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Public Goods in Federal Systems*

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ABSTRACT

We study a political-economic model of federations with both federal and supplemental regional provision of a local (impure) public good with spillover effects. Regional differences in average income levels and externalities of provision induce differences in preferences over federal and regional levels of provision. Although the voters' preferences are not single-peaked, we provide sufficient conditions for the existence of voting equilibria and characterize their properties under alternative federal institutional arrangements. We show that, under different conditions on parameters, the voting equilibria display markedly different patterns of federal vs. local provision, relying on different political coalitions for their support. We show that the inter-regional redistributive tensions present in federations lead to differences in regional support for different degrees of fiscal (de-)centralization: federation, confederation, and complete centralization.

The present paper analyzes the political and economic incentives within federal structures with public good provision on both local and federal levels and spillover effects between regions — features that often characterize contemporary federal systems. Although such systems rarely have institutional provisions that explicitly authorize the duplication of services or public goods on the federal and the local levels, in practice, public goods provided at the two levels of government are often *de facto* substitutes, with fiscal federalism emerging less as a constitutional and juridical reality and more as a dynamic economic one. Sometimes the substitution effect is immediate and direct,

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as with the financing of the creation of new stem-cell lines in the US, which, before President Bush's ban, was done with federal money, but has now been picked up by the state of California. At other times, federal and regional/state programs may have different explicit targets, e.g., different categories of the population, but their substantive closeness gives rise to explicit budgetary fungeability — as with the US federal Medicaid spending on retirees, which is used by state health care facilities to help offset their overall operating expenses (Hernandez 2005). With many other spending programs, there is what may be called "article-by-article substitutability," whereby, within a given policy area there is a division of responsibilities between the state and the federal governments, but a drop in spending by one level on an article within its purview, leads another level to "pick up the slack" by spending more on articles within its own budgetary purview. A prominent example of such a policy area is higher education financing, where the US federal government is typically understood to be responsible for the provision of low-interest student loans and faculty research funds, and state governments finance faculty and staff salaries and the physical plants of state universities. This substitutability is best thought of as a feature of what are, in effect, dual-provision fiscal federal mechanisms. Although this is less true of the EU, whose budget is still rather small relative to those of its wealthiest member-states, the range of policy areas in which the substantive policy control is, effectively, shared by the EU and its members is long and growing, including a number of economic policy areas, most notably monetary policy, agriculture and fisheries, etc., with respect to which the EU is more and more resembling a federal state with a robust policy-making and implementing apparatus (Alesina and Wacziarg 1999).

Our analysis of the dual-provision federalism focuses on the interaction between two elements of federal political economies: income heterogeneity across regions and inter-regional spillovers. Empirically, both of these elements are common place. In the US, the highest state-level median household income is nearly twice the lowest, and the highest national GDP per person in the EU (measured at PPP) is nearly three and a half times the lowest. Inter-regional spillovers in federations take different forms — some geographically contingent, others not — and turn on a number of distinct causal mechanisms.[1] In the presence of these elements, the dual structure of public good provision induces redistributive tensions within the federation, with significant empirical and theoretical implications for our understanding of federal politics.[2]

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[1] Examples of geographically contingent spillovers include many cases of federal/regional substitutability discussed in the previous paragraph. Apart from financing scientific and technological development, like stem-cell research, a particularly striking example of a public good with spillovers that are not geographically contingent is economic growth within a region. One mechanism underlying the spillovers of growth operates via its effects on the creation of new viable markets (e.g., in the EU, where the free movement of people and goods has made geographic proximity less and and less relevant, Emerson *et al.*, 1988). Another mechanism is seasonal labor migration between regions (e.g., in India, from relatively poor and remote states like Bihar to relatively technologically advanced centers like Punjab — see Ghosh and Sharma, 1995; Haberfeld *et al.* 1999).

[2] Other models of federal political economy that focus on the consequences of spillovers include Alesina *et al.* (2005), who posit primitive differences in demand for public good and Crémer and Palfrey (2000b), who analyze distributive effects of unfunded mandates. Our basic setup is also

Rather than assuming primitive differences in regional demands for public goods, we induce these differences endogenously from the interaction between the relevant spillover factors and the differences in regional incomes, which enables us to develop predictions in terms of these empirically observable regional characteristics. Our model offers a framework for analyzing a wide range of different types of dual-provision mechanisms, including the previously unexplored and important cases in which federal provision may exploit economies of scale in production, as well as the case in which federal provision is equivalent to the equal division of federal revenues among the member regions, who then produce the public good locally just as they would do using locally raised revenue (as in Alesina *et al.* 2005). It also allows us to compare the outcomes of different policy instruments available in federal systems, such as dual provision and federal mandates (Crémer and Palfrey 2000b).

Our key technical results characterize and provide sufficient conditions for the existence of a voting equilibrium with public good provision at two levels of government. Substantively, we show that the presence of spillovers in dual-provision federations results in political conflicts between the region-members of the federation and between the federal and the regional levels of government. These conflicts manifest themselves in regions' attempting to increase or decrease the role of the federal government, relative to the regional governments, in order to force other regional governments into making particular political and economic choices.[3] We show that these attempts (1) may account for the complex patterns of variation in the degree of economic conservatism on the part of wealthy and poor states within federations; (2) can give rise to politically potent ends-against-the-middle majority coalitions that often elude directly political explanations; and (3) lead to conflicts over the choices of institutional structures or organizing fiscal decision-making, though not necessarily to the instability of collective institutional choice: federal, rather than either completely centralized or confederational, institutions emerge in our analysis as invariably supportable by regional majorities.

THE MODEL

Notation and Primitives

The sequence of the game is as follows. First, the citizens of the federal union choose their federally provided level of public good. Then, citizens of each region comprising

related to Epple and Romano's (2003) model of collective public good provision with supplemental voluntary provision, which could be re-interpreted as a model of federation in which the federal and local means of provision are identical and in which the public good is a pure one. The model presented here is complicated not only by impure public goods and technological differences between the federal and local means of provision, but also by the fact that local provision must be determined by a collective choice rather than by an individual one. Surveys of formal models of federalism include Alesina *et al.* (1995) and Bolton *et al.* (1996).

[3] Thus complementing the credit claiming incentives that are highlighted in Bednar (2004) and Volden (2005).

the union simultaneously choose their respective regional levels of provision. At both levels, decisions are made by majority rule. Following the realization of the federal and the regional provision levels, citizens of the union consume their public and private goods.[4]

There are n regions in the union, where n is odd, and let l be the number of regions that choose (by domestic majority) to engage in supplemental local provision. The corresponding sets of member regions are denoted as N and L, respectively, with $L \subseteq N$. In the interests of tractability, all regions are assumed to have equal populations.[5] Although this assumption may appear restrictive initially, it does not affect the spirit of the substantive results.[6]

Let u_j^i be the utility of agent i in region j. We assume Cobb–Douglas preferences

$$u_j^i(x_j^i, y_j) = (x_j^i)^a y_j^b, \tag{1}$$

where y_j is the amount of public good enjoyed by each individual in region j, x_j^i is i's private good, $a \in (0, 1)$, and $a + b = 1$.

Additionally, we use the following notation to denote the corresponding income variables: h_j^i is the income of agent i in region j; $H_j = \sum_i h_j^i$ is the total income of a generic region j; and $H = (H_1, \ldots, H_j, \ldots, H_n)$ is the vector of total regional incomes. $H^T = \sum_j H_j$ is the total income of the federation.

We assume that the private good is numeraire, i.e., the monetary unit. One unit of private good purchases one unit of local public good, that is, one unit of an impure public good that can be enjoyed fully by citizens in the region in which it is provided, but only partly by the citizens of other regions. Let $s_L < 1$ be the proportion of that local public good that citizens of other regions are able to enjoy. The federal-level policy we analyze imposes a uniform tax rate and spends equal amounts on the provision of public goods in each region. The benefits from federal provision directed to one region spill over to other regions also at the rate of s_L. Hence, a dollar spent on public good provision by the federal government increases the amount of public good enjoyed in any given region by $\gamma\frac{(1+s_L(n-1))}{n}$, where γ is a measure of the relative efficiency of federal provision. If $\gamma < 1$, then the federal means of provision is inherently less efficient than the local means of provision, and if $\gamma > 1$ then federal provision is more efficient.[7] As noted in the introduction, our assumptions here are flexible enough to cover the cases in which

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[4] We comment on the possibility of reversing the order of provision in the section "Discussion."

[5] It is straightforward, though notationally more cumbersome, to generalize to the case in which regions are of different sizes. We comment further on the possibility of such an extension below.

[6] The mobility of voters across regions is beyond the scope of the present paper and is left to the future work extending the model developed here. Not modeling it explicitly here has the advantage of allowing us to evaluate the independent impact of spillovers on federal political economy, as well as to compare our results to the results in other papers that model federal system with spillovers and which, at the moment, do not allow for voter mobility.

[7] Note that whether or not $\gamma > 1$ is distinct from the question of whether membership in the federation is attractive, in part because the existence of spillovers may make it attractive even if $\gamma < 1$.

federal provision may exploit economies of scale in production, as well as the case in which federal provision amounts to the equal division of federal revenues among the member regions, who then produce the public good locally just as they would using locally raised revenue.

For notational convenience, we let $\gamma\frac{(1+s_L(n-1))}{n} = s_F$ represent the amount of public good enjoyed in a region for each unit the federal government spends on public goods. We thus interpret s_F to reflect both the direct impact (from the federal provision for region j) and the indirect impact (via cross-regional spillovers from the federal provision for other regions) of federal provision. Note that s_L and s_F can be treated as the rates of substitution of the individual's own region's locally provided public good for other regions' or federal public goods.[8]

The federal tax rate is t^F and the vector of local tax rates is $t = (t_1, t_2, \ldots, t_n)$, with the understanding that $t_j, t^F \in [0, 1]$. The government faces a balanced budget constraint, so all revenue is invested in public goods at the corresponding level of government. Thus, for individual i in region j,

$$\begin{aligned} y_j &= t^F H^T s_F + t_j H_j + s_L \sum_{k \in L \setminus j} t_k H_k \\ x_j^i &= h_j^i(1 - t^F - t_j). \end{aligned} \tag{2}$$

The Political-Economic Equilibrium

We begin by defining the notion of majority rule core associated with utility profile u, $C(u)$. Formally, $C(u) = \{\tau \in [0,1] : \nexists \tau' \in [0,1] \text{ s.t. } |\{j \in N : u_j^i(\tau') > u_j^i(\tau)\}| > \frac{n}{2}\}$; that is, a tax rate is in the core if there is no other admissible tax rate that a majority of citizens strictly prefers. The equilibrium of the game, then, is characterized as the pair (t^{F*}, t^*), such that for every region j, t_j^* is in the majority rule core of region j, given t^{F*} and t_{-j}^*, and the federal tax rate t^{F*} is in the majority rule core of the federation as a whole, given $t^*(t^{F*})$.

Solving by backward induction, we first find the induced preferences of a voter in region j over regional tax rates, t_j, given her expectations of the other regions' simultaneously chosen tax rates, t_{-j}, and the known federal tax rate t^F. Substituting the values from Equations 2 into Equation 1 yields

$$u_j^i(t^F, t, \cdot) = ((1 - t^F - t_j)h_j^i)^a \left(t^F H^T s_F + t_j H_j + s_L \sum_{k \in N \setminus j} t_k H_k \right)^b. \tag{3}$$

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[8] Although, in the interests of brevity, we focus our interpretation of the model on the effects of positive externalities, it is fully consistent with the presence of negative externalities as well. In the latter case, the rationale for the existence of the federal government may be thought to be the alleviation of such externalities, which is analytically equivalent to the production of positive ones.

Because both private and public goods are essential, no agent ever prefers to tax away all of her private endowment. She may, however, prefer no supplemental local provision of the public good ($t_j = 0$) if enough public good is provided at the federal level or locally in other districts. To locate any interior solution, we obtain the Kuhn–Tucker conditions, which (combined with the conditions for the corner solution) yield the most preferred local tax rate as a function of the federal tax rate:

$$t_j(t^F) = \max\left\{0,\ b(1-t^F) - a\left(t^F\frac{H^T}{H_j}s_F + s_L\sum_{k\in N\setminus j} t_k\frac{H_k}{H_j}\right)\right\}. \tag{4}$$

The first thing to note in relation to Equation 4 is that, because voters have identical marginal rates of substitution between private and public goods and the tax is a flat rate on endowment, domestic politics (i.e., the politics of determining the supplemental level of public good provided at the local level) is essentially trivial: holding constant the federal tax t^F, agents within a region have identical preferences over the internal tax financing the supplemental provision of the public good.[9] Thus the choice of regional tax rate depends on the region's total income, H_j, but does not otherwise depend on voter i's income, h_j^i. For the same reason, the equilibrium local tax rate is robust to the specification of the domestic collective choice rule. This simplicity of the intra-regional politics helps place the inter-regional politics in the spotlight, and will make it easier to see the causes and consequences of disagreements between regions over federal taxation and provision. These disputes will turn on the combination of the redistributive pressures inherent in income heterogeneity across regions and the possibility of free-riding implicit in inter-regional spillovers. We begin our analysis of these effects with the lemma that establishes the unique relationship between the set of local providers and the federal tax rate that is instrumental in proving the main results of the model.

Lemma 1 *There exists a unique equilibrium of the regional provision subgame. The equilibrium regional tax rates are monotonic in regional income* (*that is, for any pair of regions i and j such that* $H_i > H_j$, $t_i^* \geq t_j^*$).

Proof: See Appendix.[10] ■

Let $L(t^F)$ be the set of members which engage in supplemental local production at federal tax rate t^F. Let $|L(t^F)| = l(t^F)$. If we index the regions from richest to poorest, i.e., so that $H_1 \geq H_2 \geq \cdots \geq H_l \geq \cdots \geq H_n$, the key consequence of this lemma for

[9] This result should not be understood as a substantive prediction, but rather as a helpful intermediate claim in the context of our model. Still, if the individual utilities are concave in x and y, then, regardless of the individual income, receiving some positive amount of public good for free via spillovers from other regions lowers the demand for either federal or own regional public good and raises the demand for private good. Only the magnitude, not the direction, of the effect could depend on the individual income.

[10] We owe this formulation of Lemma 1 and its proof to an anonymous referee.

the present model is that $l(t^F)$ must be the highest integer such that $t_l(t^F)H_l > 0$, i.e., it has to be such that both $t_l(t^F)H_l > 0$ and $t_{l+1}(t^F)H_{l+1} = 0$ (where the lower bound on tax rate may be binding). Hence, $\sum_{j\in N} t_j(t^F)H_j = \sum_{j=1}^{l(t^F)} t_j(t^F)H_j$, and from Equation 4 we can write

$$\sum_{j\in N} t_j(t^F)H_j = \sum_{j=1}^{l(t^F)} t_j(t^F)H_j$$

$$= \frac{1}{1 + as_L(l(t^F) - 1)}\left(b(1 - t^F)\sum_{k=1}^{l(t^F)} H_k - as_F t^F H^T l(t^F)\right). \quad (5)$$

Substituting back into Equation 4 and solving for $t_j(t^F)H_j$, we get

$$t_j^*(t^F)H_j = \max\left\{0, \frac{b}{1 - as_L}(1 - t^F)H_j - \frac{1}{1 + as_L(l(t^F) - 1)}\right.$$

$$\left.\left(as_F t^F H^T + \frac{abs_L}{1 - as_L}(1 - t^F)\sum_{k=1}^{l(t^F)} H_k\right)\right\}, \quad (6)$$

where, from the above ordering of H_j and Equation 6, $l(t^F)$ and is the greatest l such that

$$(1 + as_L(l - 1))b(1 - t^F)H_l - abs_L(1 - t^F)\sum_{k=1}^{l} H_k - (1 - as_L)as_F t^F H^T \geq 0. \quad (7)$$

Together Equations 6 and 7 determine the Nash equilibrium in the simultaneous-move local provision game.

Given this solution to the local provision subgame, we next determine the agents' preferences over federal tax rates. These preferences depend critically on the relative merits, from an individual's point of view, of the federal and the local means of provision, taking into account the strategic responses of other regions to changes in each.

Two features of voters' induced preferences over federal tax rates, each of which is established formally in the appendix, play prominent roles in the characterization of the equilibrium federal tax rate. First, we show that voters within a given region j have identical induced preferences over federal tax rates even though they may have different private endowments, and hence different utilities at any given tax rate. In addition to aiding the characterization of the majority-rule core, this feature of the model facilitates the extension of the results to both a wider variety of institutions for collective choice, including representative government, and the case in which regions differ in their populations. Second, we show that voters in any two regions that do not engage in local provision for some interval of federal tax rates have identical induced preferences over

federal tax rates on that interval. Both of these properties are consequences of the fact that the citizens in question enjoy the same spillovers from other regions' local provision, the same federal provision of the public good, and the same local provision from their own regional governments. Hence, they enjoy the same amount of public good. This induced simplicity of regional politics in the model allows us to focus our attention on the consequences of inter-regional redistributive tensions.

Because 0 is a binding lower bound on all local tax rates, the expression for the indirect utility of a voter in region j as a function of federal tax rate, given the equilibrium behavior in the local provision game, is different when some other region k engages in local provision than when it does not. Let $l_{\max} = l(0)$ be the number of regions that engage in local provision when there is no federal provision of the public good, i.e., such that

$$H_{l_{\max}} > \frac{as_L}{1 + as_L(l_{\max} - 1)} \left(\sum_{k=1}^{l_{\max}} H_k \right) \geq H_{l_{\max}+1}. \tag{8}$$

Lemma 1 implies that the range of t^F can be partitioned into $[0, \hat{t}^F_{l\,\max})$, $[\hat{t}^F_{l\,\max}, \hat{t}^F_{l\,\max-1}), \ldots, [\hat{t}^F_1, 1]$, so that each of these intervals defines a range of t^F corresponding to a given value of l. These intervals are bounded by the (not necessarily optimal, from the perspective of a voter in region j) federal tax rates $\hat{t}^F_j$ at which region j ceases local provision, i.e., the lowest t^F at which $t^*_j(t^F) = 0$. (For $j > l_{\max}$, $\hat{t}^F_j = 0$.) Substituting $t^*_j = 0$ and $l = j$ into Equation 6 and solving for $t^F (\equiv \hat{t}^F_j)$, we obtain

$$\hat{t}^F_j = \frac{(1 + as_L(j-1))H_j - as_L \sum_{k=1}^{j} H_k}{(1 + as_L(j-1))H_j + (1 - as_L)\frac{a}{b}s_F H^T - as_L \sum_{k=1}^{j} H_k}, \tag{9}$$

which $\forall j \leq l_{\max}$ is greater than 0. Then, for $t^F < \hat{t}^F_j$ and $t_j \in [0, t^*_j(\hat{t}^F_j))$, voters in region j wish to procure more public good through some form of increased taxation.

Given this partition of the policy space, we proceed by identifying the extrema on each interval. In the presence of spillovers, there may exist multiple local optima, and regions' preferences are neither necessarily single-peaked nor necessarily order-restricted. Although this means that we cannot rely immediately on standard theorems for aggregating preferences over t^F, we can provide sufficient conditions for the existence of a unique stable outcome of majority rule and characterize that outcome. The next two propositions identify such conditions and characterize qualitatively different and mutually exclusive equilibria that obtain when they are satisfied. These equilibria differ dramatically in the composition of the coalitions supporting a greater or lesser degree of federal provision: in the first equilibrium, it is the citizens of the wealthiest regions (whether or not they are wealthy themselves) that prefer greater federal provision in opposition to the poorer regions' preference for local provision; in the second equilibrium, the citizens of the wealthiest and sometimes also of the poorest regions prefer local provision, in opposition to the middle-income regions' preferences for federal provision.

Recall that all citizens of a given region have identical induced preferences over tax rates and that j indexes regions in the descending order of their total (or average) income. Say that the *correspondence* $F(x)$ *is increasing* if $x' > x''$ implies that for any y' in $F(x')$ and any y'' in $F(x'')$, $\max\{y', y''\}$ is in $F(x')$ and $\min\{y', y''\}$ is in $F(x'')$. Then, letting m index the median region in this ordering, we obtain the following result:

Proposition 1 *Let H be such that for all $j \leq l_{\max}$ the citizens of j strictly prefer to increase federal provision at any t^F such that $t_j^*(t^F) > 0$. Then*:

(1) the most preferred federal tax rate correspondence is increasing in regional income;
(2) the federal majority rule core is nonempty and comprised of the set of most-preferred federal tax rates of the median income region;
(3) in a majority rule vote over any two members of the core, the higher value of the federal tax rate has a weakly larger strict-preference coalition;
(4) the m poorest regions (comprising the majority of member regions of the federation) do not engage in local provision in equilibrium.

Proof: See Appendix. ■

Proposition 1 characterizes one set of strategic incentives that heterogenous-income regions face in federal systems with two levels of public good provision. Under the specified conditions, a majority of regions (and so of the citizens of the federation) select a low federal tax rate, anticipating that a minority of wealthier regions will provide additional amounts of the public good.[11] Because members enjoy positive externalities from public goods provided by other regions, free-riding on the provision of the wealthier regions at a lower level of federal provision is sometimes more attractive for the citizens of a poorer region than is a more efficient level of provision that entails paying a higher federal tax. When the antecedent condition of the equilibrium characterized in Proposition 1 holds, the prediction is that the poorer regions/states (i.e., the net federal financing recipients) would object to the increases in federal provision, whereas the wealthier regions/states (i.e., the net federal taxpayers) prefer relatively higher levels of federal provision.

We next provide another sufficient condition for the existence of a non-empty majority-rule core for federal provision. In this case, $t_j^{F*}(t^*, \cdot)$ is weakly non-monotonic in regional income.

Proposition 2 *Let H be such that at any federal tax rate at which they are local providers, the citizens of the richest $\hat{l} \leq l_{\max}$ regions strictly prefer to decrease federal provision and (if $\hat{l} < l_{\max}$) the citizens of the $(\hat{l}+1)$th richest region strictly prefer to increase it. Then*:

[11] A similar logic underlies the equilibrium pattern of provision characterized in Alesina *et al.* (2005) for regions differing in their primitive taste for the public good, rather than in wealth as in our model.

(1) for any $t^F \in [0,1]$, *the citizens of the richest* $\hat{l}$ *regions strictly prefer a lower federal tax rate, and for any* t^F *at which poorer regions engage in local provision, their citizens strictly prefer a higher federal tax rate;*

(2) the most preferred federal tax rate correspondence is weakly decreasing in income when $\hat{l} = l_{\max}$, *and is weakly non-monotonic in regional income, with middle income regions preferring higher tax rates than do wealthier and poorer regions when* $\hat{l} < l_{\max}$;

(3) the federal majority-rule core is nonempty and is (a) $\{0\}$ *if* $m \leq \hat{l}$, *or (b) comprised of the set of most-preferred federal tax rates of region* $(m + \hat{l})$ *if* $m > \hat{l}$;

(4) the richest $\hat{l}$ *regions engage in local provision in equilibrium.*

Proof: See Appendix. ■

The preference profiles that satisfy the conditions identified in Proposition 2 may give rise to majority coalitions with very different compositions. While some profiles result in an equilibrium in which the voters are divided by differences in regional income — i.e., the richer regions, which favor no federal provision, versus the poorer ones — other profiles produce an equilibrium in which the richest and poorest regions ally against the middle-income regions. In such cases, the rich and the very poor regions prefer little or no federal provision, and the middle-income regions champion higher federal tax rates.

Figure 1 depicts the indirect utilities $u(t^F, t^*(t^F))$ over federal tax rates for the members of a federation composed of five regions, where the antecedent condition of Proposition 2 holds. Because, as already established, all the citizens of a given region have identical induced preferences over tax rates, we depict the indirect utility of one citizen in each region, with an individual income equal to the region's average per capita income. Without loss of generality, we will refer to the preferences of the citizens of a given region as "the region's preferences" throughout the following discussion.

Regions 1 and 2, which are the wealthiest members, have strictly decreasing utility. Both 1 and 2 engage in local provision at Region 3's most-preferred federal tax rate, t_3^{F*}. Regions that do not engage in local provision over a given interval of federal tax rates share the same preferences over those tax rates, e.g., regions 3, 4, and 5 all have a maximum at t_3^{F*}, and 4 and 5 both have a higher maximum at t_4^{F*}, indicating that they both prefer t_4^{F*} to t_3^{F*}. Region 3 does not have a maximum at t_4^{F*} because it engages in local provision at that tax rate, $t_4^{F*} < \hat{t}_3^F \leq t_3^{F*}$. Because poorer regions stop providing locally at lower tax rates than do richer regions, most-preferred tax rates are weakly increasing in income among all regions that strictly prefer higher tax rates when they are local providers (i.e., all local providers under the conditions of Proposition 1, and $\{\hat{l}+1, \ldots, l_{\max}\}$ under the conditions of Proposition 2) and regions that are never local providers. In the case depicted in Figure 1, the richest two regions and the poorest region prefer t_5^{F*} to any higher tax rate. To be sure, the rich and the poor regions favor low taxes for different reasons. The citizens of the richer regions oppose the redistributive effects of federal provision. The poorest regions are, in such profiles, so poor relative to the rich regions, and obtain so much public good from the local provision of other regions

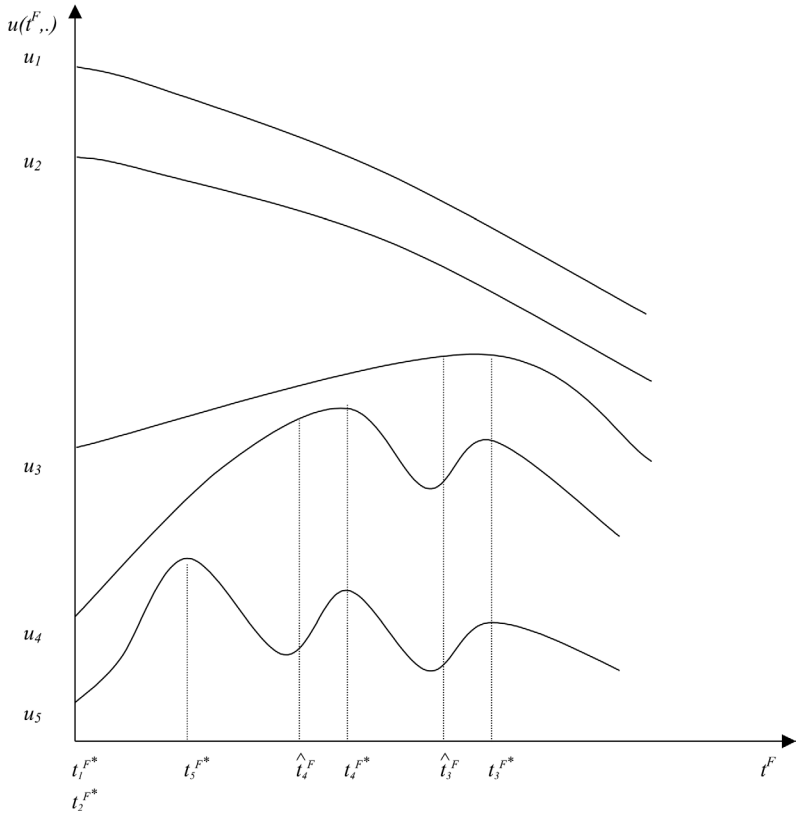

Figure 1. Induced preferences over federal tax rates satisfying conditions of proposition 2.

(in the total absence of federal provision), that their demand for additional private good is greater than their demand for additional public good.

This "ends-against-the-middle" result is the opposite of that of Epple and Romano (2003), in which the rich and poor desire higher taxation than do the middle-income voters. Because in their model, the public good is assumed to be truly global, every agent prefers additional federal provision to her own voluntary (local) provision of the good, since she bears only a fraction of the cost under the former scheme that she bears under the latter, while enjoying access to the same amount of the public good. Hence, in their model, the richest agents always demand high levels of federal taxation. Lower-income agents may demand higher levels of federal provision than middle-income agents, however, producing a coalition of rich and poor in support of higher taxation. By contrast, in the present paper, because the quantity of public good that an agent enjoys depends not only on the resources allocated to its production, but also on the location of its production in the federal system, wealthy regions may experience a decrease in

Table 1. Illustration of equilibrium existence for ($a = 0.6, b = 0.4, H = (100, 90, 80, 70, 50)$).

	S_F								
S_L	0.1	0.3	0.5	0.7	0.9	1.1	1.3	1.5	1.7
0.1	2	–	–	–	–	–	1	1	1
0.3	2	–	–	–	–	–	1	1	1
0.5	2	–	–	–	–	1	1	1	1
0.7	2	–	–	–	–	1	1	1	1
0.9	2	–	–	1	1	1	1	1	1

Note: Numbers 1 and 2 indicate the satisfaction of parameter constraints for Propositions 1 and 2, respectively.

the amount of public good that they enjoy as federal taxation increases. It is also the case that different regions engaging in local provision enjoy different amounts of the public good, giving rise to the possibility that, while some may be harmed by increases in federal provision, other local providers benefit from it. Thus citizens of middle-income regions may prefer higher levels of federal provision than do citizens of the rich and poor regions.

Table 1 illustrates some parameter values that guarantee equilibrium existence in Propositions 1 and 2.

The Effects of Spillovers

To better isolate the effects of introducing positive spillovers into the model of federations with two levels of public good provision, we next provide the properties of the equilibrium in the federation when $s_L = 0$. Let H_m be the total income in the median region of the income distribution. Then:

Proposition 3 *In the federation without inter-regional spillovers* ($s_L = 0$) :

(1) if $\gamma > \frac{nH_m}{H^T}$, *then* $t^{F*} = b$ *and all regions* j *choose* $t_j = 0$;
(2) if $\gamma < \frac{nH_m}{H^T}$, *then* $t^{F*} = 0$ *and all regions* j *choose* $t_j = b$.

Proof: See Appendix. ■

We might expect, absent strategic and redistributive effects, that members would prefer federal provision when it is more efficient, i.e., when $\gamma > 1$, and local provision when it is more efficient, i.e., when $\gamma < 1$. However, the redistributive features of federal provision complicate the effect of relative technological efficiency on induced preferences in equilibrium. Because a condition of federal provision is that each region enjoys equal

amounts of the federally–provided good, and because the regions have unequal wealth, federal provision is redistributive, dampening the richest region's enthusiasm for that means of provision. In the federation without inter-regional spillovers, if $\gamma > \frac{nH_1}{H^T}$, then all citizens of all regions prefer pure federal provision, i.e., $t^F = b$ and $t_j = 0$, for all $j \in N$; if $\gamma < \frac{nH_n}{H^T}$, then all citizens of all regions prefer pure local provision, i.e., $t^F = 0$ and $t_j = b$, for all $j \in N$. The inter-regional disagreement occurs when the efficiency of federal provision is in the middle-range, i.e., when $\gamma \in \left(\frac{nH_n}{H^T}, \frac{nH_1}{H^T}\right)$. In that case, citizens of some regions prefer federal, and citizens of others pure local, provision. Since within each region, voters share induced preferences over t^F, the outcome of majority rule in the federation as a whole is identical to the outcome of weighted voting among member states, where weights are proportional to states' populations. Moreover, since, by assumption, all regions have equal populations, we can, in this and the subsequent results, restrict our attention to simple majority rule among regions. Because the regional preferences are single-peaked, the Condorcet winner always exists, and the prediction is straightforward.

We can now ascertain the direct effects of the existence of spillovers in a federal system with joint federal and local provision, by comparing the results of Proposition 3 with those in the model with positive local (inter-regional) spillovers. The following proposition summarizes the salient features of this comparison (when the majority core is empty, we restrict the federal tax rate to be in the Pareto set):

Proposition 4 *(1) If* $\gamma < \frac{nH_m}{H^T}$, *the presence of local spillovers* ($s_L > 0$) *weakly increases federal provision, weakly decreases local provision in the richest region, and strictly decreases local provision in all other regions*;

(2) If $\gamma > \frac{nH_m}{H^T}$, *the presence of local spillovers* ($s_L > 0$) *weakly decreases federal provision and weakly increases local provision.*

Proof: See Appendix. ■

In the absence of any spillovers from local provision, i.e., for $s_L = 0$, each region's most-preferred federal tax rate is 0 or b. Thus, if, with $s_L > 0$, the majority-chosen tax rate t^{F*} is on the interval $(0, b)$, then the majority of members are free-riding on spillovers from other regions' local provision. Since local provision is monotone increasing in regional income, it is the poorer regions who are free-riding on the local provision of richer regions whenever $t^{F*} < t_1^{F*} \leq b$. Furthermore, even when the technology of federal provision is sufficiently efficient relative to the technology of local provision to compensate the richer regions for the direct redistributive effects of federal provision, the citizens of regions that provide locally may suffer from switching to federal provision because of other regions' reducing local provision in response to the increase in federal provision. Not all regions are necessarily able to reduce their provision of the public good in response to federal provision, however. When spillovers from local provision are sufficiently large, the poorest regions will not engage in local provision even in the total absence of federal provision, because they consume enough of the public good locally

provided in other regions. It follows, then, that they will not be able to respond to increases in federal provision, and thus the net effect of increasing federal taxation on the utility of the richer regions will be greater than it would be if all regions were initially engaged in local provision. For this reason, citizens of richer regions find federal provision more appealing when, *ceteris paribus*, fewer regions are engaging in local provision, i.e., when s_L is high.

Part 1 of Proposition 4 suggests that under joint federal and local provision, the presence of local spillovers produces a substitution effect similar to that which Crémer and Palfrey (2000b) characterize under the system of unfunded federal mandates. Part 2 suggests, however, the presence of another kind of substitution effect that has very different distributive properties. Whereas the use of federal mandates favors the rich regions over the poorer regions by allowing the former to enjoy the spillover effects from forcing high levels of local provision in the poorer regions, our analysis implies that the poorer regions may use the policy tools offered by joint federal and local provision to free-ride on local provision in the richer regions by shifting the burden of provision toward them and away from themselves. (Because our tax instrument is less regressive than Crémer and Palfrey's, the contrast between the distributive effects of unfunded mandates and dual provision should not be overdrawn.) Note that when the condition that ensures that t_j^{F*} is increasing in j's income is violated, either the free-riding by the poorer regions persists (whenever t_j^{F*} is increasing in j's income) or the rich regions' preferred federal tax rate is 0 and weakly below that preferred by the poorer regions. In the latter case, the distributive effects of joint provision once again favor the poorer regions, but through a more familiar directly redistributive causal mechanism. [12]

Federal (De-)centralization

Suppose that citizens could determine their preferred governance structure, choosing from three forms of government defined by their degree of centralization: reserving the public good provision authority for the national level alone (*complete centralization*), the more decentralized, dual provision, government modeled in our game (*federation*), and the completely decentralized government with provision issues resolved fully on the relevant local levels (*confederation*). As the proposition below shows, individual preferences over these alternatives will vary with their region's wealth and with political circumstances as captured by the distribution of induced preferences in the society. (Again, because citizens of the same region have the same induced preferences, we can speak meaningfully of a region's preferences.)

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[12] For any $s_L > 0$ such that the conditions of Propositions 1 or 2 are met, the presence of local spillovers increases the public good consumption of citizens in every region except those for which utility is strictly decreasing in t^F; for those regions, the presence of local spillovers increases public good consumption if they do not increase the efficiency of federal provision (s_F) too much.

Proposition 5 *(1) A federation is always majority preferred to confederation and complete centralization. If, in the federal system, provision occurs at both levels of government in equilibrium, then this preference is strict.*

(2) If H is such that at $t^F = 0$, *the citizens of the richest region strictly prefer to increase federal provision, then the majority coalition supporting federation against complete centralization includes the poorest m regions, and the coalition supporting federation against confederation includes the richest m regions.*

(3) If H is such that at any federal tax rate at which they are local providers, the citizens of the richest $\hat{l}$ *regions strictly prefer to decrease federal provision and at* $t^F = 0$, *the citizens of the* $(\hat{l}+1)$*th richest region strictly prefer to increase it, then the majority coalition supporting federation against confederation includes the middle-income regions* $\{\hat{l}+1, \hat{l}+2, \ldots, \hat{l}+m\}$, *and the coalition supporting federation against complete centralization includes the* $\hat{l}$ *richest regions and, if those regions alone do not constitute a majority, the* $n-(m+\hat{l})+1$ *poorest regions as well.*

Proof: See Appendix. ■

This proposition underscores several interesting features of federal politics. First, although preferences over institutions vary across regions by regional income, it is evident that federation is always majority preferred to both completely centralized government and to confederation. Second, since in confederation, national majorities do not have taxing and spending authority, preference for confederation under the conditions of part 2 of the proposition corresponds to implied opposition to federal spending programs even when they are redistributive toward the poor regions.

Third, of particular interest is the opposition of the wealthiest regions to the transition from fully-centralized government to federation (which occurs under the conditions of part 2 of Proposition 5). The substantive intuition can be easily conveyed. Under complete centralization, the citizens of wealthy regions can insure that their most-preferred federal tax rate of b is adopted because there is no local provision and so there is no enjoyment of local spillovers to be had. However, in a federation, if the citizens of the median region prefer a lower tax rate, then there is a majority supporting it because they can free-ride on the local provision in the wealthy regions. Since the opposition to federalization under such circumstances would come from the wealthiest regions, it seems unwise to discount its political consequences even despite the fact that it would be a minority opposition.

Finally, Proposition 5 offers an interesting comparison of federations with dual provision to federations with local provision subject to federal mandates. Whereas, as Crémer and Palfrey (2000b) show, the citizens of wealthy regions always prefer federations *qua* federal mandates to confederations, Proposition 5 shows that their preference for federations *qua* institutions with dual provision relative to the same baseline is more ambiguous: they may prefer them (under the conditions of part 2) or oppose them (under the

conditions of part 3).[13] To the extent that empirical federations often contain a mix of federal mandates and dual provision, the extent to which stronger federal government advantages wealthier regions is an open issue.[14]

DISCUSSION

Our paper offers an analysis of the political economy of federations with public good provision at both the federal and the regional levels. The key causal mechanism we characterize may be described as inter-jurisdictional free-riding, with regions exploiting spillover effects by shifting the burden of provision or its costs away from themselves and toward other members of the federation. Our results may be seen as suggesting that a conservative economic philosophy of preferring both low taxes and low government spending, which may often be observed in poorer states within federations, may have very material underpinnings. In particular, we hypothesize that it may be reinforced by expectations of indirect economic benefits (through, *inter alia*, investment and job creation in the state/region) from costly fiscal measures implemented by other states/regions (e.g., upgrading of the infrastructure, job (re-)training programs, etc.). Where we do see poor regions supporting federal spending programs and rich regions opposing them, we should expect the rich (and possibly also the poor) regions to engage in the local provision of substitutes for the federal programs, and by a means of local provision that is not too much less efficient than the means of federal provision (e.g., economies of scale are not too great).

The political economy of federal mandates provides an instructive point of comparison for our analysis of federations with joint federal and local public good provision. In particular, Crémer and Palfrey (2000b) demonstrate that incentives associated with federal mandates create the "substitution principle": in the presence of cross-regional spillovers, the high-demand regions decrease their public good production when they are able to implement federal mandates that are binding on lower-demand regions. The logic supporting something like this substitution principle is also operative in our model, but the underlying political-economic mechanism and its distributive consequences are quite distinct. In the case of federal mandates, the benefits for the rich regions of high federal mandates come largely from spillovers from public good in the poorer regions.

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[13] Note, interestingly, that although the minority coalition of poorest and wealthiest regions may unite to oppose federation in favor of decentralization, these coalition members will have different preferences on the adoption of federal mandates, with the citizens of the wealthy regions always supporting and those of the poor regions always opposing them.

[14] Proposition 5 also points to a conflict between the positive disposition in favor of federation and a normative standard of economic efficiency. That the federation is majority preferred even when federal provision is less efficient may be thought to underscore the presence of what Crémer and Palfrey (2000a) refer to as the "delimitation problem" — the difficulty of extending the (federal) constitutional structure to cover issues for which that structure is efficient while excluding those for which it is not so, when in either case there are majority coalitions in support of federalism. The diversity of those coalitions for different economic conditions further accentuates this point.

In contrast, in federations with joint provision, the benefits to the rich regions of federal over local provision come mainly from the fact that the federal level of provision allows them to force the poor regions to share the cost of provision. As the economic primitives vary, the incentives supporting the substitution principle in the model with joint provision are distributionally complex: the substitution effect is sometimes induced by an increase in local provision by the high-demanders, and at other times, by a decrease in federal provision supported by the low-demanders.

At the same time, our results indicate the political appeal of federalism relative to other institutions managing relations between regions: when the existence of majority-rule equilibrium can be assured, the "dual-provision" federalism receives majority support against confederation and fully centralized government regardless of whether the federal action would favor the wealthy or the poor member-regions. Given that the externalities that induce free-riding are the same ones that enhance the desirability of political integration in the first place, the inter-regional tensions we describe are likely to remain a central element of political economies in many countries in the foreseeable future.

We conclude with three further caveats about the modeling choices that underlie our analysis. First, it is worthwhile to note how the predictions of our model would differ if we reversed the order of provision, so that regions, rather than the federal level, became the Stackelberg leaders. Assuming that sequence, it is clear that when the wealthy regions prefer a federal tax rate above that which would be preferred by the pivotal voter, they would now have an incentive to commit to lower levels of regional provision, in expectation of the subgame-perfect behavior in the federal provision subgame. This scenario strikes us, however, as implausible. Since in federal politics proper, the federal level of provision is considerably more difficult to alter than the local, such strategic "commitments" by local governments are not likely to be viewed as credible. It seems plausible, therefore, that, quite apart from immediate descriptive verisimilitude, a model with the federal level as the Stackelberg leader may, in fact, be a better approximation to the open-horizon repeated sequence of federal and state-level provision decisions than the single such sequence with local levels as the Stackelberg leader.

Second, an increasingly important element in the EU and similar settings is voter mobility across political and economic jurisdictions. Although systematically incorporating this element into our analysis is beyond the scope of the current paper, we would expect it to reinforce the results obtained above when the Tiebout hypothesis (Tiebout 1956) holds. On the one hand, all voters would, *ceteris paribus*, prefer lower taxes and so would be interested in moving into low-tax jurisdictions; on the other, the marginal utility of additional units of public good is higher for the wealthier voters, and so the benefits of remaining in jurisdictions providing higher quantities of public good is higher. Our expectation is, then, that the first-order effect of voter mobility would be the poor voters moving into lower-tax jurisdictions in expectation of the spillover effects from wealthier regions, and the wealthy voters remaining in the higher-demand regions — reproducing, and perhaps, deepening, the distributive tensions characterized above.

Finally, although they do not figure in our model, factors such as the possibility of unwanted federal programs, credit-claiming behavior, the geographic variation in the degree of interdependence among the state-members of the federation, etc. are also,

no doubt, important contributors to the explanation of the workings of federal systems and of the empirical phenomena that characterize them. In excluding these factors, our model should not be taken to imply that the causal mechanism we delineate is solely responsible for such phenomena, only that the elements of federal systems we isolate may — via the mechanism we analyze — be in and of themselves important contributing factors to their explanation. In this sense, the model we provide is useful precisely because it focuses on one particular causal mechanism, making clear its workings, its political effects, and the conditions under which it is operative. Other elements of federal systems may, without negating the mechanism we analyze, yield both complementary and opposing causal mechanisms. The weights that should be assigned to these mechanisms in explaining particular empirical phenomena cannot be determined without a systematic empirical investigation, which must be guided by the relationships between factors that are identified in these mechanisms.

APPENDIX

Lemma 1

Proof: Monotonicity. From the Kuhn–Tucker conditions, a vector of local tax rates t is an equilibrium of the local provision subgame for a given federal tax t^F if and only if it is a fixed point of the following application:

$$\text{for all } i,\ t_i = \max\{0, \tilde{t}_i\},$$

where $\tilde{t}_i$ is given by Equations 4, 5, and 6. Algebra shows that $H_i > H_j$ implies $\tilde{t}_i > \tilde{t}_j$. Hence $H_i > H_j$ implies $t_i \geq t_j$, which means that if the equilibrium exists, it must be monotonic in regional income.

Uniqueness. Let t^* and $t^{*\prime}$ be two equilibrium vectors of regional tax rates and let L^* and $L^{*\prime}$ be the corresponding equilibrium sets of regional providers. By monotonicity, either $L^* \subset L^{*\prime}$ or $L^{*\prime} \subset L^*$. Without loss of generality, suppose $L^* \subset L^{*\prime}$. Then, there exists a region i such that $t_i^{*\prime} > 0$ and $t_i^* = 0$. However, $L^* \subset L^{*\prime}$ implies that $\sum_{k \in L^{*\prime}} H^k > \sum_{k \in L^*} H^k$ and from Equation 6, $t^{*\prime} < t^*$ — a contradiction. Finally, through Equation 6, local tax rates are uniquely identified.

Existence. Define

$$\Phi(j, l) = (1 + as_L(l-1))b(1-t^F)H_j - abs_L(1-t^F)\sum_{k=1}^{l} H_k - (1-as_L)as_F t^F H^T.$$

Simple algebra shows that this function is decreasing in l and j. $\Phi(j, l) > 0$ means that if in equilibrium the l richest regions are local contributors then region j prefers to be a local contributor as well. $\Phi(l, l)$ is decreasing and for all l, $\Phi(l, l-1) = \Phi(l, l)$. Let l_0 be the greatest l such that $\Phi(l, l) \geq 0$. Then for all $j \leq l_0$, $\Phi(j, l_0) \geq 0$. Moreover, $\Phi(l_0+1, l_0) = \Phi(l_0+1, l_0+1)$ and hence $\Phi(l_0+1, l_0) < 0$, which proves that for all $j > l_0$, $\Phi(j, l_0) < 0$. Thus the l_0 richest regions provide locally. ■

Proposition 1

The following lemma is instrumental in establishing this proposition.

Lemma 2 *(1) For every region j, every citizen i in j has a most preferred federal tax rate, and the most preferred federal tax rate is the same for all i in j.*

(2) For any regions j and k such that $j \leq k$, for any citizen of j and any citizen of k, with incomes h_j^i and h_k^i, respectively, $u_k^i(\cdot) = \left(\frac{h_k^i}{h_j^i}\right)^a u_j^i(\cdot)$ for all $t^F \in [\hat{t}_j^F, 1]$.

Proof: (1) *Existence*. Substituting values from Equations 6 and 5 into Equation 3 yields utility as a function of t^F. Because this function is real-valued and its domain [0, 1] is compact, if it is continuous on [0, 1] then it has a global maximum. From Equation 7, $l(t^F)$ is not continuous in t^F on [0, 1]. Let $\hat{t}^F$ be a point of discontinuity for $l(t^F)$. From Equation 7,

$$(1 + as_L(l(\hat{t}^F) - 1))b(1 - \hat{t}^F)H_{l(\hat{t}^F)+1} = abs_L(1 - \hat{t}^F)\sum_{k=1}^{l(\hat{t}^F)} H_k + (1 - as_L)as_F\hat{t}^F H^T.$$

From Equation 6, this implies $\lim_{t^F \to \hat{t}^F} t_j^*(t^F)H_j = t_j^*(\hat{t}^F)H_j$; thus $t_j^*(t^F)$ is continuous at every point of discontinuity of $l(t^F)$ on [0, 1]. From Equation 6, it is evident that it is also continuous at every other $t^F \in [0, 1]$. Because the sum of continuous functions is continuous, $\sum t_k^*(t^F)H_k$, x_j, and y_j are continuous functions of t^F on [0, 1]. Because the product of continuous functions is continuous, the utility of citizen i in region j is a continuous function of t^F on [0, 1].

Identity. Substituting values from Equations 6 and 5 into Equation 3 and differentiating with respect to t^F, we can see that h_j^i can be factored out of the first order condition that determines the optimal value(s) of t^F.

(2) Consider the range of t^F where neither region j nor k, $j \leq k$, is a local provider, i.e., from Equation 9, $t^F \in [\hat{t}_j^F, 1]$. Given Equation 5, Equation 3 can be re-written to get the consumption level of the individual in j, u_j^i, when region j is not a local provider:

$$u_j^i(t^F) = ((1 - t^F)h_j^i)^a \left(t^F H^T s_F + \frac{s_L}{1 + as_L(l(t^F) - 1)} \times \left(b(1 - t^F)\sum_{k=1}^{l(t^F)} H_k - as_F t^F H^T l(t^F) \right) \right)^b \tag{A.1}$$

Observe that, since the amount of public good is independent of non-providers' characteristics, the amount of public good enjoyed by the individuals in both j and k is the same.

Re-arranging the factors, we can write $u_j^i(t^F) = (h_j^i)^a Z$ and $u_k^i(t^F) = (h_k^i)^a Z$, where

$$Z = (1-t^F)^a \left(t^F H^T s_F + \frac{s_L}{1 + a s_L (l(t^F) - 1)} \left(b(1 - t^F) \sum_{k=1}^{l(t^F)} H_k - a s_F t^F H^T l(t^F) \right) \right)^b .$$

Substituting, we get $u_k^i(\,\cdot\,) = \left(\frac{h_k^i}{h_j^i}\right)^a u_j^i(\,\cdot\,)$. By implication, if $\forall t^F \in [\hat{t}_j^F, 1]$, $t_j^*(t^F) = t_k^*(t^F) = 0$, then the citizens of j and the citizens of k have identical induced preferences over $[\hat{t}_j^F, 1]$. ■

Proof of Proposition 1

(1) Let $T^F(j)$ be the set of most preferred t^F for the citizens of j. (For notational convenience, we suppress the other arguments of $T^F(\,\cdot\,)$.) From part 1 of Lemma 2, $T^F(j) \neq \emptyset$. By assumption, $\forall t^F \in T^F(j)$, $t^F \geq \hat{t}^F$. From part 2 of Lemma 2, $\forall k > j$, the induced preferences over $[\,\hat{t}_j^F, 1]$ are identical for all citizens of j and k. From Lemma 1 and the definition of $\hat{t}_j^F$ (Equation 9), $\hat{t}_j^F > \hat{t}_k^F$ $\forall j, k$ s.t. $j < k \leq l_{\max}$. Thus, either (1) $T^F(j) \subseteq T^F(k)$ and $\forall t^F \in T^F(k) \backslash T^F(j)$, $t^F < \hat{t}_j^F$; or (2) $T^F(j) \cap T^F(k) = \emptyset$ and $\forall t^F \in T^F(k)$, $t^F < \hat{t}_j^F$. Thus, both min $T^F(j)$ and max $T^F(j)$ are nonincreasing in j.

(2) Let $T^F(m)$ represent the set of most preferred federal tax rates of the median-income region. Suppose $t_m^F \in T^F(m)$. From part 2 of Lemma 2, $\forall j \geq m$, t_m^F is strictly preferred by j to any $t^F > t_m^F$ s.t. $t^F \notin T^F(m)$ and is weakly preferred to any $t^F > t_m^F$ s.t. $t^F \in T^F(m)$. $\forall j \leq m$, if $t_m^F > \hat{t}_j^F$, then Lemma 1, the definition of $\hat{t}_j^F$, and part 2 of Lemma 2 imply that t_m^F is strictly preferred to any $t^F \in [\hat{t}_j^F, t_m^F) \backslash T^F(m)$ and weakly preferred to any $t^F \in [\,\hat{t}_j^F, t_m^F) \cap T^F(m)$. By assumption, j strictly prefers $\hat{t}_j^F$ to any $t^F < \hat{t}_j^F$, and thus t_m^F to any $t^F < \hat{t}_j^F$. If $t_m^F \leq \hat{t}_j^F$, then by assumption j strictly prefers t_m^F to any $t^F < t_m^F$.

(3) If $|T^F(m)| > 1$ and min $T^F(m) < \hat{t}_1^F$, then $\frac{\partial u_j}{\partial t^F} > 0$ for all $t^F < \hat{t}_j^F$ for all $j \leq l_{\max}$ and part 2 of Lemma 2 imply that there are more regions whose citizens strictly prefer max $T^F(m)$ to min $T^F(m)$ than there are regions whose citizens strictly prefer min $T^F(m)$ to max $T^F(m)$.

(4) Immediate from $\frac{\partial u_j}{\partial t^F} > 0$ for all $t^F < \hat{t}_j^F$ for all $j \leq l_{\max}$ and part 2 of this proposition. ■

Proposition 2

Proof: (1) By the definition of $\hat{t}_j^F$, for all $t^F < \hat{t}_j^F$, $t_j^*(t^F) > 0$. Thus, $\frac{\partial u_j}{\partial t^F} < 0$ for all $t^F < \hat{t}_j^F$ implies that for j, the return on additional local provision is higher than the

return on additional federal provision for all $t^F < \hat{t}_j^F$, given $t_{-j}^*(t^F)$. Because $s_L > 0$, $\frac{\partial}{\partial t^F}\left(\sum_{k=1}^{j=1} t_k(t^F)H_k\right)|_{t^F \in [\hat{t}_j^F, \hat{t}_{j-1}^F)} < \frac{\partial}{\partial t^F}\left(\sum_{k=1}^{j=1} t_k(t^F)H_k\right)|_{t^F \in [\hat{t}_{j+1}^F, \hat{t}_j^F)}$; thus, the return on additional local provision is higher than the return on additional federal provision $\forall t^F \in [\hat{t}_j^F, \hat{t}_{j-1}^F)$. From $t_j^*(t^F) = 0$ for $t^F \in [\hat{t}_j^F, \hat{t}_{j-1}^F)$, j obtains a higher return on additional private good than on additional local provision of the public good for $t^F \in [\hat{t}_j^F, \hat{t}_{j-1}^F)$, given $t_{-j}^*(t^F)$. Thus, j obtains a higher return on private good than on additional federal provision of the public good for $t^F \in [\hat{t}_j^F, \hat{t}_{j-1}^F)$ and $\frac{\partial u_j}{\partial t^F} < 0 \; \forall t^F \in [\hat{t}_j^F, \hat{t}_{j-1}^F)$.

From part 2 of Lemma 2, $\frac{\partial u_j}{\partial t^F} < 0 \; \forall t^F \in [\hat{t}_j^F, \hat{t}_{j-1}^F)$ implies $\frac{\partial u_k}{\partial t^F} < 0 \; \forall t^F \in [\hat{t}_j^F, \hat{t}_{j-1}^F)$ $\forall k \geq j$. By assumption, $\frac{\partial u_j}{\partial t^F} < 0$ for $t^F \in [\hat{t}_{j+1}^F, \hat{t}_j^F) \; \forall j \leq \hat{l}$; thus $\frac{\partial u_k}{\partial t^F} < 0 \; \forall t^F \in [\hat{t}_k^F, 1]$ $\forall k$. If $\frac{\partial u_j}{\partial t^F} > 0$ at $t^F < \hat{t}_j^F$, then $\frac{\partial u_k}{\partial t^F} > 0 \; \forall t^F < \hat{t}_k^F \; \forall k \geq j$.

(2) Once again, let $T^F(j)$ be the set of most preferred t^F for the citizens of j, and, for notational convenience, suppress the other arguments of $T^F(\cdot)$. From part 1 of Lemma 2, $T^F(j) \neq \emptyset$. From part 1 of this proposition, $T^F(j) = \{0\} \; \forall j \leq \hat{l}$, and $\min T^F(\hat{l}+1) > \hat{t}_{\hat{l}+1}^F$. By assumption, $\forall j > \hat{l}$, $\forall t^F \in T^F(j)$, $t^F \geq \hat{t}_j^F$. From part 2 of Lemma 2, $\forall k > j$, the induced preferences over $[\hat{t}_j^F, 1]$ are identical for all citizens of j and k. From Lemma 1 and the definition of $\hat{t}_j^F$ (Equation 9), $\hat{t}_j^F > \hat{t}_k^F \; \forall j, k$ s.t. $j < k \leq l_{\max}$. Thus, either (1) $T^F(j) \subseteq T^F(k)$ and $\forall t^F \in T^F(k) \backslash T^F(j)$, $t^F < \hat{t}_j^F$; or (2) $T^F(j) \cap T^F(k) = \emptyset$ and $\forall t^F \in T^F(k)$, $t^F < \hat{t}_j^F$. Thus, both $\min T^F(j)$ and $\max T^F(j)$ are nonincreasing in j.

(3) Suppose $m \leq \hat{l}$. From part 1 of this proposition, $T_j^F = \{0\} \; \forall j \leq \hat{l}$. Thus a majority strictly prefers $t^F = 0$ to any $t^F > 0$.

Suppose $m > \hat{l}$, and let $t_{m+\hat{l}}^F \in T^F(m+\hat{l})$. From part 1 of this proposition, every citizen of $j \leq \hat{l}$ strictly prefers $t_{m+\hat{l}}^F$ to any $t^F > t_{m+\hat{l}}^F$. From part 2 of Lemma 2, every citizen of $j \geq m + \hat{l}$ strictly prefers $t_{m+\hat{l}}^F$ to any $t^F \in (t_{m+\hat{l}}^F, 1] \backslash T^F(m+\hat{l})$ and weakly prefers $t_{m+\hat{l}}^F$ to any $t^F \in T^F(m+\hat{l})$. Thus, a majority prefers $t_{m+\hat{l}}^F$ to any $t^F > t_{m+\hat{l}}^F$.

For $j \in \{\hat{l}+1, \ldots, m+\hat{l}-1\}$ s.t. $\hat{t}_j^F \geq t_{m+\hat{l}}^F$, part 1 of this proposition implies j strictly prefers $t_{m+\hat{l}}^F$ to any $t^F < t_{m+\hat{l}}^F$. For $j \in \{\hat{l}+1, \ldots, m+\hat{l}-1\}$ s.t. $\hat{t}_j^F < t_{m+\hat{l}}^F$, part 2 of Lemma 2 implies that j strictly prefers $t_{m+\hat{l}}^F$ to any $t^F \in [\hat{t}_j^F, t_{m+\hat{l}}^F] \backslash T^F(m+\hat{l})$ and weakly prefers it to any $t^F \in [\hat{t}_j^F, t_{m+\hat{l}}^F] \cap T^F(m+\hat{l})$. By assumption, j strictly prefers $\hat{t}_j^F$ to any $t^F < \hat{t}_j^F$. Thus, a majority prefers $t_{m+\hat{l}}^F$ to any $t^F < t_{m+\hat{l}}^F$.

(4) From part 1 of this proposition and part 2 of Lemma 2 , $\frac{\partial u_j}{\partial t^F} < 0 \, \forall \, t^F > \hat{t}_{\hat{l}}^F \; \forall j \in N$. Thus, $\forall t_{m+\hat{l}}^F \in T^F(m+\hat{l})$, $t_{m+\hat{l}}^F < \hat{t}_{\hat{l}}^F$. ■

Proposition 3

Proof: Suppose $H_j < \frac{\gamma}{n} H^T$. Then from Equation 3, $\frac{\partial u_j^i}{\partial t_j} < \frac{\partial u_j^i}{\partial t^F}$; thus the optimal local tax rate for i in j is $t_j = 0$. From Equations 3, 4, and $s_L = 0$, the consumption of i in j is strictly increasing in t^F $\forall t^F < b$ and strictly decreasing in t^F $\forall t^F \geq b$; thus $t_j^{F*} = b$.

Suppose $H_j > \frac{\gamma}{n} H^T$. Then $\frac{\partial u_j^i}{\partial t_j} > \frac{\partial u_j^i}{\partial t^F}$; thus the optimal federal tax rate for i in j is $t_j^{F*} = 0$. From Equations 3, 4, and $s_L = 0$, the consumption of i in j is strictly decreasing in t^F. It follows that $\forall j \in N$, the preferences of all i in region j are identical and single-peaked with respect to t^F. Given that all regions have the same population, the application of the median-voter theorem yields the conditions in the statement of the proposition. From Equation 3 and $s_L = 0$, if $t^F = 0$ then $t_j^* = b$ $\forall j \in N$. ■

Proposition 4

Proof: (1) From Proposition 3, $\gamma < \frac{nH_m}{H^T}$ and $s_L = 0$ imply that $t^{F*} = 0$ and $t_j^* = b$ $\forall j \in N$. If $s_L > 0$, $t^F \geq 0 = t^{F*}$. If $t^F = 0$, then from Equation 6, higher s_L implies lower t_j $\forall j > 1$. If $t^F > 0$, then from Equation 6, higher s_L implies lower t_j $\forall j \in N$.

(2) From Proposition 3, $\gamma > \frac{nH_m}{H^T}$ and $s_L = 0$ imply that $t^{F*} = b$ and $t_j^* = 0$ $\forall j \in N$. If $s_L > 0$, $t_j \geq 0$ $\forall j \in N$. If $l(b) = 0$ then $\frac{\partial u_j}{\partial t^F} < 0$ $\forall t^F > b$. If $l(b) > 0$ then from Lemma 1 and Equation 9, $\hat{t}_1^F > b$. But $\hat{t}_1^F > b$ iff $s_F H^T < H_1$. Hence $\frac{\partial u_1}{\partial t^F} < 0$ $\forall t^F \in [0, 1]$. Because other regions are obtaining spillovers from one's local provision, they prefer $t^{F**} < b$ so that they can obtain more of the (complementary) private good, and $\frac{\partial u_j}{\partial t^F} < 0$ $\forall t^F > t^{F**}$. Thus b is strictly preferred to any $t^F > b$ $\forall j \in N$. Thus, the Pareto set is bounded above by b. ■

Proposition 5

Proof: (1) Denote the outcome of a federation $(t^{F*}, t^*(t^{F*}))$, of confederation $(0, t^*(0))$, and of complete centralization $(b, 0)$. By definition of t^{F*}, $(t^{F*}, t^*(t^{F*}))$ is preferred to $(b, t^*(b))$ by all citizens in a majority of regions, and is strictly preferred if $t^{F*} \neq b$. For every citizen of every region, $(b, t^*(b))$ is at least as good as $(b, 0)$, and it is strictly better if $t^*(t^F) > 0$. Thus a majority always prefers federation to complete centralization. By definition of t^{F*}, $(t^{F*}, t^*(t^{F*}))$ is preferred to $(0, t^*(0))$ by all citizens of a majority of regions, and is strictly preferred if $t^{F*} \neq 0$. Thus a majority always prefers federation to confederation.

(2) From part 2 of Proposition 1, $t_m^{F*} \in T^F(m)$ and $t = t^*(t_m^{F*})$ are the equilibrium outcomes in a federation. From part 1 of Proposition 1, citizens of $\{m, \ldots, n\}$ prefer $t^F = t_m^{F*}$ and $t = t^*(t_m^{F*})$ to $t^F = b$ and $t = t^*(b) = 0$, and strictly prefer the former if $t_m^{F*} \neq b$. From part 4 of Proposition 1, $t_m^*(t_m^{F*}) = 0$. From Lemmata 1 and 2 (part 2), citizens of $\{1, \ldots, m\}$ strictly prefer $t^F = t_m^{F*}$ and $t = t^*(t_m^{F*})$ to $t^F = 0$ and $t = t^*(0)$.

(3) From part 3 of Proposition 2, $t^F = t^{F*}_{m+\hat{l}}$ and $t = t^*(t^{F*}_{m+\hat{l}})$ are the equilibrium outcomes in a federation if $m > \hat{l}$, and $t^{F*} = 0$ and $t = t^*(0)$ otherwise. From parts 1 and 2 of Proposition 2, if $m > \hat{l}$, citizens of $\{\hat{l}+1, \ldots, m+\hat{l}\}$ prefer $t^F = t^{F*}_{m+\hat{l}}$ and $t = t^*(t^{F*}_{m+\hat{l}})$ to $t^F = 0$ and $t = t^*(0)$, and strictly prefer the former if $t^{F*}_{m+\hat{l}} \neq 0$. If $m \leq \hat{l}$, they are indifferent. If $m > \hat{l}$ and $t^{F*}_{m+\hat{l}} \neq 0$, then $t^*_{m+\hat{l}}(t^{F*}_{m+\hat{l}}) = 0$. From Lemmata 1 and 2 (part 2), citizens of $\{m+\hat{l}, \ldots, n\}$ strictly prefer $t^F = t^{F*}_{m+\hat{l}}$ and $t = t^*(t^{F*}_{m+\hat{l}})$ to $t^F = b$ and $t = t^*(b) \neq 0$. From part 1 of Proposition 2, citizens of $\{1, \ldots, \hat{l}\}$ also strictly prefer $t^F = t^{F*}_{m+\hat{l}}$ and $t = t^*(t^{F*}_{m+\hat{l}})$ to $t^F = b$ and $t = t^*(b) \neq 0$. All citizens in $\{1, \ldots, \hat{l}\} \cup \{m+\hat{l}, \ldots, n\}$ strictly prefer the latter to $t^F = b$ and $t = 0$. ■

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