, 85^{a}15–16; see Philop. *In An. Post.* 273. 26–9). The latter phrase suggests that direct demonstration was taken to be superior to demonstration by *reductio* not only by Aristotle but also by other thinkers at his time.

[121] Burnyeat, 'Understanding Knowledge', 126.

such demonstrations by *reductio* as inferior to direct ones—but if so, he does not explain them in *Posterior Analytics* 1. 26.

By contrast, many later theorists have endorsed the universal thesis that all proofs by *reductio* fail to be explanatory. Such a universal thesis is clearly evinced, for example, in the passages from the *Port-Royal Logic* and the *Critique of Pure Reason* quoted above. It is also maintained by Bernard Bolzano, who argues in his *Theory of Science* that no proof by *reductio* can exhibit the 'objective ground' of the *demonstrandum*. Following Aristotle's lead in *Prior Analytics* 2. 14, Bolzano holds that every proof by *reductio* can be transformed into a direct proof.[122] In his view, proofs by *reductio* cannot exhibit the objective ground of the *demonstrandum* because they involve superfluous steps and redundant assumptions that are absent from the corresponding direct proofs.[123] Whatever the merits of Bolzano's account, it differs significantly from Aristotle's argument in *Posterior Analytics* 1. 26. For, as we have seen, this argument relies on a deductive system in which not every proof by *reductio* can be transformed into a direct one, and it does not turn on the fact that proofs by *reductio* involve an assumption for *reductio* and other steps that are absent from direct proofs. Unlike Bolzano's account, Aristotle's argument does not imply that all demonstrations by *reductio* are inferior to direct ones.

In his *Theory of Science*, Bolzano takes a keen interest in the relation of priority in nature introduced by Aristotle in *Posterior Analytics* 1. 2–3. Bolzano argues that this relation can be viewed as a relation of 'grounding' whereby one truth is grounded in one or more other truths.[124] In accordance with this suggestion, the results concerning priority in nature stated in Theorems 1 and 2 have close analogues in modern theories of grounding. These theories explore, among other things, how the relation of grounding interacts with the rules of inference governing logical connectives in deductive systems. Consider, for example, a deductive system for the language of propositional logic using the connectives of negation ($\neg$)

[122] Bolzano, *Wissenschaftslehre*, iv. 271–8 (§530. 2–4 and n. 2); see Mancosu, *Mathematics*, 110–15.

[123] Bolzano, *Wissenschaftslehre*, iv. 270–1 (§530. 1). See S. Centrone, 'Das Problem der apagogischen Beweise in Bolzanos *Beyträgen* und seiner *Wissenschaftslehre*', *History and Philosophy of Logic*, 33 (2012), 127–57 at 142–3; S. Roski, *Bolzano's Conception of Grounding* (Frankfurt a.M., 2017), 159–60.

[124] Bolzano, *Wissenschaftslehre*, ii. 364 (§209 n. 1); see also ii. 341 (§198 n. 1).

and conjunction ($\wedge$). The system contains four direct rules of inference, where ϕ and ψ are any propositions:

ϕ, ψ, therefore $\phi\wedge\psi$
ϕ, therefore $\neg\neg\phi$
$\neg\phi$, therefore $\neg(\phi\wedge\psi)$
$\neg\psi$, therefore $\neg(\phi\wedge\psi)$

In addition, the system contains the following rule of *reductio ad impossibile*, where ϕ and ψ are any propositions and Γ any set of propositions:

$$\frac{\Gamma,\ \neg\phi,\ \text{therefore } \psi\wedge\neg\psi}{\Gamma,\ \text{therefore } \phi}$$

Taken together, these rules suffice to derive all the laws of classical propositional logic. The four direct rules are special in that, when applied to true propositions, the conclusion not only is a logical consequence of the premisses but also is grounded in these premisses. For example, if ϕ and ψ are true, then $\phi\wedge\psi$ is grounded in these two propositions (or the fact that $\phi\wedge\psi$ obtains in virtue of the fact that ϕ and the fact that ψ). More generally, the relation of grounding is usually taken to obey the following laws corresponding to the four direct rules of inference:[125]

If ϕ and ψ, then ϕ, ψ ground $\phi\wedge\psi$
If ϕ, then ϕ grounds $\neg\neg\phi$
If $\neg\phi$, then $\neg\phi$ grounds $\neg(\phi\wedge\psi)$
If $\neg\psi$, then $\neg\psi$ grounds $\neg(\phi\wedge\psi)$

By contrast, there is no such agreement between the laws of grounding and the derived rules of inference that rely on *reductio ad impossibile* in the deductive system. Consider, for example, the following derived rules which are licensed by the above rule of *reductio*:

[125] K. Fine, 'Guide to Ground', in F. Correia and B. Schnieder (eds.), *Metaphysical Grounding: Understanding the Structure of Reality* (Cambridge, 2012), 37–80 at 58 and 62–3; F. Correia, 'Logical Grounds' ['Grounds'], *Review of Symbolic Logic*, 7 (2014), 31–59 at 33–6; id., 'An Impure Logic of Representational Grounding', *Journal of Philosophical Logic*, 46 (2017), 507–38 at 530; similarly, B. Schnieder, 'A Logic for "Because"', *Review of Symbolic Logic*, 4 (2011), 445–65 at 449; J. Korbmacher, 'Axiomatic Theories of Partial Ground I: The Base Theory', *Journal of Philosophical Logic*, 47 (2018), 161–91 at 170.

$\phi \wedge \psi$, therefore ϕ
$\phi \wedge \psi$, therefore ψ
$\neg\neg\phi$, therefore ϕ

When these rules are applied to a true proposition, this proposition does not ground the conclusion. For example, a true conjunction $\phi \wedge \psi$ does not ground either conjunct (the fact that ϕ does not obtain in virtue of the fact that $\phi \wedge \psi$). Thus, while sound deductions instantiating the four direct rules proceed from what grounds to what is grounded, sound deductions using the rule of *reductio* do not.

The four direct rules of inference are analogous to the four universal syllogistic moods in Aristotle's system. When the former are applied to true propositions, the premisses ground the conclusion; when the latter are applied to scientific propositions, the premisses are prior in nature to the conclusion. In both cases, direct deductions proceed in the direction of increasing complexity.[126] In the first case, the conclusion is syntactically more complex than the premisses.[127] In the second case, the conclusion is path-theoretically more complex than the premisses, in the sense that every path for a premiss is a proper part of a path for the conclusion.

In both systems, the direct rules of inference are especially useful in contexts in which preference is given to deductions that proceed from what grounds to what is grounded, or from what is prior in nature to what is posterior in nature. However, these rules and moods do not suffice to generate all deductive consequences of a given set of premisses. To this end, one needs to employ the rule of *reductio ad impossibile* in the two systems. Once this rule is admitted, deductions from true propositions no longer follow the order of grounding, and deductions from scientific propositions no longer follow the order of priority in nature. Instead, the rule of *reductio* licenses deductions in which this order is inverted, inferring the ground from the grounded, or what is prior in nature from what is posterior in nature.

[126] This is in accordance with Fine's suggestion that relations of grounding proceed in the direction of increasing complexity for some suitable complexity measure on propositions; K. Fine, 'Some Remarks on Bolzano on Ground', in S. Roski and B. Schnieder (eds.), *Priority Among Truths: Bernard Bolzano's Philosophy of Grounding* (Oxford, *forthcoming*), §5.

[127] See Correia, 'Grounds', 35–7.

Thus, Bolzano's suggestion that the relation of priority in nature employed by Aristotle in the *Posterior Analytics* can be viewed as a relation of grounding is borne out by the parallels between Aristotle's argument in 1. 26 and modern developments in the logic of ground. More specifically, Aristotle's relation of priority in nature corresponds to what is known as strict partial ground. Kit Fine characterizes the latter relation in terms of the idea of a fact verifying a proposition. Facts can be fused to form more complex facts of which they are parts. On Fine's account, if a true proposition is a strict partial ground of another true proposition, every fact verifying the former is part of a fact verifying the latter.[128] Similarly, the present account of priority in nature is based on the idea that paths underwrite scientific propositions. Paths can be combined to form more complex paths of which they are proper parts. If a scientific proposition is prior in nature to another scientific proposition, every path for the former is a proper part of a path for the latter.[129]

7. Parts and wholes

Aristotle's argument in *Posterior Analytics* 1. 26 rests on the claim that the major premiss of the direct negative demonstration, AeB, is prior in nature to the conclusion AeC (87ª17–18). In support of this claim, Aristotle states that the latter proposition is a conclusion while the former is among the propositions 'from which the conclusion derives' (*ἐξ ὧν τὸ συμπέρασμα*):

> *πρότερα γάρ ἐστι τοῦ συμπεράσματος ἐξ ὧν τὸ συμπέρασμα. ἔστι δὲ τὸ μὲν Α τῷ Γ μὴ ὑπάρχειν συμπέρασμα, τὸ δὲ Α τῷ Β ἐξ οὗ τὸ συμπέρασμα.* (*Post. An.* 1. 26, 87ª18–20)

[128] Fine, 'Guide to Ground', 71–3; id., 'The Pure Logic of Ground' ['Pure Logic'], *Review of Symbolic Logic*, 5 (2012), 1–25 at 7–9.

[129] Thus, the account of priority of nature given in Definition 8 can be viewed as a special case of Fine's factual semantics for ground, when a 'fact' is taken to be any set of atomic a- and e-paths, and the operation of 'factual fusion' is taken to be set-theoretic union (see Fine, 'Pure Logic', 7–9). However, while Fine's account applies to any true propositions and the facts that make them true, Definition 8 applies only to scientific truths and the paths that underwrite their status as scientific truths in acyclic demonstrative structures.

For the things from which a conclusion derives are prior to the conclusion; and that A does not belong to C is a conclusion, whereas that A does not belong to B is that from which the conclusion derives.

According to this passage, the propositions 'from which a conclusion derives' are prior in nature to the conclusion. Given that AeC is a conclusion that derives from AeB in the direct negative demonstration, the latter proposition is prior in nature to the former. But what about the demonstration by *reductio*, in which the conclusion AoB is derived from the premisses AeC and BaC? If these premisses are propositions 'from which the conclusion derives', it would seem to follow that AeC is prior in nature to AoB, thereby undermining Aristotle's argument in 1. 26. Aristotle responds to this objection as follows:

οὐ γὰρ εἰ συμβαίνει ἀναιρεῖσθαί τι, τοῦτο συμπέρασμά ἐστιν, ἐκεῖνα δὲ ἐξ ὧν. ἀλλὰ τὸ μὲν ἐξ οὗ συλλογισμός ἐστιν ὃ ἂν οὕτως ἔχῃ ὥστε ἢ ὅλον πρὸς μέρος ἢ μέρος πρὸς ὅλον ἔχειν, αἱ δὲ τὸ ΑΓ καὶ ΑΒ προτάσεις οὐκ ἔχουσιν οὕτω πρὸς ἀλλήλας. (*Post. An.* 1. 26, 87ᵃ20–5)

For it is not the case that if something happens to be rejected, this is a conclusion and the other things are that from which the conclusion derives. Rather, that from which a deduction proceeds is what is related to one another either as whole to part or as part to whole.[130] But the propositions AC and AB are not related to one another in this way.

In this passage, demonstrations by *reductio* are described as arguments in which something is rejected. The thing rejected in them is the assumption for *reductio*.[131] Aristotle emphasizes that the

[130] Crivelli ('Hypothesis', 168) takes *ἔχειν* at 87ᵃ23 to be transitive, translating the clause as follows: 'what is in such a condition as to have a whole in relation to a part or a part in relation to a whole'. Elsewhere Aristotle states that the premisses of a deduction are 'so related as to be one a whole and the other a part' (*ἔχει οὕτως ὥστ' εἶναι τὸ μὲν ὡς ὅλον τὸ δ' ὡς μέρος*, *Pr. An.* 1. 25, 42ᵃ15–16; similarly, 42ᵃ10; 1. 41, 49ᵇ37–8); see H. Maier, *Die Syllogistik des Aristoteles*, ii/2: Die Entstehung der Aristotelischen Logik [*Entstehung*] (Tübingen, 1900), 152 n. 1. Since there is no other passage in which Aristotle states that the premisses of a deduction have a whole in relation to a part, it seems preferable to take *ἔχειν* at 87ᵃ23 to be intransitive; Philop. *In An. Post.* 298. 7–8; Zabarella, *Opera logica*, 978 E–980 A; Kirchmann, *Erläuterungen*, 116; Mure, *Posteriora*, ad loc.; Tredennick, *Posterior Analytics*, 153; Mignucci, *L'argomentazione dimostrativa*, 564; Seidl, *Zweite Analytiken*, 127 and 266; Barnes, *Posterior Analytics 2nd edn.*, 41; Detel, *Analytica posteriora*, i. 54; Pellegrin, *Seconds Analytiques*, 211; Tricot, *Seconds Analytiques*, 146.

[131] 'A demonstration reducing to the impossible differs from an ostensive demonstration in that it posits what it wishes to reject by reducing it to an agreed falsehood' (*διαφέρει δ' ἡ εἰς τὸ ἀδύνατον ἀπόδειξις τῆς δεικτικῆς τῷ τιθέναι ὃ βούλεται ἀναιρεῖν,*

proposition expressing this rejection, the contradictory opposite of the assumption for *reductio*, is not always a 'conclusion' (*συμπέρασμα*).[132] Thus, he denies that in every demonstration by *reductio* the proposition inferred in the final step is a 'conclusion'. Accordingly, he denies that the premisses of every such demonstration are 'that from which' this proposition derives. With respect to Aristotle's demonstration by *reductio*, this means that AoB is not a conclusion, and the premisses AeC and BaC are not that from which AoB derives.

In denying that AoB is a conclusion, Aristotle departs from his terminology in the *Prior Analytics*, where he refers to any proposition inferred in the final step of a deduction by *reductio* as a 'conclusion'.[133] In the passage just quoted, he adopts a stricter use of the term, in which not every proposition deduced by *reductio* counts as a conclusion. Grosseteste argues that Aristotle uses 'conclusion' here specifically for those propositions that are posterior in nature to the premisses from which they are deduced.[134] If this is correct, the proposition inferred in any direct demonstration is a conclusion and the premisses are 'that from which the conclusion derives'. Aristotle suggests that, in such a demonstration, the conclusion 'derives from' the premisses because the premisses are related to one another 'as whole to part or as part to whole'. Thus, while the premisses of the direct negative demonstration, AeB and BaC, are related to one another 'as whole to part or as part to whole', this is not the case for the premisses of the demonstration by *reductio*, AeC and BaC.

In the last sentence of the passage, Aristotle states that 'the propositions AC and AB are not related to one another in this way'. The first of these propositions is the negative premiss of the demonstration by *reductio*, AeC, and the second is the proposition inferred in this demonstration, AoB. Since AeC and AoB do not form a pair of premisses for any deduction, it makes little sense to point out that they are not premisses related to one another 'as whole to part or as part to whole'. In view of this difficulty, Ross

ἀπάγουσα εἰς ὁμολογούμενον ψεῦδος, *Pr. An.* 2. 14, 62ᵇ29–31). See also the use of *ἀναιρεῖν* at *SE* 5, 167ᵇ22–4.

[132] See F. Biese, *Die Philosophie des Aristoteles,* i: Logik und Metaphysik (Berlin, 1835), 270–1; Waitz, *Organon*, ii. 370; Ross, *Analytics*, 594; Crivelli, 'Hypothesis', 168–9.

[133] *Pr. An.* 2. 11, 61ᵃ20, 61ᵇ17; 2. 14 62ᵇ34–5, 62ᵇ38, 63ᵇ16.

[134] Grosseteste, *Commentarius*, 254.

emends the text of the sentence, printing 'AC and BC' instead of 'AC and AB' at 87^a24.[135] The sentence then asserts that the premisses of the demonstration by *reductio*, AeC and BaC, do not stand in the requisite part-whole relation. While this is a natural reading, there is no textual evidence for Ross's emendation.[136] Barnes has suggested reading 'BC and AB' instead of 'AC and AB', but there is no textual support for this emendation either.[137] Instead, all manuscripts have 'AC and AB'.

If we wish to follow the manuscripts, the sentence can be taken to assert that AeC and AoB are not related in such a way that the latter can be derived from AeC and another premiss which stands to AeC 'as whole to part or as part to whole'. On this reading, Aristotle's focus shifts from the part-whole relations that obtain between the premisses of a demonstration to the relation that obtains between one of the premisses and the conclusion. This shift may be facilitated by the fact that Aristotle recognizes part-whole relations not only between the premisses of demonstrations but also between their premisses and the conclusion. For example, in a demonstration by Barbara, Aristotle takes the conclusion to be a part of the major premiss, noting that 'the one is a part and the other a whole' (*μέρος γάρ, τὸ δ' ὅλον*, 2. 3, 91^a4–5).[138] Maier has argued that, for Aristotle, this is just another way of expressing the same thought he has in mind when he says that the minor premiss is related to the major premiss 'as part to whole'.[139] Thus, according

[135] Ross, *Analytics*, 595; followed by Tredennick, *Posterior Analytics*, 152; Mignucci, *L'argomentazione dimostrativa*, 564–5; Barnes, *Posterior Analytics 2nd edn.*, 41 and 189; Detel, *Analytica posteriora*, i. 54 and ii. 458; Pellegrin, *Seconds Analytiques*, 210 and 388–9 n. 7; Crivelli, 'Hypothesis', 167.

[136] In manuscript c, the phrase *καὶ* AB at 87^a24 is accompanied by the marginal note '*γρ ΒΓ*' (see Waitz, *Organon*, ii. 43). Here *γρ* may stand either for *γράφεται*, indicating a variant reading, or for *γράφε*, indicating an emendation; see N. G. Wilson, 'More about *γράφεται* Variants', *Acta Antiqua Academiae Scientiarum Hungaricae*, 48 (2008), 79–81. Since *ΒΓ* is not attested in any other manuscripts, it seems more likely that the marginal note in c is meant to indicate an emendation. Ross (Analytics, ad 87^a24) and Williams (*Studies in the Manuscript Tradition of Aristotle's Analytica* (Königstein, 1984), 64) claim that *ΒΓ* is found in a secondary hand in manuscript C. This claim, however, is not correct (the error may be due to a misprint in Ross's apparatus, writing 'C' in place of 'c'). I am grateful to Michel Crubellier and Pieter Sjoerd Hasper for discussion of the manuscript situation at 87^a24.

[137] Barnes, *Posterior Analytics 1st edn.*, 45 and 181; followed by Striker, 'Review of Barnes', 317, but retracted in id. *Posterior Analytics 2nd edn.*, 41 and 189.

[138] See 91^a2–5 and the explanations by Maier, *Entstehung*, 152–3 n. 3; Barnes, *Posterior Analytics 2nd edn.*, 208.

[139] Maier, *Entstehung*, 151–5.

to Maier, a premiss of a demonstration is part of another premiss just in case the conclusion is part of the latter premiss. Hence, asserting that BaC is part of AeB is equivalent to asserting that AeC is part of AeB. Likewise, denying that BaC is part of AeC is equivalent to denying that AoB is part of AeC. This may help to explain why Aristotle transitions from the one denial to the other in the passage just quoted. On this reading, the point of the last sentence is equivalent to the one expressed by Ross's emendation. Thus, whether or not we accept the emendation, Aristotle denies that the premisses of the demonstration by *reductio*, AeC and BaC, are related to one another 'as whole to part or as part to whole'.

In the *Analytics*, Aristotle repeatedly claims that the premisses of a deduction are related to one another 'as whole to part'.[140] He does not explain what it is for two premisses to be related in this way, nor is it clear whether he intends the claim to hold for all deductions in the three figures.[141] In *Posterior Analytics* 1. 26, at any rate, the claim seems to apply only to direct demonstrations but not to those by *reductio*. In all direct demonstrations, the premisses are related to one another 'as whole to part or as part to whole', whereas this is not the case in all demonstrations by *reductio*. Given the deductive framework employed by Aristotle in 1. 26, this means that the claim applies to demonstrations instantiating the four universal moods, but not to other demonstrations in the three figures.[142]

To see how the claim applies to demonstrations in the four universal moods, consider the case of Barbara: AaB, BaC, therefore AaC. The major premiss asserts that, as Aristotle puts it, 'A belongs to the whole of B' (*τὸ Α ὅλῳ τῷ Β ὑπάρχει*).[143] The minor premiss

[140] *Pr. An.* 1. 25, 42ᵃ9–18; 1. 41, 49ᵇ37–50ᵃ1; *Post. An.* 1. 10, 77ᵃ3–4; 2. 6, 92ᵃ12–13. See Maier, *Entstehung*, 152–5; Barnes, *Posterior Analytics 2nd edn.*, 189; Striker, *Prior Analytics*, 183–4.

[141] Some commentators take the claim to apply to all deductions in the three figures (Mignucci, *L'argomentazione dimostrativa*, 565; Barnes, *Posterior Analytics 2nd edn.*, 189; Crivelli, 'Hypothesis', 169–70). However, it is not easy to see how the claim applies to deductions in the third figure, e.g. to those of the form Darapti. Thus, Alexander takes Aristotle's claim at *Prior Analytics* 1. 25, 42ᵃ9–17 to apply only to deductions in the first figure (Alex. Aphr. *In An. Pr.* 277. 5–23 Wallies). Similarly, Ross (*Analytics*, 379) holds that the claim 'is most clearly true' of deductions in the first figure.

[142] Kirchmann (*Erläuterungen*, 116) takes Aristotle's claim at 87ᵃ22–3 to be restricted to deductions in the first figure. On the present account, by contrast, the claim applies not only to first-figure deductions in Barbara and Celarent, but also to second-figure deductions in Cesare and Camestres.

[143] *Pr. An.* 2. 2, 53ᵇ30, 54ᵇ4–5, 54ᵇ25–6, 54ᵇ28–9, 55ᵃ6, 55ᵃ37; 2. 3, 55ᵇ27–8, 55ᵇ35–6, 56ᵃ26, 56ᵃ29–30, 56ᵃ33–4, 56ᵇ1; 2. 4, 56ᵇ38, 57ᵃ13–14, 57ᵃ19, 57ᵃ21; 2. 21, 67ᵃ33–4;

asserts that 'C is in B as in a whole' (*τὸ Γ ἐν ὅλῳ ἐστὶ τῷ Β*).[144] Thus, the major premiss makes a universal claim about B as a whole while the minor premiss identifies a part of B that is in B as in a whole. In this sense, the major premiss can be viewed as a 'whole' and the minor premiss as a 'part'.[145] Similarly, in the case of Celarent, the major premiss AeB asserts that 'A does not belong to the whole of B' (*τὸ Α ὅλῳ τῷ Β οὐχ ὑπάρχει*).[146] Thus, the major premiss makes a universal negative claim about B as a whole. Likewise, in the case of Cesare and Camestres, the e-premiss asserts that the major or minor term does not belong to the middle term as a whole.

In all four universal moods, then, one premiss makes a universal affirmative or negative claim about the middle term B as a whole, while the other premiss states that the minor term is a part of B which is in B as in a whole. This is not the case for any of the other moods in the three figures. It is therefore natural for Aristotle to characterize the four universal moods as those in which the premisses are related to one another 'as whole to part or as part to whole'. In Barbara, Celarent, and Cesare, the major premiss is related to the minor premiss as whole to part, and in Camestres they are related the other way around. Whenever the premisses of a deduction are scientific propositions that stand in this part-whole relation, they are prior in nature to the conclusion.[147] Grosseteste explains this point in his commentary on *Posterior Analytics* 1. 26 for the cases of Barbara and Celarent as follows:

Cum igitur in demonstrationibus sit semper predicatio directa naturaliter ordinata et minor propositio sit pars, maior vero totum, palam quoniam conclusio erit natura posterior et ea ex quibus est sillogismus erunt priora

2. 22, 68^{a}16–17, 68^{a}22; 2. 23, 68^{b}21; *Post. An.* 1. 16, 80^{a}40–^{b}1, 80^{b}4, 80^{b}8–9; 1. 17, 80^{b}37–8.

[144] *Pr. An.* 1. 1, 24^{a}13–14, 24^{b}26–7; 1. 4, 25^{b}32–4; 2. 1, 53^{a}21–4; *Post. An.* 1. 15, 79^{a}36–^{b}18; 1. 17, 80^{b}27–8.

[145] Maier, *Entstehung*, 152–5.

[146] *Pr. An.* 2. 3, 55^{b}33; 2. 4, 56^{b}38, 57^{a}3; *Post. An.* 1. 16, 80^{b}1, 80^{b}8.

[147] Lotze attributes to Aristotle the view that the best way to prove a conclusion from its explanatory grounds is by means of a first-figure deduction in Barbara, since 'it is only here that we find the subordination of a given idea under a general truth which enables us to understand not only that [the conclusion] holds, but also why it holds' ('nur hier findet die Unterordnung eines gegebenen Inhalts unter eine allgemeine Wahrheit statt, aus welcher nicht blos begriffen wird, daß [das zu Beweisende] gilt, sondern auch warum es gilt', Lotze, *Logik*, 265). On the present account, explanatory proofs of this sort are found not only in Barbara but in all four universal moods. Whenever these moods are applied to scientific propositions, the premisses stand to the conclusion in the explanatory relation described by Lotze.

natura. . . . Cum autem in hac: omne C est B, sit predicatio directa secundum ordinem naturalem et ita C sit pars B, et hec propositio: omne C est B, sit sicut pars ad hanc: nullum B est A, manifestum quoniam hec: nullum C est A, est posterior natura quam hec: nullum B est A. (Grosseteste, *Commentarius*, 253–4)

Since in demonstrations the predication is always direct and in natural order, and the minor premiss is a part while the major premiss is a whole, it is plain that the conclusion will be posterior in nature and that from which the deduction proceeds will be prior in nature. . . . When in the proposition *All C is B* the predication is direct and in accordance with natural order and, in this way, C is a part of B and the proposition *All C is B* is related as a part to the proposition *No B is A*, it is evident that the proposition *No C is A* is posterior in nature to the proposition *No B is A*.

Grosseteste's argument does not apply to deductions in which the premisses do not stand in the part-whole relation exhibited by the four universal moods. When such deductions proceed from scientific propositions, the premisses may fail to be prior in nature to conclusion.

Throughout the first book of the *Posterior Analytics*, Aristotle appeals, more or less directly, to the deductive framework of the assertoric syllogistic. In doing so, he usually focuses on the four universal moods Barbara, Celarent, Cesare, and Camestres, but often does not mention any of the other moods in the three figures.[148] Robin Smith takes this to be an indication that the *Posterior Analytics* was largely written before Aristotle developed the full syllogistic theory presented in the *Prior Analytics*. In Smith's view, 'the *Posterior Analytics* as we have it does not presuppose the *Prior Analytics* but something decidedly simpler'.[149] By contrast, the preceding considerations suggest a different explanation of Aristotle's focus in the *Posterior Analytics* on the four universal moods. These moods constitute the core of the deductive framework employed in *Posterior Analytics* 1. 26, and they are the only ones that license direct demonstrations in this framework. Whenever they are applied to scientific propositions, the premisses are prior in nature to the conclusion. All other moods in the three figures rely on the rule of *reductio ad impossibile*, and they do not guarantee that scientific premisses are prior in nature to the conclusion. As we have seen, Aristotle clearly recognizes such moods in the *Posterior Analytics*.[150]

148 See Smith, 'Relationship', 328–35.

149 Smith, 'Relationship', 327; see also id., 'Syllogism', 114–35.

150 See n. 43.

But they are less suitable for the purposes of scientific demonstration than the four universal ones, and therefore play a less prominent role in this treatise. Thus, Aristotle is focusing on the four universal moods in the *Posterior Analytics* not because he was not sufficiently aware of the other moods when he wrote the treatise, but because these are the ones that are most important and useful for the purposes of scientific demonstration.

Smith argues that *Posterior Analytics* 1. 26 was written before *Prior Analytics* 2. 11–14 because otherwise one 'must suppose that Aristotle, after having acquired the sort of understanding of *reductio ad impossibile* reflected in *Pr. An.* 2. 11–14, . . . somehow produced the very unsatisfactory treatment of this subject in *Post. An.* 1. 26'.[151] In my view, this argument is not convincing. As we have seen, Aristotle's discussion in *Posterior Analytics* 1. 26 is not incompatible with, or inferior to, his treatment of *reductio* in the *Prior Analytics*.[152] Instead, his aims are different in the two parts. In the *Prior Analytics*, Aristotle studies the nature of deductions by *reductio* in general and their relationship to direct deductions in the assertoric syllogistic. This investigation deals with features common to all deductions by *reductio*, and hence does not appeal to any considerations concerning priority in nature, since these pertain only to scientific demonstrations and not to the other kinds of deduction countenanced by Aristotle. In *Posterior Analytics* 1. 26, on the other hand, he deals specifically with demonstrations by *reductio* and the relations of priority in nature that obtain between their constituent propositions. The latter discussion relies on special assumptions of his theory of demonstration that are absent

[151] Smith, 'Syllogism', 134.

[152] Smith ('Syllogism', 119) claims that 'none of [*Posterior Analytics* 1. 26] makes any sense unless Aristotle somehow supposes that the conclusion of an indirect proof must be negative'. In the *Prior Analytics*, on the other hand, Aristotle is clear that deductions by *reductio* can establish not only negative but also affirmative conclusions (*Pr. An.* 2. 14, 62^b37–8; see also 1. 6, 28^a22–3, 28^b14; 1. 7, 29^a32–$^b11$). It is true that Aristotle's focus in *Posterior Analytics* 1. 26 is on demonstrations by *reductio* that establish a negative conclusion, but this does not mean that he excludes demonstrations by *reductio* which establish an affirmative conclusion. Given his claim in 1. 25 that direct affirmative demonstrations are better than direct negative ones, it is obvious that direct affirmative demonstrations are also better than demonstrations by *reductio* which establish an affirmative conclusion. For the latter not only involve an application of the rule of *reductio*, but also employ a direct negative deduction (unless they involve nested applications of *reductio*). Thus, it makes sense for Aristotle in chapter 1. 26 to focus on the more interesting question of whether direct negative demonstrations are better than those demonstrations by *reductio* that establish a negative conclusion.

from the *Prior Analytics*. In particular, it relies on the assumption, introduced in *Posterior Analytics* 1. 19–22, that demonstrations are based on structures of acyclic a-paths. It is therefore not surprising that the discussion in *Posterior Analytics* 1. 26 takes a different form from the treatment of *reductio* in the *Prior Analytics*. Of course, Aristotle's argument in 1. 26, like in other parts of the *Posterior Analytics*, is highly compressed and stands in need of extensive interpretation.[153] Nonetheless, the argument is coherent and well suited to the aims of the chapter. It succeeds in establishing that all direct demonstrations, but not all demonstrations by *reductio*, proceed from premisses that are prior in nature to the conclusion.

Friedrich Solmsen has argued that the account of demonstration presented in the first book of the *Posterior Analytics* is based on what he calls *Eidosketten*, chains of universal terms arranged in order of generality.[154] In the same vein, Smith maintains that Aristotle's 'theory of demonstration is the theory of the structure of a system of terms arranged in ordered "chains" (συστοιχίαι)'.[155] In accordance with this view, I have argued that any Aristotelian science determines a structure of a-paths in which terms are connected by immediate universal affirmations. Given that a-paths are acyclic, they give rise to a well-defined relation of priority in nature among scientific propositions. It is this relation of priority in nature, I submit, that lies at the heart of Aristotle's argument in *Posterior Analytics* 1. 26.[156]

New York University

[153] Kirchmann, *Erläuterungen*, 116–17.

[154] Solmsen, *Entwicklung*, 53–7 and 81–122; id., 'The Discovery of the Syllogism', *Philosophical Review*, 50 (1941), 410–21 at 410 and 420.

[155] Smith, 'Syllogism', 122.

[156] Earlier versions of this paper were presented at Fudan University, Humboldt University Berlin, Peking University, Princeton University, Shandong University Jinan, Tufts University, UCLA, UC San Diego, University of Bologna, University of Campinas, University of Utah, University of Vienna, and Williams College. I would like to thank all those in attendance for their stimulating questions and helpful comments, especially Francesco Ademollo, Lucas Angioni, Rob Bolton, David Charles, Adam Crager, Michel Crubellier, Paolo Fait, Mary-Louise Gill, Katerina Ierodiakonou, Monte Johnson, George Karamanolis, Hendrik Lorenz, Henry Mendell, Stephen Menn, Calvin Normore, Christof Rapp, Samuel Rickless, Anne Siebels Peterson, Gisela Striker, Wei Wang, Eric Watkins, and Breno Zuppolini. In preparing the final version of the paper, I have benefited from generous and detailed written comments by Victor Caston, Paolo Crivelli, Riccardo Strobino, and two anonymous referees for *Oxford Studies in Ancient Philosophy*. Finally, I am very grateful to Kit Fine, Ben Morison, and Jacob Rosen for invaluable discussions and suggestions which led to significant improvements in the paper.

BIBLIOGRAPHY

Albertus Magnus, *Opera omnia*, vol. ii, ed. A. Borgnet (Paris, 1890).

Arnauld, A. and Nicole, P., *La Logique ou l'art de penser* [*Logique*], ed. P. Clair and F. Girbal, 2nd edn. (Paris, 1993).

Barnes, J., 'Property in Aristotle's *Topics*', *Archiv für Geschichte der Philosophie*, 52 (1970), 136–55.

Barnes, J., *Aristotle's* Posterior Analytics, 1st edn. [*Posterior Analytics 1st edn.*] (Oxford, 1975).

Barnes, J. (ed.), *The Complete Works of Aristotle: The Revised Oxford Translation* [*Complete Works*], 2 vols. (Princeton, 1984).

Barnes, J., *Aristotle's* Posterior Analytics, 2nd edn. [*Posterior Analytics 2nd edn.*] (Oxford, 1993).

Barnes, J., *Truth, etc.: Six Lectures on Ancient Logic* (Oxford, 2007).

Barrow, I., *The Mathematical Works of Isaac Barrow*, ed. W. Whewell (Cambridge, 1860).

Biancani, G., *De mathematicarum natura dissertatio* (Bologna, 1615).

Biese, F., *Die Philosophie des Aristoteles,* i: Logik und Metaphysik (Berlin, 1835).

Bolzano, B., *Wissenschaftslehre: Versuch einer ausführlichen und größtentheils neuen Darstellung der Logik mit steter Rücksicht auf deren bisherige Bearbeiter* [*Wissenschaftslehre*], 4 vols. (Sulzbach, 1837).

Bronstein, D., *Aristotle on Knowledge and Learning: The* Posterior Analytics [*Knowledge and Learning*] (Oxford, 2016).

Brunschwig, J., *Aristote,* Topiques*, Livres I–IV* (Paris, 1967).

Burnyeat, M. F., 'Aristotle on Understanding Knowledge' ['Understanding Knowledge'], in E. Berti (ed.), *Aristotle on Science: The* Posterior Analytics, *Proceedings of the Eighth Symposium Aristotelicum* (Padua, 1981), 97–139.

Centrone, S., 'Das Problem der apagogischen Beweise in Bolzanos *Beyträgen* und seiner *Wissenschaftslehre*', *History and Philosophy of Logic*, 33 (2012), 127–57.

Colyvan, M., *An Introduction to the Philosophy of Mathematics* (Cambridge, 2012).

Corcoran, J., 'Completeness of an Ancient Logic', *Journal of Symbolic Logic*, 37 (1972), 696–702.

Corcoran, J., 'Aristotle's Natural Deduction System', in id. (ed.), *Ancient Logic and its Modern Interpretations* (Dordrecht, 1974), 85–131.

Correia, F., 'Logical Grounds' ['Grounds'], *Review of Symbolic Logic*, 7 (2014), 31–59.

Correia, F., 'An Impure Logic of Representational Grounding', *Journal of Philosophical Logic*, 46 (2017), 507–38.

Crager, A. D., *Meta-Logic in Aristotle's Epistemology* (diss. Princeton, 2015).

Crivelli, P., 'Aristotle on Syllogisms from a Hypothesis' ['Hypothesis'], in A. Longo (ed.), *Argument from Hypothesis in Ancient Philosophy* (Naples, 2011), 95–184.

Detel, W., *Aristoteles:* Analytica posteriora, 2 vols. [*Analytica posteriora*] (Berlin, 1993).

Ebert, T. and Nortmann, U., *Aristoteles:* Analytica priora*, Buch I* [*Analytica priora*] (Berlin, 2007).

Edlow, R. B., *Galen on Language and Ambiguity* (Leiden, 1977).

Fine, K., 'Guide to Ground', in F. Correia and B. Schnieder (eds.), *Metaphysical Grounding: Understanding the Structure of Reality* (Cambridge, 2012), 37–80.

Fine, K., 'The Pure Logic of Ground' ['Pure Logic'], *Review of Symbolic Logic*, 5 (2012), 1–25.

Fine, K., 'Some Remarks on Bolzano on Ground', in S. Roski and B. Schnieder (eds.), *Priority Among Truths: Bernard Bolzano's Philosophy of Grounding* (Oxford, *forthcoming*).

Frede, M., 'Stoic vs. Aristotelian Syllogistic', *Archiv für Geschichte der Philosophie*, 56 (1974), 1–32.

Galilei, G., *Tractatus de praecognitionibus et praecognitis and Tractatio de demonstratione*, ed. W. F. Edwards (Padua, 1988).

Goldin, O. and Martijn, M., *Philoponus:* On Aristotle, *Posterior Analytics* 1. 19–34 (London, 2012).

Grosseteste, R., *Commentarius in Posteriorum Analyticorum libros* [*Commentarius*], ed. P. Rossi (Florence, 1981).

Hamlyn, D. W., 'Aristotle on Predication' ['Predication'], *Phronesis*, 6 (1961), 110–26.

Hanna, G., 'Reflections on Proof as Explanation', in A. J. Stylianides and G. Harel (eds.), *Advances in Mathematics Education Research on Proof and Proving* (Cham, 2018), 3–18.

Heath, T. L., *The Thirteen Books of Euclid's* Elements, vol. i: Introduction and Books I, II [*Elements*] (Cambridge, 1908).

Helmig, C., 'Proclus' Criticism of Aristotle's Theory of Abstraction and Concept Formation in *Analytica Posteriora* II 19', in F. de Haas, M. Leunissen, and M. Martijn (eds.), *Interpreting Aristotle's* Posterior Analytics *in Late Antiquity and Beyond* (Leiden, 2010), 27–54.

Jakob, L. H., *Grundriß der allgemeinen Logik und kritische Anfangsgründe der allgemeinen Metaphysik*, 4th edn. (Halle/Saale, 1800).

Kant, I., *Kritik der reinen Vernunft*, in *Kant's gesammelte Schriften I*, Königlich Preußische Akademie der Wissenschaften (Berlin, 1911).

Kiesewetter, J. G. C. C., *Grundriß einer allgemeinen Logik nach Kantischen Grundsätzen,* 1. Theil: Reine allgemeine Logik, 4th edn. (Leipzig, 1824).

Kilwardby, R., *Notule libri Posteriorum* [*Notule*], ed. D. Cannone (Cassino, 2003–4).

Kirchmann, J. H. von, *Erläuterungen zu den zweiten Analytiken des Aristoteles* [*Erläuterungen*] (Leipzig, 1878).

Korbmacher, J., 'Axiomatic Theories of Partial Ground I: The Base Theory', *Journal of Philosophical Logic*, 47 (2018), 161–91.
Krug, W. T., *System der theoretischen Philosophie,* 1. Theil: Denklehre oder Logik (Königsberg, 1806).
Lange, M., *Because Without Cause: Non-Causal Explanations in Science and Mathematics* (Oxford, 2017).
Lear, J., 'Aristotle's Compactness Proof' ['Compactness'], *Journal of Philosophy*, 76 (1979), 198–215.
Lear, J., *Aristotle and Logical Theory* [*Logical Theory*] (Cambridge, 1980).
Lipton, P., 'Understanding without Explanation', in H. W. de Regt, S. Leonelli, and K. Eigner (eds.), *Scientific Understanding: Philosophical Perspectives* (Pittsburgh, 2009), 43–63.
Lipton, P., 'Mathematical Understanding', in J. Polkinghorne (ed.), *Meaning in Mathematics* (Oxford, 2011), 49–54.
Lotze, H., *System der Philosophie,* 1. Theil: Drei Bücher der Logik [*Logik*] (Leipzig, 1874).
Löwenheim, L., 'On Making Indirect Proofs Direct', *Scripta Mathematica*, 12 (1946), 125–39.
Łukasiewicz, J., *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, 2nd edn. (Oxford, 1957).
Maier, H., *Die Syllogistik des Aristoteles,* ii/1: Formenlehre und Technik des Syllogismus (Tübingen, 1900).
Maier, H., *Die Syllogistik des Aristoteles*, ii/2: Die Entstehung der Aristotelischen Logik [*Entstehung*] (Tübingen, 1900).
Malink, M., 'Aristotle on Principles as Elements' ['Elements'], *Oxford Studies in Ancient Philosophy*, 53 (2017), 163–213.
Mancosu, P., 'On the Status of Proofs by Contradiction in the Seventeenth Century', *Synthese*, 88 (1991), 15–41.
Mancosu, P., *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* [*Mathematics*] (New York, 1996).
Mancosu, P., 'On Mathematical Explanation', in E. Grosholz and H. Berger (eds.), *The Growth of Mathematical Knowledge* (Dordrecht, 2000), 103–19.
McKirahan, R. D., *Principles and Proofs: Aristotle's Theory of Demonstrative Science* [*Principles*] (Princeton, 1992).
Mignucci, M., *L'argomentazione dimostrativa in Aristotele: commento agli* Analitici secondi *I* [*L'argomentazione dimostrativa*] (Padua, 1975).
Mignucci, M., *Aristotele*: Analitici secondi (Rome, 2007).
Morrow, G. R., *Proclus: A Commentary on the First Book of Euclid's* Elements, 2nd edn. [*Proclus*] (Princeton, 1992).
Mure, G. R. G., *Aristotle's* Analytica posteriora [*Posteriora*], in *The Works of Aristotle Translated into English*, ed. W. D. Ross and J. A. Smith, vol. i (Oxford, 1928).
Owen, O. F., *The* Organon*, or Logical Treatises, of Aristotle*, vol. i [*Organon*] (London, 1889).

Pacius, J., *Aristotelis Stagiritae Peripateticorum principis Organum*, 2nd edn. [*Organum*] (Frankfurt a.M., 1597).
Parsons, T., *Articulating Medieval Logic* (Oxford, 2014).
Pellegrin, P., *Aristote:* Seconds Analytiques [*Seconds Analytiques*] (Paris, 2005).
Poston, T., *Reason and Explanation: A Defense of Explanatory Coherentism* (New York, 2014).
Primavesi, O., *Die Aristotelische Topik: Ein Interpretationsmodell und seine Erprobung am Beispiel von* Topik *B* (München 1996).
Reinhold, E., *Die Logik oder die allgemeine Denkformenlehre* (Jena, 1827).
Roski, S., *Bolzano's Conception of Grounding* (Frankfurt a.M., 2017).
Ross, W. D., *Aristotle's* Prior *and* Posterior Analytics*: A Revised Text with Introduction and Commentary* [*Analytics*] (Oxford, 1949).
Schnieder, B., 'A Logic for "Because"', *Review of Symbolic Logic*, 4 (2011), 445–65.
Seidl, H., *Aristoteles:* Zweite Analytiken, 2nd edn. [*Zweite Analytiken*] (Würzburg, 1987).
Smiley, T. J., 'What is a Syllogism?', *Journal of Philosophical Logic*, 2 (1973), 136–54.
Smiley, T. J., 'Aristotle's Completeness Proof', *Ancient Philosophy*, 14 (1994), 25–38.
Smith, R., 'The Relationship of Aristotle's Two *Analytics*' ['Relationship'], *Classical Quarterly*, NS, 32 (1982), 327–35.
Smith, R., 'The Syllogism in *Posterior Analytics* I' ['Syllogism'], *Archiv für Geschichte der Philosophie*, 64 (1982), 113–35.
Smith, R., 'Aristotle as Proof Theorist', *Philosophia Naturalis*, 27 (1984), 590–7.
Smith, R., 'Immediate Propositions and Aristotle's Proof Theory' ['Immediate Propositions'], *Ancient Philosophy*, 6 (1986), 47–68.
Smith, R., *Aristotle:* Prior Analytics [*Prior Analytics*] (Indianapolis, 1989).
Smith, R., 'Predication and Deduction in Aristotle: Aspirations to Completeness', *Topoi*, 10 (1991), 43–52.
Smith, R., 'Aristotle's Regress Argument' ['Regress'], in I. Angelelli and M. Cerezo (eds.), *Studies on the History of Logic: Proceedings of the III. Symposium on the History of Logic* (Berlin, 1996), 21–32.
Smith, R., 'Aristotle's Theory of Demonstration', in G. Anagnostopoulos (ed.), *A Companion to Aristotle* (Oxford, 2009), 51–65.
Solmsen, F., *Die Entwicklung der Aristotelischen Logik und Rhetorik* [*Entwicklung*] (Berlin, 1929).
Solmsen, F., 'The Discovery of the Syllogism', *Philosophical Review*, 50 (1941), 410–21.
Soto, D., *In libros Posteriorum Aristotelis sive de demonstratione absolutissima commentaria* (Venice, 1574).

Striker, G., 'Review of J. Barnes, *Aristotle's* Posterior Analytics' ['Review of Barnes'], *Zeitschrift für philosophische Forschung*, 31 (1977), 316–20.

Striker, G., 'Perfection and Reduction in Aristotle's *Prior Analytics*', in M. Frede and G. Striker (eds.), *Rationality in Greek Thought* (Oxford, 1996), 203–19.

Striker, G., *Aristotle's* Prior Analytics*, Book 1* [*Prior Analytics*] (Oxford, 2009).

Strobach, N., *Aristoteles:* Analytica priora*, Buch II* (Berlin, 2015).

Thom, P., *The Syllogism* [*Syllogism*] (Munich, 1981).

Tredennick, H., *Aristotle:* Posterior Analytics [*Posterior Analytics*] (Cambridge, MA, 1960).

Trendelenburg, F. A., *Elementa logices Aristoteleae*, 5th edn. (Berlin, 1862).

Trendelenburg, F. A., *Logische Untersuchungen,* 2. Band, 3rd edn. (Leipzig, 1870).

Tricot, J., *Aristote:* Seconds Analytiques [*Seconds Analytiques*] (Paris, 2012).

Waitz, T., *Aristotelis Organon Graece* [*Organon*], 2 vols. (Leipzig, 1844–6).

Weidemann, H., 'Aristotle on the Reducibility of All Valid Syllogistic Moods to the Two Universal Moods of the First Figure (*APr* A7, $29^{b}1$–25)', *History and Philosophy of Logic*, 25 (2004), 73–8.

Williams, M., *Studies in the Manuscript Tradition of Aristotle's* Analytica (Königstein, 1984).

Wilson, N. G., 'More about *γράφεται* Variants', *Acta Antiqua Academiae Scientiarum Hungaricae*, 48 (2008), 79–81.

Zabarella, J., *Opera logica* [*Opera logica*] (Frankfurt a.M., 1608).