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Economic Theory

The political economy of redistribution under democracy*

Jess Benhabib[1] and Adam Przeworski[2]

[1] Department of Economics, New York University, New York, NY 10003, USA
(e-mail: jess.benhabib@nyu.edu)
[2] Department of Politics, New York University, New York, NY 10003, USA
(e-mail: adam.przeworski@nyu.edu)

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Summary. We ask what redistributions of income and assets are feasible in a democracy, given the initial assets and their distribution. The question is motivated by the possibility that if redistribution is insufficient for the poor or excessive for the rich, they may turn against democracy. In turn, if no redistribution simultaneously satisfies the poor and the wealthy, democracy cannot be sustained. Hence, the corollary question concerns the conditions under which democracy is sustainable. We find that democracies survive in wealthy societies. Conditional on the initial income distribution and the capacity of the poor and the wealthy to overthrow democracy, each country has a threshold of capital stock above which democracy survives. This threshold is lower when the distribution of initial endowments is more equal and when the revolutionary prowess of these groups is lower. Yet in poor unequal countries there exist no redistribution scheme which would be accepted both by the poor and the wealthy. Hence, democracy cannot survive. As endowments increase, redistribution schemes that satisfy both the poor and the wealthy emerge. Moreover, as capital stock grows the wealthy tolerate more and the poor less redistribution, so that the set of feasible redistributions becomes larger. Since the median voter prefers one such scheme to the dictatorship of either group, democracy survives.

Keywords and Phrases: Redistribution, Democracy, Dictatorship.

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Correspondence to: J. Benhabib

1 Introduction

We ask what redistributions of income and assets are feasible in a democracy, given the initial assets, their distribution, and some features of the political environment. The question is motivated by the possibility that if the redistribution is insufficient for the poor or excessive for the wealthy, they may turn against democracy. Moreover, if no redistribution simultaneously satisfies the poor and the wealthy and if either group has any chance to establish its dictatorship, democracy cannot be sustained. Hence, the corollary question concerns the conditions under which democracy is sustainable.

In a simple model of production and accumulation, where agents are heterogeneous in their initial wealth, the median voter chooses a sequence of redistributive tax rates. Decisions to save are endogenous, which means that they depend on future tax rates and thus future growth rates. We assume that decisions about redistribution are made in elections and show that no majority coalition of poor and wealthy leaves both better off than the decision of the median voter (see Theorem 2). Moreover, given a linear redistribution scheme, the identity of the median voter does not change over time. Hence, the same median voter is decisive at each time with regard to the entire path of future redistribution. To be accepted, however, the decision of the median voter must leave the poor and the wealthy at least as well off as they expect to be if they sought to establish their respective dictatorships, where they would choose their best redistribution scheme unilaterally. Hence, if democracy is to survive, any redistribution must satisfy the constraints originating from the possibility of rebellion. While we do not solve for the optimal redistribution scheme of the median voter, we investigate these constraints.

We find that democracies survive in wealthy countries. Conditional on the initial income distribution and the capacity of the poor and the wealthy to overthrow democracy, each country has a threshold of capital stock above which democracy survives. This threshold is lower when the distribution of initial endowments is more equal and when the revolutionary prowess of these groups is lower. In the extreme, democracy survives at any income if its distribution is sufficiently egalitarian or if neither group can establish dictatorship. Yet in poor unequal countries there exists no redistribution scheme that would be accepted both by the poor and the wealthy. Hence, democracy cannot survive. As endowments increase, redistribution schemes that satisfy both the poor and the wealthy emerge. Moreover, as capital stock grows the wealthy tolerate more and the poor less redistribution, so that the set of feasible redistributions becomes larger. Since the median voter prefers one such scheme to the dictatorship of either group, the outcome of electoral competition is obeyed by everyone and democracy survives.

These results are driven by an assumption about preferences. The cost of dictatorship is the loss of freedom. We follow the argument of Sen (1991) that people suffer disutility when they are not free to live the lives of their choosing. Specifically, even if we allow that the losers in the conflict over dictatorship suffer more, we also allow that everyone may dislike dictatorship to some extent. This preference against dictatorship (or for democracy) is independent of income: as Dasgupta (1993, p. 47) put it, the view that the poor do not care about freedoms associated with democracy "is a piece of insolence that only those who don't suffer from their lack seem to entertain" (see also Sen, 1994). Yet since the marginal utility of income declines in income, while the dislike of dictatorship is independent of income, at a sufficiently high income the additional gain that would accrue from being able to dictate tax rates becomes too small to overcome the loss of freedom.[1] However, this straightforward intuition valid for a static model is a significant oversimplification. First, to show that when income is low, there exists no redistribution profile over time that can sustain democracy, we must rule out the possibility that some redistribution sequence may generate future growth and well-being sufficient to forestall revolt today. Second, to show that when incomes are high there always exists redistribution profiles that sustain democracy, we must demonstrate that democracy is sustainable not only today but at every moment along the growth path, which of course depends on the redistribution scheme that is implemented. The prospects of growth that depends on current as well as future redistributions complicate the analysis, and therefore the proofs are relegated to the appendix.

Explanations in terms of preferences are justifiably suspect. We are driven to it because of two facts discussed in the next section: democracy is more likely, indeed certain, to survive in wealthy countries, and no plausible rival hypothesis eliminates the role of income in sustaining democracy. Hence, income matters and income is not a proxy for something else.[2]

2 Per capita income and the survival of democracy

The probability that a democracy[3] would survive rises steeply in per capita income. Between 1950 and 1999, the probability that a democracy would die during any year in countries with per capita income under $1,000 (1985 PPP dollars) was 0.0845, so that one in twelve died. In countries with incomes between $1,001 and $3,000, this probability was 0.0362, for one in twenty-eight. Between $3,001 and $6,055, this probability was 0.0163, one in sixty-one. And no democracy ever fell in a country with per capita income higher than that of Argentina in 1975, $6,055. This is a startling fact, given that throughout history about seventy democracies collapsed in poorer countries, while thirty-seven democracies spent over 1000 years in more developed countries and not one died.

Is income a proxy for something else? Following the classical book of Lipset (1960), an enormous literature sought to explain the observed prevalence of democracies in the developed countries and their paucity in the less developed ones. This literature did not distinguish the factors that lead to the emergence of democracy

[1] For a general game-theoretic approach to the dependence of social conflict and cooperation on wealth see Benhabib and Rustichini (1996).

[2] An alternative possibility may be to directly impose a cost of revolt. If such costs are proportional to income, we can show that we loose the income dependence of the sustainability of democracy. If the costs are fixed, the dependence on income, if it exists, will go the wrong way: revolt becomes more likely in rich countries than in poor.

[3] Democracy is defined here as a regime in which incumbents lose elections and leave office if they lose, but alternative definitions generate the same conclusion. The data set we use to identify democracies can be found at http://pantheon.yale.edu/˜jac236/.

from those that cause it to survive once established, and these factors are different (Przeworski and Limongi, 1997; Przeworski et al., 2000; Przeworski, 2004). Yet several arguments offered in this literature apply to the role of factors other than per capita income in sustaining democracy.

In Table 1 we show probit regressions in which the dependent variable are deaths of democracies and the column headings specify the rival hypotheses. The conclusion is clear: While some of the rival factors do matter in the presence of income,[4] none of them eliminates the crucial role of income in sustaining democracy.

The most obvious candidate for a rival explanation is education. We take the years of education of an average member of the labor force (from Bhalla, 1994), and learn that education plays some additional rule in sustaining democracy, but it does not reduce the importance of income.

Coser (1956) argued, and many sociologists following him agreed, that democracy is easier to sustain if a country has a complex social structure. Coser's argument was that when social structure is complex, cleavages overlap, rather than pit one large group against another (see also Ross, 1960). We test this argument by calculating labor force fractionalization, that is, the probability that two random members of the labor force do not work in the same of nine one-digit sectors (This variable is called COMPLEXITY, from Kim, 2004). Complexity strongly reduces the probability that a democracy would die, but income still matters.[5]

In 1860 J.S. Mill (1991, p. 230) had already argued that democracy is more difficult to sustain in countries ridden with ethno-linguistic divisions. With all the caveats about measuring ethnicity across cultures, we take the index of ethno-linguistic fractionalization, ELF60 (from Easterly and Levine, 1997). Again, while democracy is less likely to die in the more homogeneous countries, the role of income continues to be important.

The theory of political obligation asserts that people feel duty to accept the results of a process in which they participated. Democracy, the argument goes, requires the "willingness to accept outcomes of as yet undetermined content" (Lamounier, 1979, p. 13). We find that electoral participation, measured as voters as a proportion of adults,[6] plays no role in addition to income.

Unfortunately, data on inequality are scarce, unreliable, and not easily compared across countries. All we can do is to take the high quality data from Deininger and Squire (1996) and extend them by attributing the same degree of inequality to two years before and after each observation. The resulting sample is still heavily biased in favor of wealthy countries, an additional reason to take the results with a grain of salt. With all these caveats, income distribution appears not to matter in regression, while income continues to do so. When, however, the observations of inequality are dichotomized by $GINI = 0.35$, the odds of democracy falling are 4.7 higher in the more unequal countries. The difference is even more pronounced when the

[4] For income we take GDP/cap in thousands of 1985 PPP, from PWT, extended by Easterly, http://nyu.edu/fas/institute/dri/Easterly.

[5] To measure complexity, we also used the standard deviation of the proportions working in the nine-digit sectors, with the same result. In addition, we tested the impact of the proportion of labor force in agriculture, which appears to play no role.

[6] The data is from IDEA, http://www.idea.int/

observations are dichotomized by $Q1/Q5 = 9$, since no democracy fell in the cases more equal by this criterion.[7] Hence, there are reasons to suspect that democracy is more brittle in unequal societies.

In the end, then, none of the rival hypotheses for which data exist eliminates the role of income. But why would income matter, independently of everything else? Lipset (1960, p. 51) offered a hypothesis which we find convincing:

> The general income level of a nation also affects its receptivity to democratic norms. If there is enough wealth in the country so that it does make too much difference whether some redistribution takes place, it is easier to accept the idea that it does not matter greatly which side is in power. But if loss of office means serious losses for major groups, they will seek to retain office by any means available.

Table 1. Transitions to dictatorship, as a function of per capita income and rival variables

	None	Education	Complexity	ELF	Participation	Inequality
Constant	−13066	−0.7771	2.5750	−1.0137	−0.7488	−0.8037
	(0.1161)	(0.2002)	(1.1970)	(0.1528)	(0.4334)	(0.6409)
GDP/cap	−0.2262	−0.1820	−0.1959	−0.1755	−0.2273	−0.2734
	(0.0426)	(0.0633)	(0.1103)	(0.0404)	(0.0950)	(0.0867)
Rival		−0.0816	−5.5095	−0.6373	−0.7150	−0.0050
		(0.0504)	(1.7709)	(0.2518)	(0.7344)	(0.0140)
N	2423	1085	1201	2234	581	771
TDA	47	30	10	46	12	14

3 The economy

The output y_t at time t is produced with capital k_t according to the linear production function $y_t = rk_t$ with $r > 1$. There are n agents, indexed by i. In the initial period t_0, they each own a share of the capital stock, $v^i_{t_0}$, with $\sum_{i=1}^{n} v^i_{t_0} = 1$. The shares of capital owned by agent i at time s are denoted by v^i_s, and the capital stock of this agent is $k^i_s = v^i_s k_s$. Taxes are redistributive. The tax rate on assets at time t is denoted as τ_t, and in each period tax collections are distributed to the agents in proportion n^{-1} of the total. Hence, the tax rate uniquely determines the extent of redistribution. The post-redistribution income is $y^i_t = (1-\tau_t) r v^i_t k_t + n^{-1}\tau_t r k_t = (1-\tau_t) r k^i_t + n^{-1}\tau_t r k_t$, where we assume that $\tau_s \in [0, \tilde{\tau}]$, where $\tilde{\tau} \leq 1$ for all s.

Agents use their capital to produce income, pay proportional taxes on assets, and consume. They have CRRA preferences and the value function is

$$V^i(k^i_t) = \max_{c_i} \frac{(c^i_t)^{1-\sigma} - 1}{(1-\sigma)} + (1-\sigma)^{-1} b + \beta V^i(r(1-\tau_t)k^i_t + q^i_t - c^i_t)$$

[7] These numbers are taken from Przeworski et al. (2000, Table 2.15).

where q_s^i is the redistributive transfer agent i receives at time s. The constant term $(1-\sigma)^{-1}b$ is zero under a democratic regime and non-positive under a dictatorial regime. The utility function is discussed further in section 4.

The first-order condition of the agent for an interior solution is:

$$c_{t+1}^i = c_t^i(\beta r(1-\tau_t))^{\frac{1}{\sigma}} \tag{1}$$

Forward iteration of the budget constraint $k_{t+1}^i = r(1-\tau_t)k_t^i - (c_t^i - q_t^i)$ implies, provided $\tau_{t+s} < 1$ for $s = 1, 2...$, that:

$$\begin{aligned} & c_0^i - q_0^i + \sum_{j=1}^{t}(c_j^i - q_j^i)\left[\prod_{s=1}^{j}(r(1-\tau_s))^{-1}\right] \\ & + \prod_{s=1}^{t}(r(1-\tau_s))^{-1}k_{t+1}^i = (r(1-\tau_0))k_0^i \end{aligned} \tag{2}$$

From the no Ponzi and transversality conditions,

$$\lim_{t\to\infty}\left(\prod_{s=1}^{t}(r(1-\tau_s))^{-1}\right)k_{t+1}^i = 0. \tag{3}$$

Iterating 1 forward, substituting into 2, using 3 and solving for c_0^i, we obtain:

$$c_t^i = \lambda_t^i\left((r(1-\tau_t))k_t^i + \left(q_t^i + \sum_{j=t+1}^{\infty} q_j^i \prod_{s=t+1}^{j}\left((r(1-\tau_s))^{-1}\right)\right)\right)$$

$$\lambda_t^i = \left(1 + \sum_{j=t+1}^{\infty}\prod_{s=t+1}^{j}\beta^{\frac{1}{\sigma}}(r(1-\tau_s))^{\frac{1-\sigma}{\sigma}}\right)^{-1}$$

The following assumptions assure that λ_t is bounded away from zero and one, that is $0 < \lambda^l \leq \lambda_t \leq \lambda^h < 1$ for all $t \geq t_0$. Note that the assumption places no further restrictions on the tax rate in the initial period t_0.

Assumption 1 $\tau_s \leq \tilde{\tau}_s < 1$ *for all* $s = t_0+1, t_0+2, \ldots$ *where* t_0 *is the initial period.*

Assumption 2 $\beta^{\frac{1}{\sigma}}(r(1-\tilde{\tau}_s))^{\frac{1-\sigma}{\sigma}} < 1$, $\beta^{\frac{1}{\sigma}}r^{\frac{1-\sigma}{\sigma}} < 1$.

3.1 Endogenizing transfers

Without loss of generality we define growth rates as $g_s = \frac{k_s}{k_{s-1}}$ so that $k_t = (\prod_{s=t_0+1}^{t} g_s)k_{t_0}$. Let the transfers be defined as $q_t^i = n^{-1}\tau_t r k_t = n^{-1}\tau_t r\left(\prod_{s=t_0+1}^{t} g_s\right)k_{t_0}$. Using the definition of the growth rates and transfers,

$$c_t^i = \lambda_t\left((1-\tau_t)\,rk_t^i + n^{-1}\left(\tau_t + \sum_{j=t+1}^{\infty}\tau_j\prod_{s=t+1}^{j}g_s(r(1-\tau_s))^{-1}\right)rk_t\right) \tag{4}$$

Each agent's budget constraint implies

$$k_{t+1}^{i}=(1-\tau_{t})\,rk_{t}^{i}+n^{-1}\tau_{t}rk_{t}-c_{t}^{i}=(1-\tau_{t})\,(1-\lambda_{t})\,rk_{t}^{i}$$
$$+\left(\tau_{t}\,(1-\lambda_{t})-\lambda_{t}\left(\sum_{j=t+1}^{\infty}\tau_{j}\prod_{s=t+1}^{j}g_{s}(r\,(1-\tau_{s}))^{-1}\right)\right)n^{-1}rk_{t} \tag{5}$$

Summing over agents,

$$k_{t+1}=\sum_{i=1}^{n}k_{t+1}^{i}=r\left(1-\lambda_{t}-\lambda_{t}\left(\sum_{j=t+1}^{\infty}\tau_{j}\prod_{s=t+1}^{j}g_{s}(r\,(1-\tau_{s}))^{-1}\right)\right)k_{t}$$

Therefore, the equilibrium relation describing growth rates for our redistributive economy is:

$$g_{t+1}=r\left(1-\lambda_{t}-\lambda_{t}\left(\sum_{j=t+1}^{\infty}\tau_{j}\prod_{s=t+1}^{j}g_{s}(r\,(1-\tau_{s}))^{-1}\right)\right) \tag{6}$$

Note right away that if we confine ourselves to a tax sequence that remains constant after the first period, $\tau_s=\tau$ for $s>t_0$, the solution of the above equation is simply

$$g_{s}=r\,(1-\lambda_{s})\,(1-\tau) \tag{7}$$

3.2 Dynamics of shares $\frac{k_t^i}{k_t}$

To characterize the equilibrium dynamics of the economy, to be used in the next section, we first describe the evolution of asset shares from an initial distribution, given the redistribution scheme. If we express asset shares as $v_t^i=\frac{k_t^i}{k_t}$, then equation 5 yields:

$$v_{t+1}^{i}=\left((g_{t+1})^{-1}\,r\,(1-\lambda_{t})\,(1-\tau_{t})\right)v_{t}^{i}$$
$$+\,(g_{t+1})^{-1}\,rn^{-1}\left(\tau_{t}\,(1-\lambda_{t})-\lambda_{t}\sum_{j=t+1}^{\infty}\tau_{j}\prod_{s=t+1}^{j}g_{s}(r\,(1-\tau_{s}))^{-1}\right) \tag{8}$$

where the law of motion for g_{t+1} is given by (6).

Note that the last term on the right-hand side of equation 8 describing the evolution of shares is independent of i, while the first term is proportional to the endowment v_t^i. Therefore even if shares change, their ordering is unaffected, and the median voter will be the same agent in each period. Note that if $\tau_s=\tau$, then $\lambda_s=\lambda$ and, using (7) and (8), we can solve for the evolution of shares as $v_{t+1}^i=g^{-1}r\,(1-\tau)\,(1-\lambda)\,v_t^i=v_t^i$.[8]

[8] Bertola (1993) derives a vey similar result where he allows for differentially productive labor by introducing increasing returns to scale.

4 A political model of the sustainability of democracy

We study the political constraints on the median voter that prevent him from implementing his preferred tax scheme: if the median voter is poorer than the average, he prefers the tax sequence $\tau_{t_0} = 1$ and $\tau_{t_0+s} = 0$ for $s = 1, 2, \ldots$, whereas if the median voter is richer than the average, he wants $\tau_{t_0+s} = 0$ for $s = 0, 1, 2, \ldots$. We will proceed under the assumption that the median voter is poor.

There is a wealthy pivotal agent w, whose share of initial capital, larger than the average share, is denoted by $v^w_{t_0}$. He prefers the tax scheme $\tau_{t_0+s} = 0$ for $s = 0, 1, 2, \ldots$. In turn, the poor pivotal agent, p, has an initial share of capital smaller than or equal to the share of the median voter: $v^p_{t_0} \leq v^i_{t_0}$. This agent wants $\tau_{t_0} = 1$ and $\tau_{t_0+s} = 0$ for $s = 1, 2, \ldots$, a complete redistribution resulting in equal shares in the first period, followed by zero taxes afterwards. If in any period the pivotal agents receive less discounted utility under democracy than the expected value of a revolt aimed at instituting an authoritarian regime, they will revolt.

Assumption 3 *Let t_a be the first period in which an authoritarian regime is established. Then $\tau_{t_a} \in [0, 1]$, and $\tau_s \in [0, \tilde{\tau}_s]$, where $\tilde{\tau}_s < 1$ for all $s > t_a$.*

This assumption allows the pivotal agent who initiates a successful revolt to reset initial taxes when she reverts to an authoritarian regime. We assume, for simplicity, that once established, an authoritarian regime lasts forever. The success of a revolt, however, is probabilistic. If the median voter chooses a tax sequence under which the wealthy pivotal agent finds it optimal to revolt, the poor pivotal agent will also want to revolt, rather than passively accept a right-wing rule with zero taxes and bear the costs of the autocratic regime: it is better to suffer autocracy under one's preferred tax sequence than under the tax sequence set by the other class. So we assume that if the tax sequence chosen by the median voter induces the right-wing wealthy agents to revolt, the revolution will succeed with probability π, but the left-wing poor agents will counter-revolt and may come to power with probability $1 - \pi$. Similarly, if the tax sequence chosen by the median voter induces the left to revolt, the revolution will succeed with probability $1 - \pi'$, but the right will counter-revolt and may come to power with probability π'. Of course it may be reasonable to assume that it makes no difference whether the right or the left initiates the revolution, in which case we can set $\pi = \pi'$. Democracy is sustained if the median voter accommodates the right and the left by setting taxes that deter both of the pivotal agents from attempting to establish an authoritarian regime.

We also assume that the agents suffer a loss of utility under dictatorship. The utilities of the agents are given by $(1 - \sigma)^{-1}(c^{1-\sigma} - 1) + (1 - \sigma)^{-1}b^j$, where $j = s, u$, and $\sigma > 1$.[9] We set the parameters $b^u > 0$, $b^s \geq 0$ under dictatorship,

[9] For the case $\sigma < 1$, we must set $b^u < 0$, $b^s \leq 0$ so that the costs of reverting to dictatorship are positive. However in this case the proof of Theorem 1 given in the appendix may fail if σ is too low. The intuition for this is that there may be enough growth in the system so that there is always a healthy gain to imposing one's preferred tax rate which, despite the mild concavity of the utility function, can outweigh the disutility of dictatorship and makes democracy unsustainable. Theorem 1 shows that this possibility is ruled out if $\sigma > 1$, which is the case that empirical evidence supports. Modifying the algebra, we can also show that the proofs of Theorem 1 go through with log utility,that is if $\sigma = 1$.

and we set them to zero under democracy, where b^s is the per period utility cost to the group that successfully establishes a dictatorship, and b^u is the per period utility cost to the other group. We also assume the losers suffer at least as much under autocracy than the winners, and perhaps more.

Assumption 4 *$\sigma > 1$, and $b^u \geq b^s \geq 0$, $b^u > 0$ under dictatorship, while $b^u = b^s = 0$ under democracy.*

To preserve democracy starting at time t_0, the median voter M must set the sequence of taxes τ_{t_0+s}, $s = 0, 1, ...$ to maximize his value function,[10] for all $t = t_0 + s$,

$$V\left(k_{t_0}, k_{t_0}^M, t_0\right) = Max_{\{\tau_t\}_{t=t_0}^{\infty}} \frac{1}{(1-\sigma)}$$

$$\left(\begin{array}{c}\lambda_{t_0}^{1-\sigma}\left(\left((1-\tau_{t_0})\, v_{t_0}^M + n^{-1}\tau_{t_0} + n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j \prod_{s=t_0+1}^{j} g_s\left(r\left(1-\tau_s\right)\right)^{-1}\right) r k_{t_0}\right)^{1-\sigma} \\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}\left(\beta r\left(1-\tau_s\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right) - (1-\beta)^{-1}\end{array}\right)$$

subject to, for all $t \geq t_0$,

$$\begin{aligned} 0 \leq & \left(\begin{array}{c}(1-\sigma)^{-1}\lambda_t^{1-\sigma}\left(\left((1-\tau_t) r v_t^w + n^{-1}\tau_t + n^{-1}\sum_{j=t+1}^{\infty}\tau_j \prod_{s=t+1}^{j} g_s\left(r(1-\tau_s)\right)^{-1}\right) r k_t\right)^{1-\sigma} \\ \cdot\left(1+\sum_{n=t+1}^{\infty}\beta^{n-t}\left(\prod_{s=t+1}^{n}\left(\beta r\left(1-\tau_s\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right) - (1-\sigma)^{-1}(1-\beta)^{-1}\end{array}\right) \\ & -\pi(1-\sigma)^{-1}\left(\tilde{\lambda}_t^{-\sigma}\left(r v_t^w\right)^{1-\sigma} k_t^{1-\sigma} - (1-\beta)^{-1} + B^s\right) \\ & -(1-\pi)(1-\sigma)^{-1}\left(\tilde{\lambda}_t^{-\sigma}\left(r n^{-1}\right)^{1-\sigma} k_t^{1-\sigma} - (1-\beta)^{-1} + B^u\right) \end{aligned}$$

and

$$\begin{aligned} 0 \leq & \left(\begin{array}{c}(1-\sigma)^{-1}\lambda_t^{1-\sigma}\left(\left((1-\tau_t) r v_t^p + n^{-1}\tau_t + n^{-1}\sum_{j=t+1}^{\infty}\tau_j \prod_{s=t+1}^{j} g_s\left(r(1-\tau_s)\right)^{-1}\right) r k_t\right)^{1-\sigma} \\ \cdot\left(1+\sum_{n=t+1}^{\infty}\beta^{n-t}\left(\prod_{s=t+1}^{n}\left(\beta r\left(1-\tau_s\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right) - (1-\sigma)^{-1}(1-\beta)^{-1}\end{array}\right) \\ & -(1-\pi')(1-\sigma)^{-1}\left(\tilde{\lambda}_t^{-\sigma}\left(r n^{-1}\right)^{1-\sigma} k_t^{1-\sigma} - (1-\beta)^{-1} + B^s\right) \\ & -\pi'(1-\sigma)^{-1}(\tilde{\lambda}_t^{-\sigma}(r v_t^p)^{1-\sigma} k_t^{1-\sigma} - (1-\beta)^{-1} + B^u) \end{aligned}$$

where $\tilde{\lambda}_t = (1 + \sum_{n=t+1}^{\infty}\beta^{n-t}(\prod_{s=t+1}^{n}(\beta r)^{\frac{1}{\sigma}})^{1-\sigma})^{-1} = 1 - \beta^{\frac{1}{\sigma}} r^{\frac{1-\sigma}{\sigma}} = \tilde{\lambda}$ because under dictatorship $\{0, 0, ..\}$ is the preferred tax sequence of the rich and $\{1, 0, ..\}$ is the preferred tax sequence of the poor. Also, $B^j = (1-\beta)^{-1} b^j$, $j = s, u$,: the group successfully establishing dictatorship looses $(1-\sigma)^{-1} B^s$ and the other group looses $(1-\sigma)^{-1} B^u$. The constraints restrict the taxes implemented by the median voter to yield utilities to the pivotal agents that exceed the expected

We thank the anonymous referee for drawing our attention to the additional issues arising in the case of $\sigma < 1$.

[10] The expressions for value functions below follow from 1 which requires consumption at time t to grow at the rate $(\beta r(1-\tau_t))^{\frac{1}{\sigma}}$.

utility that they would obtain by revolting. Since we consider the case where $\sigma > 1$, rearranging the constraints for the rich and the poor agent for $t \geq t_0$:

$$
\begin{aligned}
&0 \leq \\
&(1-\sigma)^{-1} \lambda_t^{-\sigma} \left(\left((1-\tau_t)\, r v_t^w + n^{-1} \tau_t + n^{-1} \sum_{j=t+1}^{\infty} \tau_j \prod_{s=t+1}^{j} g_s (r(1-\tau_s))^{-1} \right) r k_t \right)^{1-\sigma} \\
&- (1-\sigma)^{-1} (1-\beta)^{-1} \\
&- (1-\sigma)^{-1} \left[\left(\pi \left(v_t^w\right)^{1-\sigma} + (1-\pi) \left(n^{-1}\right)^{1-\sigma} \right) r^{1-\sigma} \tilde{\lambda}_t^{-\sigma} k_t^{1-\sigma} - (1-\beta)^{-1} + \pi B^s + (1-\pi) B^u \right]
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
&0 \leq \\
&(1-\sigma)^{-1} \lambda_t^{-\sigma} \left(\left((1-\tau_t)\, r v_t^p + n^{-1} \tau_t + n^{-1} \sum_{j=t+1}^{\infty} \tau_j \prod_{s=t+1}^{j} g_s (r(1-\tau_s))^{-1} \right) r k_t \right)^{1-\sigma} \\
&- (1-\sigma)^{-1} (1-\beta)^{-1} \\
&- (1-\sigma)^{-1} \left[\left((1-\pi') \left(n^{-1}\right)^{1-\sigma} + \pi' \left(v_t^p\right)^{1-\sigma} \right) r^{1-\sigma} \tilde{\lambda}_t^{-\sigma} k_t^{1-\sigma} - (1-\beta)^{-1} + (1-\pi') B^s + (1-\pi') B^u \right]
\end{aligned} \tag{10}
$$

Assumption 5 $\beta r > 1$.

Theorem 1 *There exists $\hat{k}(\pi, \pi')$ and $\tilde{k}(\pi, \pi')$, $\tilde{k}(\pi, \pi') \geq \hat{k}(\pi, \pi') \geq 0$, such that democracy is sustainable for $k_{t_0} \geq \tilde{k}(\pi, \pi')$, and democracy is unsustainable for $k_{t_0} < \hat{k}(\pi, \pi')$.*

Proof See Appendix. □

For a given an initial capital stock, democracy will be sustainable if wealth is sufficiently equally distributed, or if the probability of a successful revolution is sufficiently small:

Corollary 1 *Democracy is always sustainable (i) if v^w and v^p are sufficiently close to n^{-1}, that is, if income distribution is sufficiently equal, or (ii) if distribution is unequal, that is $v^w > v^p > n^{-1}$, but π and $1 - \pi'$ are sufficiently small, that is, if the probability of a successful revolt for both of the pivotal agents is small.*

Proof See Appendix. □

To inquire whether higher stocks of capital allow for higher levels of redistribution. We will say that the tax sequence $\{\tau'_{t_0}, \tau'_{t_0+1}, \tau'_{t_0+2}, ...\}$ is "more" than $\{\tau_{t_0}, \tau_{t_0+1}, \tau_{t_0+2}, ...\}$ if $1 \geq \tau'_{t_0+i} \geq \tau_{t_0+i}$, $i = 0, 1, ...$ This of course is not a complete ranking of tax sequences, but sufficient for our purposes.

Corollary 2 *Let $\{\tau_{t_0}, \tau_{t_0+1}, \tau_{t_0+2}, ...\}$ be a tax sequence with $\tau_{t_0} < 1$ for which democracy is sustainable from initial stock k_{t_0}, and let $\{\tau'_{t_0}, \tau_{t_0+1}, \tau_{t_0+2}, ...\}$, $\tau'_{t_0} > \tau_{t_0}$, be a "more redistributive" sequence that is not sustainable from initial stock k_{t_0}. Then democracy is sustainable with the more redistributive tax sequence $\{\tau'_{t_0}, \tau_{t_0+1}, \tau_{t_0+2}, ...\}$ for some $k'(\tau'_{t_0}) > k_{t_0}$.*

Proof See Appendix. □

Finally, we can ask whether the tax sequences in the feasible set for which democracy is sustainable contain sequences that are also time consistent from the perspective of the median voter. We can show that the feasible set of tax sequences contains a time consistent sequence. In proving the Theorem 1 above, we showed that for capital stocks that are sufficiently high, the sequence $\{\tau_{t_0}, 0, 0, ...\}$ is always sustainable, and in particular $\{1, 0, 0, ...\}$ is sustainable: if the stock is high enough, the rich agent will accept a full redistribution without revolt. This sequence is clearly time consistent, so even if the feasible set of tax sequences sustaining democracy are further constrained to time-consistent sequences, democracy is still sustainable for high enough initial capital.

4.1 The median voter

If the median voter is to be decisive at each time with regard to the entire path of future taxes, then at no time can a majority coalition of the poor and the rich make at least one party better off and the other no worse off, relative to the preferred tax proposal of the median voter.[11] Hence we need to check whether under democracy, the poor and rich pivotal agents, by proposing a tax sequence that draws all the voters poorer than the poor pivotal agent and those richer than the rich pivotal agent, can improve their utility by forming a majority coalition against the median voter. Note that the initial wealth share of a poor (rich) pivotal agent, other than being below (above) the median, is arbitrary, so that our Theorem 2 below implies that there is no tax sequence that a majority coalition of the rich and poor could propose to Pareto improve the utilities of the coalition over the tax sequence proposed by the median voter. Here a coalition will require that the incentive constraint for the two pivotal agents hold period by period, and that the maximized discounted utility of the poor agent, subject to the constraint that the rich agent's utility is at least as large as what he gets under the tax sequence chosen by the median voter, exceeds the discounted utility the poor agent receives under tax sequence chosen by the median voter.

The value functions of the poor pivotal agent, the median voter, and the rich pivotal agent are given by

$$V\left(k_{t_0}, k_{t_0}^i, t_0\right) = Max_{\{\tau_{t_0}\}_{t_0}^{\infty}} \frac{1}{(1-\sigma)} \tag{11}$$

$$\left(\begin{array}{c} \lambda_{t_0}^{1-\sigma} \left(\left((1-\tau_{t_0}) v_{t_0}^i + n^{-1}\tau_{t_0} + n^{-1} \sum_{j=t_0+1}^{\infty} \tau_j \prod_{s=t_0+1}^{j} g_s (r(1-\tau_s))^{-1} \right) r k_{t_0} \right)^{1-\sigma} \\ \cdot \left(1 + \sum_{n=t_0+1}^{\infty} \beta^{n-t_0} \left(\prod_{s=t_0+1}^{n} (\beta r (1-\tau_s))^{\frac{1}{\sigma}} \right)^{1-\sigma} \right) - (1-\beta)^{-1} \end{array} \right)$$

where $i = p, M, w$ for the poor, median, and rich agents. Note that the value functions of the poor agent, the median voter, and the rich agent are identical except with respect to the terms $(1-\tau_{t_0})v_{t_0}^i$. Changing taxes for periods after t_0 affects

[11] The standard median voter theorems apply when there is a single issue to be voted on. In our case there is an infinite sequence of tax rates, so the decisiveness of the median voter under simple assumptions is no longer assured. See however Gans and Smart (1996). We thank Marco Basetto for this reference.

consumptions of these agents identically, by changing first period consumption as well as its rate of growth but changing τ_{t_0} affects first period consumptions differentially. Since we have decreasing marginal utility, changes in consumption will have larger effects on the utility of the poor agent and smaller effect on the utility of the rich agent.

Assume that the poor agent implements a tax sequence $\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\}$ that improves his payoff relative to the taxes chosen by the median voter $\{\tau_{t_0}^M, (\tau_s^M)_{s=t_0+1}^{\infty}\}$, that respects all incentive constraints, and leaves the rich agent no worse off. If this change makes the median voter better off as well, we have a contradiction, since the median voter could have chosen those tax rates to start with. Suppose then that these tax rates, $\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0}^{\infty}\}$, make the median voter worse off. Then, it follows that:

$$V\left(k_{t_0}, v_{t_0}^p, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right) - V\left(k_{t_0}, v_{t_0}^p, t_0; \left\{\tau_{t_0}^M, (\tau_s^M)_{s=t_0+1}^{\infty}\right\}\right) > 0$$
$$V\left(k_{t_0}, v_{t_0}^M, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right) - V\left(k_{t_0}, v_{t_0}^M, t_0; \left\{\tau_{t_0}^M, (\tau_s^M)_{s=t_0+1}^{\infty}\right\}\right) < 0$$
$$V\left(k_{t_0}, v_{t_0}^w, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right) - V\left(k_{t_0}, v_{t_0}^w, t_0; \left\{\tau_{t_0}^M, (\tau_s^M)_{s=t_0+1}^{\infty}\right\}\right) \geq 0$$

Since the value functions are continuous in the shares $v_{t_0}^i$, from the Intermediate Value Theorem, there exist $v_{t_0}^{pM} \in (v_{t_0}^p, v_{t_0}^M)$, $v_{t_0}^{wM} \in (v_{t_0}^M, v_{t_0}^w)$ such that

$$V\left(k_{t_0}, v_{t_0}^{pM}, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right) = V\left(k_{t_0}, v_{t_0}^{pM}, t_0; \left\{\tau_{t_0}^M, (\tau_s^M)_{s=t_0+1}^{\infty}\right\}\right) \tag{12}$$

$$V\left(k_{t_0}, v_{t_0}^M, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right) < V\left(k_{t_0}, v_{t_0}^M, t_0; \left\{\tau_{t_0}^M, (\tau_s^M)_{s=t_0+1}^{\infty}\right\}\right) \tag{13}$$

$$V\left(k_{t_0}, v_{t_0}^{wM}, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right) = V\left(k_{t_0}, v_{t_0}^{wM}, t_0; \left\{\tau_{t_0}^M, (\tau_s^M)_{s=t_0+1}^{\infty}\right\}\right) \tag{14}$$

Let $V^p(v)$ be the discounted utility value of the poor pivotal agent as a function of his share under his preferred taxes $\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\}$, and $V^m(v)$ be the same under the taxes $\{\tau_{t_0}^M, (\tau_s^M)_{s=t_0+1}^{\infty}\}$ preferred by the median voter. Then the conditions above can be graphically illustrated in Figure 1:

$V^m(v)$ and $V^p(v)$ must intersect at least twice, as drawn, to satisfy the conditions above, but of course they may cross more than twice. If there are more than two intersections, we take the smallest $v_{t_0}^{pM}$ and $v_{t_0}^{wM}$ for which the above holds.[12] If the agents below $v_{t_0}^{pM}$ together with those above $v_{t_0}^{wM}$ can form a majority, then the median voter's proposed tax sequence would be defeated. Theorem 2 proves that under democracy, there is no coalition of the rich and poor that can Pareto

[12] If the smallest $v_{t_0}^{pM}$ belongs to an interval over which 13 holds, we can redefine $v_{t_0}^{pM}$ as the sup v_{t_0} over that interval.

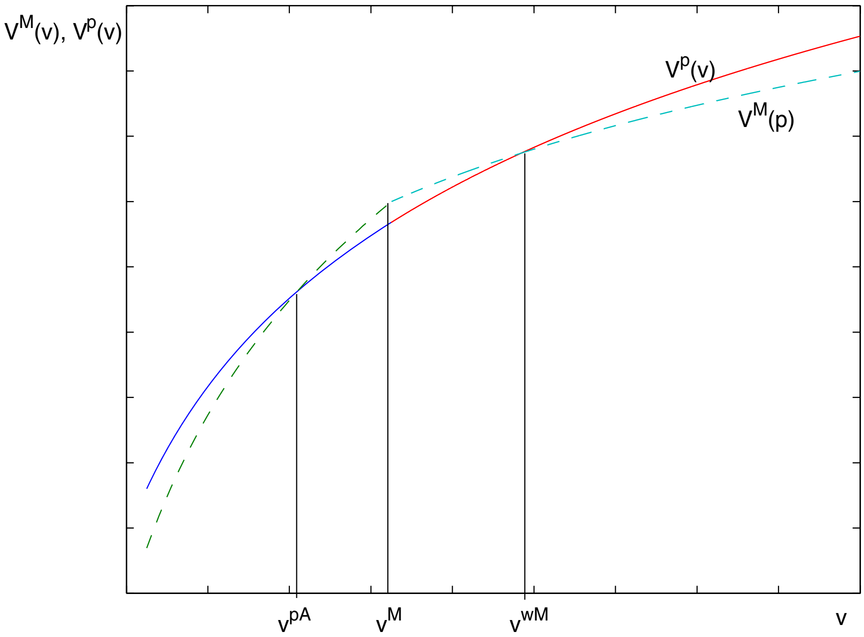

Figure 1. Double-crossing

improve their utility by proposing an alternative tax sequence to defeat the median voter, as depicted in Figure 3 above.[13]

Theorem 2 *There is no feasible tax sequence that a majority coalition of the rich and poor could propose to Pareto improve the utilities of the coalition over the tax sequence proposed by the median voter.*

Proof See Appendix. □

5 Appendix

Proof of Theorem 1. Let $G^p = (1-\pi')B^s + \pi' B^u$ and $G^w = \pi B^s + (1-\pi)B^u$. Consider the tax sequence $\{\tau_{t_0}, \tau, \tau, ...\}$. The constraints 10 and 11 for period t_0

[13] We note that while the result follows in a straighforward manner once the single crossing property above is obtained, the contribution of the theorem lies in establishing the single crossing for the case where preferences are defined over many policy variables, in our case over an infinity of tax rates across time. Clearly the standard assumption of "single peaked preferences" in one dimension, typically required for median voter theorems, does not apply here. Long-lived agent models with intertemporal redistribution that utilize median voter results necessarily encounter this difficulty, and circumvent the problem by assumptions like constant taxes or two period lives (see for example Bertola, 1993). Our Theorem 2 may be useful more generally for establishing median voter results in intertemporal growth models.

in the optimization problem of the median voter become (assuming $\sigma > 1$):

$$G^w \left(rk_{t_0}\right)^{\sigma-1} \geq \lambda_{t_0}^{-\sigma} \left(\left(1-\tau_{t_0}\right) v_{t_0}^w + n^{-1}\tau_{t_0} + n^{-1}\left(\lambda^{-1}-1\right)\tau\right)^{1-\sigma} \quad (15)$$
$$-\tilde{\lambda}^{-\sigma}\left(\pi\left(v_{t_0}^w\right)^{1-\sigma} + (1-\pi)\left(n^{-1}\right)^{1-\sigma}\right)$$
$$G^p \left(rk_{t_0}\right)^{\sigma-1} \geq \lambda_{t_0}^{-\sigma} \left(\left(1-\tau_{t_0}\right) v_{t_0}^p + n^{-1}\tau_{t_0} + n^{-1}\left(\lambda^{-1}-1\right)\tau\right)^{1-\sigma} \quad (16)$$
$$-\tilde{\lambda}^{-\sigma}\left(\left(1-\pi'\right)\left(n^{-1}\right)^{1-\sigma} + \pi'\left(v_{t_0}^p\right)^{1-\sigma}\right)$$

To show that democracy is sustainable, we must show that the feasible set satisfying the constraints is not empty. We have to show that the inequalities above will hold for a feasible tax sequence at every $t_0+s,\ s=0,1,...$ for k_{t_0} sufficiently large. For the sequence of $\{\tau_{t_0},\tau,\tau,...\}$, $\lambda_{t_0+s}=\lambda,\ s=0,1,...$, and the growth rate is $g=r(1-\lambda)(1-\tau)$. Since taxes are constant after the initial period, from equation 8 the shares $v_{t_0}^j$, $j=w,p$ become, for $s=1,2,...$,

$$v_{t_0+1+s}^j = v_{t_0+1}^j = (1-\tau)^{-1}\left(\left(1-\tau_{t_0}\right)v_{t_0}^i + n^{-1}\left(\tau_{t_0}-\tau\right)\right) \quad (17)$$

We note from 8 that if $\tau_{t_0}>\tau$, $(v_{t_0+1}^j - v_{t_0}^j)(v_{t_0}^j - n^{-1})<0,$ and if $\tau_{t_0}<\tau$, then $(v_{t_0+1}^j - v_{t_0}^j)(v_{t_0}^j - n^{-1})>0$. Using 17, the incentive constraints at t_0+1 are:

$$G^w\left(r^2(1-\lambda)(1-\tau)k_{t_0}\right)^{\sigma-1} \quad (18)$$
$$\geq \lambda_{t_0}^{-\sigma}\left(\left(1-\tau_{t_0}\right)v_{t_0}^w + n^{-1}\tau_{t_0} + n^{-1}\tau\left(\lambda^{-1}-1\right)\right)^{1-\sigma}$$
$$-\tilde{\lambda}^{-\sigma}\left(\pi\left(v_{t_0+1}^w\right)^{1-\sigma} + (1-\pi)\left(n^{-1}\right)^{1-\sigma}\right)$$
$$G^p\left(r^2(1-\lambda)(1-\tau)k_{t_0}\right)^{\sigma-1} \quad (19)$$
$$\geq \lambda_{t_0}^{-\sigma}\left(\left(1-\tau_{t_0}\right)v_{t_0}^p + n^{-1}\tau + n^{-1}\tau\left(\lambda^{-1}-1\right)\right)^{1-\sigma}$$
$$-\tilde{\lambda}^{-\sigma}\left(\left(1-\pi'\right)\left(n^{-1}\right)^{1-\sigma} + \pi'\left(\left(v_{t_0+1}^p\right)\right)^{1-\sigma}\right)$$

Let $\tau_{t_0}>\tau$. Then, from 17 it follows that $v_{t_0+1}^p > v_{t_0}^p$ and $v_{t_0+1}^w < v_{t_0}^w$, To assure that the incentive constraints hold from t_0 on, we define the inequalities:

$$G^w \left(rk_{t_0}\right)^{\sigma-1} \geq \lambda_{t_0}^{-\sigma} \left(\left(1-\tau_{t_0}\right) v_{t_0}^w + n^{-1}\tau_{t_0} + n^{-1}\tau\left(\lambda^{-1}-1\right)\right)^{1-\sigma} \quad (20)$$
$$-\tilde{\lambda}^{-\sigma}\left(\pi\left(v_{t_0}^w\right)^{1-\sigma} + (1-\pi)\left(n^{-1}\right)^{1-\sigma}\right)$$
$$G^p \left(rk_{t_0}\right)^{\sigma-1} \geq \lambda_{t_0}^{-\sigma} \left(\left(1-\tau_{t_0}\right) v_{t_0}^p + n^{-1}\tau_{t_0} + n^{-1}\tau\left(\lambda^{-1}-1\right)\right)^{1-\sigma} \quad (21)$$
$$-\tilde{\lambda}^{-\sigma}\left(\left(1-\pi'\right)\left(n^{-1}\right)^{1-\sigma} + \pi'\left(v_{t_0+1}^p\right)^{1-\sigma}\right)$$

Under assumptions 1 and 2, $0<\lambda^m\leq\lambda_{t_0}\leq\lambda^M<1$, and the right hand sides of 20 and 21 are bounded. Note that the right side of 20 is identical to the right side of 16, and since $\sigma>1$ and $v_{t_0+1}^p > v_{t_0}^p$, the right side of 21 is at least as large the right side of 17. Furthermore, the right side of 21 is identical to the right side of 20 and 21 is at least as large 19. Therefore, if 20 and 20 hold, the incentive constraints will be satisfied from time t_0 onwards. Let $\varsigma=\{\tau:$

$g = r(1-\lambda)(1-\tau) > 1$ and $\tau < \tau_{t_0}\}$. Note that $0 \in \varsigma$. Given π, for $\tau \in \varsigma$, there is a $\tilde{k}^w(\pi)$ such that for $k_{t_0} \geq \tilde{k}^w(\pi)$ 20 is satisfied with equality, and given π', there is a $\tilde{k}^p(\pi')$ such that for $k_{t_0} \geq \tilde{k}^p(\pi')$ 21 is satisfied with equality. Let $\tilde{k}(\pi,\pi') = Min(Max(\tilde{k}^p(\pi',0)),\ Max(\tilde{k}^w(\pi),0))$. Then if $\tilde{k}(\pi,\pi') < k_{t_0}$ democracy is sustainable for the feasible sequence $\{\tau_{t_0}, \tau, \tau, ...\},\ \tau \in \varsigma$. Of course there may be other feasible tax sequences that the median voter would prefer, but this establishes that democracy is sustainable by showing that the feasible set that satisfies the constraints and the assumptions is not empty.

To characterize unsustainability we show that for k_{t_0} sufficiently small, the feasible set is empty. Let $k \to 0$, so that $(rk_{t_0})^{\sigma-1} \to 0$. Then the constraints 16 and 17 become:

$$\begin{aligned}
0 \geq\ & \lambda_{t_0}^{-\sigma}\left(\left(1-\tau_{t_0}\right)v_{t_0}^w + n^{-1}\tau_{t_0} + n^{-1}x_{t_0}\right)^{1-\sigma} \\
& -\tilde{\lambda}_{t_0}^{-\sigma}\left(\pi\left(v_{t_0}^w\right)^{1-\sigma} + (1-\pi)\left(n^{-1}\right)^{1-\sigma}\right) \\
0 \geq\ & \lambda_{t_0}^{-\sigma}\left(\left(1-\tau_{t_0}\right)v_{t_0}^p + n^{-1}\tau_{t_0} + n^{-1}x_{t_0}\right)^{1-\sigma} \\
& -\tilde{\lambda}_{t_0}^{-\sigma}\left(\left(1-\pi'\right)\left(n^{-1}\right)^{1-\sigma} + \pi'\left(v_{t_0}^p\right)^{1-\sigma}\right)
\end{aligned}$$

where $x_{t_0} = \sum_{j=t_0+1}^{\infty}\tau_j\prod_{s=t_0+1}^{j}g_s(r(1-\tau_s))^{-1}$. Note that for $k_{t_0} \to 0$, we have $G^j(rk_{t_0})^{\sigma-1} \to 0, j = w, p$, which is equivalent to the case where $G^w = G^p = 0$, that is where there is no cost to an autocratic regime. In that case however, for any given tax sequence other than $\{\tau\} = \{0, 0, ...\}$, there exists $\tilde{\pi} \leq 1$ such that for $\pi > \tilde{\pi}$,

$$\begin{aligned}
0 >\ & \left(\begin{array}{l}(1-\sigma)^{-1}\lambda_{t_0}^{1-\sigma}\left((1-\tau_{t_0})v_{t_0}^w + n^{-1}\tau_{t_0} + n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s\right)\right)^{-1}rk_{t_0}\right)^{1-\sigma} \\ \quad\cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}(\beta r\left(1-\tau_s\right))^{\frac{1}{\sigma}}\right)^{1-\sigma}\right) - (1-\sigma)^{-1}(1-\beta)^{-1}\end{array}\right) \\
& -(1-\sigma)^{-1}\left[\tilde{\lambda}_{t_0}^{-\sigma}\left(\pi\left(v_{t_0}^w\right)^{1-\sigma} + (1-\pi)\left(n^{-1}\right)^{1-\sigma}\right)r^{1-\sigma}k_{t_0}^{1-\sigma} - (1-\beta)^{-1}\right]
\end{aligned}$$

or

$$0 < \lambda_{t_0}^{-\sigma}\left(\left(1-\tau_{t_0}\right)v_{t_0}^w + n^{-1}\tau_{t_0} + n^{-1}x_{t_0}\right)^{1-\sigma} - \tilde{\lambda}_{t_0}^{-\sigma}\left(\pi\left(v_{t_0}^w\right)^{1-\sigma} + (1-\pi)\left(n^{-1}\right)^{1-\sigma}\right) \tag{22}$$

This follows because, when $\pi = 1$, the wealthy agent always prefers $\{\tau\} = \{0, 0, ...\}$, which is the same as the expected utility $(1-\sigma)^{-1}[\tilde{\lambda}_{t_0}^{-\sigma}(v_{t_0}^w)^{1-\sigma}r^{1-\sigma}k_{t_0}^{1-\sigma} - (1-\beta)^{-1}]$ of revolting. Thus for $\pi = 1$, there is no tax sequence that satisfies the constraint for the rich for $G^w(k_{t_0})^{\sigma-1} = 0$, other than the sequence $\{\tau\} = \{0, 0, ...\}$. Furthermore, the wealthy agent prefers any tax sequence to $\{\tau\} = \{1, 0, 0, ...\}$ which establishes full wealth equality in the first period, so that the inequality 22 is reversed for $\pi = 0$. Therefore, since the right side of 22 is increasing in π, for any tax sequence there will be a $\tilde{\pi} \in (0, 1)$, which under defection gives a convex combination of utilities under $\{\tau\} = \{0, 0, ...\}$ and

$\{\tau\} = \{1, 0, 0, ...\}$, such that 22 is satisfied with equality. Consider now

$$G^w (rk_{t_0})^{\sigma-1} = \lambda_{t_0}^{-\sigma} \left((1-\tau_{t_0}) v_{t_0}^w + n^{-1}\tau_{t_0} + n^{-1}x_{t_0}\right)^{1-\sigma} - \tilde{\lambda}_{t_0}^{-\sigma} \left(\pi \left(v_{t_0}^w\right)^{1-\sigma} + (1-\pi) \left(n^{-1}\right)^{1-\sigma}\right) \tag{23}$$

and define $\bar{k}^w$ as the value of k_{t_0} that satisfies 24 when $\pi = 1$. We can, since $\frac{G^w(rk_{t_0})^{\sigma-1}}{dk} > 0$, define $\hat{k}^w(\pi) : [0, 1] \to [0, \bar{k}]$ such that 24 is satisfied, where we have $\bar{k}^w = \hat{k}^w(1)$. Note that if $b^u > b^s$ so that $\frac{dG^w}{d\pi} < 0$, it also follows that $\frac{d\tilde{k}(\pi)}{d\tilde{\pi}} > 0$. Therefore, given π there exists a unique $\hat{k}^w(\pi)$ such that revolt is preferred for $k < \hat{k}^w(\pi)$ because 16 cannot hold.

Now consider the poor agent. Again, if there is no cost to dictatorship $(G^p(rk_{t_0})^{\sigma-1} = 0)$, and the probability that the revolt fails is zero $(\pi' = 0)$, the poor pivotal agent will revolt because he can implement his preferred tax scheme, $\{1, 0, 0, ...\}$ costlessly and for sure. Under these assumptions, for any other tax scheme than $\{1, 0, 0, ...\}$ put forth by the median voter, we have

$$0 > \left(\begin{array}{c}(1-\sigma)^{-1}\lambda_{t_0}^{1-\sigma}\left((1-\tau_{t_0})rv_{t_0}^p + n^{-1}\tau_{t_0} + n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j\prod_{s=t_0+1}^{j} g_s\left(r\,(1-\tau_s)\right)^{-1} rk_{t_0}\right)^{1-\sigma} \\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}(\beta r\,(1-\tau_s))^{\frac{1}{\sigma}}\right)^{1-\sigma}\right) - (1-\sigma)^{-1}(1-\beta)^{-1}\end{array}\right) - (1-\sigma)^{-1}\left[\left((1-\pi')\left(n^{-1}\right)^{1-\sigma} + \pi'\left(v_{t_0}^p\right)^{1-\sigma}\right) r^{1-\sigma}\lambda_{t_0}^{-\sigma}k_{t_0}^{1-\sigma} - (1-\beta)^{-1} + G^p\right]$$

or, using $G^p(rk_{t_0})^{\sigma-1} = 0$,

$$0 < \lambda_{t_0}^{-\sigma} \left((1-\tau_{t_0}) v_{t_0}^p + n^{-1}\tau_{t_0} + n^{-1}x_{t_0}\right)^{1-\sigma} - \tilde{\lambda}_{t_0}^{-\sigma} \left((1-\pi') \left(n^{-1}\right)^{1-\sigma} + \pi' \left(v_{t_0}^p\right)^{1-\sigma}\right) \tag{24}$$

Furthermore, the poor agent prefers any tax sequence to $\{\tau\} = \{0, 0, ...\}$ which establishes full wealth equality in the first period, so that the inequality 25 is reversed for $\pi' = 1$. Therefore, since the right side of 25 is decreasing in π', for any tax sequence $\{\tau\}$ there will be a $\hat{\pi} > 0$, which under defection gives a convex combination of utilities under $\{\tau\} = \{0, 0, ...\}$ and $\{\tau\} = \{1, 0, 0, ...\}$, such that 25, is satisfied with equality. Consider now

$$G^p (rk_{t_0})^{\sigma-1} = \lambda_{t_0}^{-\sigma} \left((1-\tau_{t_0}) v_{t_0}^p + n^{-1}\tau_{t_0} + n^{-1}x_{t_0}\right)^{1-\sigma} - \tilde{\lambda}_{t_0}^{-\sigma} \left((1-\pi') \left(n^{-1}\right)^{1-\sigma} + \pi' \left(v_{t_0}^p\right)^{1-\sigma}\right) \tag{25}$$

and define $\bar{k}^p$ as the value of k_{t_0} that satisfies 26 when $\pi' = 0$. We can, since $\frac{G^p(rk_{t_0})^{\sigma-1}}{dk} > 0$, define $\hat{k}^p(\pi) : [0, 1] \to [0, \bar{k}_{t_0}^p]$ such that 26 is satisfied, where we have $\bar{k}^p = \hat{k}^p(0)$. If $b^u > b^s$ so that $\frac{dG^p}{d\pi'} > 0$, it also follows that $\frac{d\tilde{k}^p(\pi)}{d\tilde{\pi}'} < 0$. Therefore, given π' there exists a unique $\hat{k}^p(\pi')$ such that revolt is preferred for $k < \hat{k}^p(\pi)$ because 17 cannot hold.

For the sustainability of democracy both 16 and 17 must hold, so we define $\hat{k}(\pi, \pi') = Max\ (\hat{k}^p(\pi'), \hat{k}^w(\pi))$. Democracy then will not be sustainable if $k_{t_0} < \hat{k}(\pi)$. Finally, note that it must be true that $\tilde{k}(\pi, \pi') \geq \hat{k}(\pi, \pi')$, for otherwise there would be values of k_{t_0} which would be both sustainable and unsustainable, which is a contradiction. □

Proof of Corollary 1. For $\{\tau\} = \{\tau_{t_0}, \tau, \tau, ...\}$, the growth rate is $g = r(1-\lambda)(1-\tau)$ for all t_0, and $\sum_{j=t_0+1}^{\infty} \tau_j \prod_{s=t_0+1}^{j} g_s (r(1-\tau_s))^{-1} = \tau \sum_{j=t_0+1}^{\infty} (1-\lambda)^{j-t_0} = \tau\lambda^{-1}$. Then the incentive constraints are:

$$G^w\left(rk_{t_0}\right)^{\sigma-1} \geq \lambda^{-\sigma}\left(\left(1-\tau_{t_0}\right)v_{t_0}^w + n^{-1}\tau_{t_0} + \lambda^{-1}\left(1-\lambda\right)\tau\right)^{1-\sigma} - \tilde{\lambda}^{-\sigma}\left(\pi\left(v_{t_0}^w\right)^{1-\sigma} + \left(1-\pi\right)\left(n^{-1}\right)^{1-\sigma}\right) \tag{26}$$

$$G^p\left(rk_{t_0}\right)^{\sigma-1} \geq \lambda^{-\sigma}\left(\left(1-\tau_{t_0}\right)v_{t_0}^p + n^{-1}\tau_{t_0} + \lambda^{-1}\left(1-\lambda\right)\tau\right)^{1-\sigma} - \tilde{\lambda}^{-\sigma}\left(\left(1-\pi'\right)\left(n^{-1}\right)^{1-\sigma} + \pi'\left(v_{t_0}^p\right)^{1-\sigma}\right) \tag{27}$$

Note that for $\{\tau\} = \{\tau_{t_0}, 0, 0, ...\}$, $\tilde{\lambda} = \lambda = 1 - \beta^{\frac{1}{\sigma}} r^{\frac{1-\sigma}{\sigma}} < 1$. If $v^w = v^p = n^{-1}$, the tax sequence $\{\tau\} = \{\tau_{t_0}, 0, 0, ...\}$ is sustainable if

$$G^w\left(rk_{t_0}\right)^{\sigma-1} \geq \lambda^{-\sigma}\left(\left(n^{-1}\right)^{1-\sigma} - 1\right)\left(v^w\right)^{1-\sigma}\right)$$

$$G^p\left(rk_{t_0}\right)^{\sigma-1} \geq \lambda^{-\sigma}\left(\left(\left(n^{-1}\right)^{1-\sigma} - 1\right)\left(v^p\right)^{1-\sigma}\right)$$

But $((n^{-1})^{1-\sigma} - 1) < 0$ if $\sigma > 1$, so the inequalities above hold strictly for any $k_{t_0} \geq 0$. It follows by continuity that the constraints hold if v^w and v^p are sufficiently close to n^{-1}.

If $\pi = 1 - \pi' = 0$, that is if a revolt never succeeds, for $\{\tau\} = \{\tau_{t_0}, 0, 0, ...\}$

$$G^w\left(rk_{t_0}\right)^{\sigma-1} \geq \lambda^{-\sigma}\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^w + n^{-1}\tau_{t_0}\right)^{1-\sigma} - \left(n^{-1}\right)^{1-\sigma}\right) \tag{28}$$

$$G^p\left(rk_{t_0}\right)^{\sigma-1} \geq \lambda^{-\sigma}\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^p + n^{-1}\tau_{t_0}\right)^{1-\sigma} - \left(v_{t_0}^p\right)^{1-\sigma}\right) \tag{29}$$

For $\sigma > 1$, the inequalities above will hold for any k_{t_0}, since the right sides of 28 and 29 are negative if $v^w > v^p > n^{-1}$. Furthermore the inequalities will continue to hold after period t_0 because the capital stock grows and the shares remain constant at $v^j = v_{t_0+s}^j = (1-\tau_{t_0})\, v_{t_0}^j + \tau_{t_0} n^{-1}$ for $s = 1, 2, ...$ and $j = w, p$, with $v^p \leq n^{-1}$, and $v^w \geq n^{-1}$, so that the right sides of 28 and 29, with v^w and v^p replacing $v_{t_0}^w$ and $v_{t_0}^p$ are still negative. Thus democracy will be sustainable for any k_{t_0} under taxes $\{\tau\} = \{\tau_{t_0}, 0, 0, ...\}$. Therefore, if π and $1 - \pi'$ are suffcently close to 0, the set of tax sequences for which democracy is sustainable is not empty. Of course the median voter will then choose from the feasible set the tax sequence that maximizes his utility. □

Proof of Corollary 2. By hypothesis for $\{\tau_{t_0}, \tau_{t_0+1}, \tau_{t_0+2}, ...\}$, the incentive constraints for period t_0,

$$G^w\left(rk_{t_0+s}\right)^{\sigma-1} \geq \lambda_{t_0+s}^{-\sigma}\left(\left(1-\tau_{t_0+s}\right)v_{t_0+s}^{w}+n^{-1}\tau_{t_0+s}+n^{-1}\left(\lambda_{t_0+s}^{-1}-1\right)\tau_{t_0+s}\right)^{1-\sigma} - \tilde{\lambda}^{-\sigma}\left(\pi\left(v_{t_0+s}^{w}\right)^{1-\sigma}+\left(1-\pi\right)\left(n^{-1}\right)^{1-\sigma}\right) \tag{30}$$

$$G^p\left(rk_{t_0+s}\right)^{\sigma-1} \geq \lambda_{t_0+s}^{-\sigma}\left(\left(1-\tau_{t_0+s}\right)v_{t_0+s}^{p}+n^{-1}\tau_{t_0+s}+n^{-1}\left(\lambda_{t_0+s}^{-1}-1\right)\tau_{t_0+s}\right)^{1-\sigma} - \tilde{\lambda}^{-\sigma}\left(\left(1-\pi'\right)\left(n^{-1}\right)^{1-\sigma}+\pi'\left(v_{t_0+s}^{p}\right)^{1-\sigma}\right) \tag{31}$$

are satisfied for $s = 0, 1, ...$ Under assumptions 1 and 2, $0 < \lambda^m \leq \lambda_{t_0} \leq \lambda^M < 1$, and the right hand sides of 30 and 31 are bounded. Suppose the period t_0 incentive constraint is not satisfied for $\{\tau'_{t_0}, \tau_{t_0+1}, \tau_{t_0+2}, ...\}$. Since $G^p(rk_{t_0})^{\sigma-1}$ is increasing in k_{t_0}, there is a $k_{t_0}^{\prime 0}(\tau'_{t_0}) > k_{t_0}$ such that the period t_0 constraint holds. Suppose instead that the period t_0+s incentive constraint is not satisfied for $\{\tau'_{t_0}, \tau_{t_0+1}, \tau_{t_0+2}, ...\}$. Since λ_s and g_s depend only on $\{\tau_{t_0+1}, \tau_{t_0+2}, ...\}$, they are unaffected by τ_{t_0}. Thus $k_{t_0+s} = (\prod_{j=t_0+1}^{\tau+s-1} g_j)k_{t_0}$ is strictly increasing in k_{t_0}. Then again there is a $k_{t_0}^{\prime s}(\tau'_{t_0}) > k_{t_0}$ such that the period t_0+s constraint is satisfied. Let $S = \{s|$ the period t_0+s constraint is not satisfied for $\{\tau'_{t_0}, \tau_{t_0+1}, \tau_{t_0+2}, ...\}\}$. Let $k'_{t_0}(\tau') = \sup_{s\in S}\{k_{t_0}^{\prime s}\}$. Then all the incentive constraints are satisfied for initial stock $k'_{t_0}(\tau'_{t_0})$. □

Proof of Theorem 2. If there is a coalition of the rich and poor that can make the poor better off than under the tax scheme chosen by the median voter, while making the rich no worse off, then 13-15 holds, and we show that this leads to a contradiction. From 13,

$$\begin{aligned}&\left(\begin{array}{c}\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^p\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^p\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}\\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}(\beta r\left(1-\tau_s^p\right))^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}\end{array}\right)\\ =&\left(\begin{array}{c}\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^M\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^M\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}\\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}(\beta r\left(1-\tau_s^M\right))^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}\end{array}\right)\end{aligned}$$

and from 15,

$$\begin{aligned}&\left(\begin{array}{c}\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^p\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^p\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}\\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}(\beta r\left(1-\tau_s^p\right))^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}\end{array}\right)\\ =&\left(\begin{array}{c}\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^M\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^M\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}\\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}(\beta r\left(1-\tau_s^M\right))^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}\end{array}\right)\end{aligned}$$

Rearranging these yields:

$$\left(\frac{\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}\left(\beta r\left(1-\tau_s^M\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}}{\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}\left(\beta r\left(1-\tau_s^p\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}}\right) \tag{32}$$

$$= \left(\frac{\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^p\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^p\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}}{\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^M\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^M\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}}\right)$$

$$= \left(\frac{\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^p\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^p\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}}{\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^M\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^M\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}}\right)$$

We also have:

$$\frac{\partial V^i}{\partial v_{t_0}^i} = \left(1-\tau_{t_0}\right)\left(\begin{array}{c}\left(\left(\left(1-\tau_{t_0}\right)v_{t_0}^{i}+n^{-1}\tau_{t_0}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s\right)\right)^{-1}\right)rk_{t_0}\right)^{-\sigma} \\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}\left(\beta r\left(1-\tau_s\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}\end{array}\right)$$

Since $V(k_{t_0}, v_{t_0}^p, t_0; \{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\}) > V(k_{t_0}, v_{t_0}^p, t_0; \{\tau_{t_0}^M, (\tau_s^M)_{s=t_0+1}^{\infty}\})$, if the first intersection of value functions for $v_{t_0} \geq v_{t_0}^p$ is at $v_{t_0}^{pM}$,

$$\frac{\partial V\left(k_{t_0}, v_{t_0}^{pM}, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right)}{\partial v_{t_0}} < \frac{\partial V\left(k_{t_0}, v_{t_0}^{M}, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right)}{\partial v_{t_0}}$$

and if the first intersection of value functions for $v_{t_0} \geq v_{t_0}^M$ is at $v_{t_0}^{wM}$,

$$\frac{\partial V\left(k_{t_0}, v_{t_0}^{wM}, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right)}{\partial v_{t_0}} \geq \frac{\partial V\left(k_{t_0}, v_{t_0}^{M}, t_0; \left\{\tau_{t_0}^p, (\tau_s^p)_{s=t_0+1}^{\infty}\right\}\right)}{\partial v_{t_0}}$$

Evaluating the derivatives at $v_{t_0}^{pM}$ implies:

$$\frac{\left(\begin{array}{c}\left(\left(\left(1-\tau_{t_0}^M\right)v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^M\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^M\right)\right)^{-1}\right)rk_{t_0}\right)^{-\sigma} \\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}\left(\beta r\left(1-\tau_s^M\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}\end{array}\right)}{\left(\begin{array}{c}\left(\left(\left(1-\tau_{t_0}^p\right)v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^p\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^p\right)\right)^{-1}\right)rk_{t_0}\right)^{-\sigma} \\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}\left(\beta r\left(1-\tau_s^p\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}\end{array}\right)} > \frac{\left(1-\tau_{t_0}^p\right)}{\left(1-\tau_{t_0}^M\right)}$$

Similarly, evaluating the derivatives at $v_{t_0}^{wM}$ implies

$$\frac{\left(\begin{array}{c}\left(\left(\left(1-\tau_{t_0}^{M}\right)v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{M}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{M}\right)\right)^{-1}\right)rk_{t_0}\right)^{-\sigma}\\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}\left(\beta r\left(1-\tau_s^{M}\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}\end{array}\right)}{\left(\begin{array}{c}\left(\left(\left(1-\tau_{t_0}^{p}\right)v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{p}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{p}\right)\right)^{-1}\right)rk_{t_0}\right)^{-\sigma}\\ \cdot\left(1+\sum_{n=t_0+1}^{\infty}\beta^{n-t_0}\left(\prod_{s=t_0+1}^{n}\left(\beta r\left(1-\tau_s^{p}\right)\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}\right)^{\sigma}\end{array}\right)}$$
$$\leq\frac{\left(1-\tau_{t_0}^{p}\right)}{\left(1-\tau_{t_0}^{M}\right)}$$

This is only possible if

$$\left(\frac{\left(\left(1-\tau_{t_0}^{p}\right)v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{p}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{p}\right)\right)^{-1}\right)}{\left(\left(1-\tau_{t_0}^{M}\right)v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{M}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{M}\right)\right)^{-1}\right)}\right)^{\sigma}$$
$$>$$
$$\left(\frac{\left(\left(1-\tau_{t_0}^{p}\right)v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{p}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{p}\right)\right)^{-1}\right)}{\left(\left(1-\tau_{t_0}^{M}\right)v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{M}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{M}\right)\right)^{-1}\right)}\right)^{\sigma}$$

But, from equation 33,

$$\frac{\left(\left((1-\tau_{t_0})v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{p}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{p}\right)\right)^{-1}\right)\right)^{1-\sigma}}{\left(\left((1-\tau_{t_0})v_{t_0}^{pM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{M}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{M}\right)\right)^{-1}\right)\right)^{1-\sigma}}$$
$$=\frac{\left(\left((1-\tau_{t_0})v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{p}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{p}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{p}\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}}{\left(\left((1-\tau_{t_0})v_{t_0}^{wM}+n^{-1}\tau_{t_0}^{M}+n^{-1}\sum_{j=t_0+1}^{\infty}\tau_j^{M}\prod_{s=t_0+1}^{j}g_s\left(r\left(1-\tau_s^{M}\right)\right)^{-1}\right)rk_{t_0}\right)^{1-\sigma}}$$

which leads to a contradiction. □

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