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On the Origins of Property Rights: Conflict and Production in the State of Nature

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I analyse the emergence of property rights in a model of conflict and production in the absence of institutions of enforcement. The population of agents evolves dynamically through conflicts for possession of factor goods among pairs of randomly matched agents. Conflicts are incomplete information wars of attrition with an agent's type consisting of his or her independently drawn valuation of the prize and cost of competing for it. I prove equilibrium existence and show that in the steady state of the game, the population sorts into two stable groups (resource "haves" and "have-nots") in accordance with a known function of their primitive types.

1. INTRODUCTION

This paper develops a dynamic model of the emergence of property rights in a decentralized political and economic environment, that is, an environment in which there are no pre-existing socio-political institutions of enforcement, either because the existing authorities are weak or ineffective, or because the external enforcers lack information about or the authorization of the direct participants.[1] Although such descriptions of the role of government are fair depictions of the status quo in a large number of less developed countries, many of these countries have managed to develop complex and relatively stable networks of customary property rights (*e.g.* Sub-Saharan and West Africa—see Ensminger, 1992; Firmin-Sellers, 1996; Kaplow and Shavell, 2002) or conduct large proportions of economic transactions in the informal economic sector (*e.g.* post-communist and post-authoritarian transition economies—see De Soto, 1989; Frye and Shleifer, 1997, etc.). Similar conditions characterized various economic domains of medieval and early modern European societies and the economies of the American frontier as late as the first part of the 19-th century (see, for example, De Roover, 1965; Umbeck, 1977; Libecap, 1978; Berman, 1983).

What enables economic transactions and what conditions support the norms of property exploitation under such circumstances and, more broadly, what gives rise to an enforcement authority where it is present have remained open questions which cannot be addressed within the third-party enforcer model. These questions are the point of departure for the present model, which analyses them in the context of the aggregate effects of individual costly conflicts over scarce inputs to production. The nature of the conflict and production equilibrium and the allocation of factor goods across agents as a function of their primitive skills supply critical elements

1. See, for example, the empirical studies of institutions in such contexts by Umbeck (1977, 1981), De Soto (1989), Ostrom (1990), and Ellickson (1991). The theoretical approaches explicitly addressed to these contexts include Shleifer (1994) and Skaperdas and Syropoulos (1995).

for the analysis of the emergence of property rights and shed light on how particular elements of secure property rights may emerge as a result of conflict.

In the absence of institutions of enforcement, security, if it exists, arises from some commonly known structural advantage of the incumbent property holder that makes defeating him or her through conflict too unlikely or too costly to warrant the attempt. In existing models of conflict and production, differences in agents' willingness to devote resources to conflict are derived from assumed structural asymmetries—for example, the ability of one agent to be the first to commit resources publicly to conflict (Grossman and Kim, 1996; Hotte, 2001), the ability of one agent to exclude the other unilaterally (De Meza and Gould, 1992), or the superiority of defensive technology over offensive technology or vice versa (Grossman and Kim, 1995, 1996; Grossman, 2001). In contrast, the model presented here treats agents who do and do not currently control the resource symmetrically and shows how, even in the absence of savings or investment, it becomes easier over time for an incumbent to prevail in conflict than for a challenger of the same type to do so. The emergence of such a bias depends on a derived function of individual productivities and on allowing the outcomes of prior conflicts to determine the current distribution of the resource across agents, so that possession of the resource becomes a credible indicator of past success in conflict. Thus, agents who possess the resource come to enjoy a group reputation for fierceness that discourages their potential attackers.

In order to establish this and other results, I first completely characterize the solution to the two-player game when the distribution of types is the same for both players, that is, when the players have common beliefs, and prove the existence and some properties of the equilibrium when the players' beliefs differ. I then present a large-population infinite-horizon dynamic game in which players are randomly matched in each period to play the war of attrition, and as a consequence of those interactions, the distribution of types among the winners and the losers changes with each round of conflict.

I show that in the steady-state equilibrium of this game, attained in finite time, the population of heterogeneous agents separates into two permanent groups (of resource "haves" and "have-nots") in accordance with a known function of their primitive types. Moreover, once the separation is sufficiently complete, no agent finds it in his or her interest to challenge another agent for possession of the resource; that is, in the steady-state equilibrium there is no conflict, and the agents enjoy security of property. These results, which depart from the Hobbesian intuition of "perpetual war of every man against every man" shared by the recent economic literature on conflict and appropriation, suggest a causal model for the emergence of property rights proper. Furthermore, the allocation of factor goods in the steady-state equilibrium is inefficient, and although agents enjoy security, the form of security they enjoy cannot be transferred and hence they cannot meaningfully transfer the resource itself to another agent through voluntary exchange.

"State of nature" models of conflict and production (Skaperdas, 1992; Grossman and Kim, 1995, 1996; Hirshleifer, 1995; Skaperdas and Syropoulos, 1995, 1996*a*, *b*; Grossman, 2001; Muthoo, 2003) capture the basic notion of an "institutionless" state of nature in which there is no enforcement of claims to property except individually provided coercive force, and thus redistribution can be achieved through aggression. In most of these models, agents allocate their individual endowments between directly productive activity and attempts to acquire or retain another input necessary for the productive activity, that is, attempts to increase their success in conflict as determined by a commonly known conflict technology function.[2]

2. The exception is Muthoo (2003), in which agents allocate their endowments between producing a consumption good and leisure in expectation of conflict with exogenous probabilities of winning.

While many of these models portray a single interaction between two agents, some provide a dynamic analysis. Each of Hirshleifer (1995) and Baker (2003) model a continuing struggle for resources between two or more symmetrically positioned agents. In both cases, however, the analysis is effectively limited to a steady state and does not address the cumulative effects of agents' behaviour on their strategic environment.[3] Muthoo (2003) combines a "state of nature" model with the repeated games approach to studying institutional stability. His analysis provides sufficient conditions for the emergence of security of property as well as conditions that guarantee that no equilibrium supporting such rights exists. However, the use of an infinitely repeated stage game, in which the strategic environment is identical in each period, prevents the examination of the dynamically evolving cumulative effects of conflict on the distribution of resources across players and the consequences of that change for the emergence of property rights. Grossman and Kim (1996), on the other hand, do incorporate the cumulative effects of conflict in an examination of how the rate of capital accumulation and the security of property evolve over time in a general equilibrium model of production and predation. However, they assign the players the asymmetric roles of "predator" and "prey", which are assumed to be fixed over time. To the extent that it is desirable to understand the conditions under which some agents become predators, or, equivalently, under which some agents' property is subject to incursion, it is desirable to develop a dynamic model that captures the cumulative effects of agent behaviour on their strategic environment, but which assumes symmetric initial conditions.

This paper presents and analyses just such a model. The distribution of factor goods at the beginning of each period reflects the outcomes of the previous period's conflicts. Because agents dispute the possession of a durable capital good, the outcomes of conflicts in one period affect the expected pay-offs that agents face in the next period. This evolving dynamic element of the model proves critical in determining the emergence of property rights. Modelling it explicitly in the context of conflict over and production with factor goods permits the direct analysis of changes in the allocation of time to conflict and in the posterior distributions of goods. In so doing, it provides a natural way to explore the emergence of *de facto* property rights, emphasized by Sugden (1986) and Skaperdas and Syropoulos (1995) and allows a measure of the extent of (unrealized) potential gains from trade, and so of the long-term effects of the inability to secure property and, separately, of the inability to exchange it.

In order to be able to study the changing distribution of property rights over the physical resource and to provide potential gains from trade, making the efficient distribution of factor goods a meaningful issue, this model incorporates several features that distinguish it from the models of conflict discussed above. First, unlike the existing models, I assume that the population of agents is large and heterogeneous in two different types of abilities. These abilities pertain to two different modes of production, one of which requires a complementary capital input ("land") and one of which requires only labour. Conflict occurs over the possession and use of land, and the variation in agents' skills, combined with the complementarity of land and labour, ensures that agents value the prize differently. Further, because skill in using one technology is uncorrelated with skill in using another, the opportunity cost of devoting time to conflict is uncorrelated with the agents' valuation of the prize. Because I distinguish between the skills used in different technologies, I am able to obtain more subtle efficiency results, addressing not only the inefficient allocation of labour to conflict but also the inefficient allocation of land across differently skilled agents.

Second, I model conflicts over land as simultaneous incomplete information wars of attrition between pairs of randomly matched players. Both agents attempt to claim the good by committing

3. In a related but distinct context, De Meza and Gould (1992) consider owners' private decisions to undertake a costly appeal to the state to enforce their rights. They examine the efficiency of steady-state enforcement decisions in a general equilibrium framework.

their time to “conflict” over it; the conflict ends when one of the agents surrenders his or her claim to the prize by devoting time to something else. The remaining player wins the prize and then also devotes his or her time to another activity, for example, production. This model of conflict contrasts with the “conflict success function” (CSF) used in most of the state-of-nature models cited above. In models using the CSF, agents’ allocations of resources to conflict, once made, are irrevocable, and no further resources can be committed to that purpose after conflict begins. In this sense, they suggest that, although conflict requires some existing capital infrastructure, it does not require labour in its actual execution in that it does not require the sort of resources that can be readily transferred to productive activities. In contrast, in a war of attrition, at every moment in time each agent must choose whether to allocate more time to the conflict. Even if the winning agent would be willing to commit still more time to the conflict when the opponent quits, he or she nonetheless pays only the loser’s bid; superfluous resources devoted to conflict can be diverted to production. The war of attrition thus better captures labour-intensive modes of conflict. Such cases are of particular interest, both because they are theoretically prior to cases in which the agents have accumulated wealth, and because it corresponds to descriptions of some empirical cases (see, for example, Umbeck, 1977; De Soto, 1989; Field, 2002). Although the choice of the model of conflict affects the allocation of time to conflict in the steady state, it does not affect the substantive results on the efficiency of the distribution of land, as discussed in greater detail in Section 3.

The introduction of types that vary on two dimensions rather than one complicates the usual incomplete information war of attrition. I prove the existence of a stationary perfect Bayesian equilibrium and partially characterize its properties; for the one-shot game, I provide a complete solution. The closest existing model is that of Fudenberg and Tirole (1986) in which firms are heterogeneous in both, the value they place on winning and in their costs of competing, as in the present work, but in which each firm’s value of winning and cost of competing is derived from the same underlying feature of the firm. Assuming such a relationship between the costs of conflict and the rewards of winning is unwarranted here, because in the state of nature, the means of acquiring land is entirely independent of the means of using it. Thus, the model presented here is more general in the sense that the costs of engaging in conflict need not be correlated to the value of the prize; indeed, here they are taken to be independent. Krishna and Morgan (1997) find sufficient conditions for equilibrium existence in an incomplete information war of attrition when player’s types are affiliated and symmetrically distributed. The game presented here violates both conditions, and in particular, because of the dynamic evolution of the distribution of the physical resource across types, the distribution of types is asymmetric in all but the first period of play. Athey (2001) obtains sufficient conditions for equilibrium existence in a variety of incomplete information games, including auctions, but restricts attention to auctions in which the value of winning is independent of one’s opponents’ actions. This assumption is not satisfied in the war of attrition, in which the winner must pay the loser’s bid.

The remainder of the paper is organized as follows. Section 2, which presents the analysis of the model, begins with the consideration of the model of individual conflict, a variant of the one-shot incomplete information war of attrition game in isolation (Section 2.1). This game is not a stage game, since the evolution of the distributions of agent types among those who hold land and those who do not changes the pay-offs that agents face from period to period. However, the sequence of play described in the one-shot game is identical to the sequence of play repeated in each period of the dynamic infinite sequential game, which is then analysed in Section 2.2. In Section 3, I discuss the implications of these results for understanding the emergence of property rights and the roles of these institutions in promoting aggregate economic production. Section 4, then, contains the formal proofs of the results in the paper.

2. THE MODEL

2.1. *The one-shot game*

Before characterizing the solution of the one-shot war of attrition, I present a specific example, in which both time and the set of possible types are discrete, in order to develop several key intuitions.

2.1.1. A discrete example. Suppose that there are two players but only one, indivisible, parcel of land which may be used to produce a consumption good. Let each agent's utility be simply the amount that he or she consumes.

Two production technologies are available. Only technology A requires land; both A and B require labour, although they require labour of different kinds. Each agent is endowed with four discrete units of time. Agent i's possession of land is indicated by $l_i = 1$, where $l_i \in \{0, 1\}$, and his or her type is characterized by the vector $(\alpha_i, \beta_i) \in \{2, 5\} \times \{1, 3\}$, where α_i is the marginal productivity of the agent's labour using technology A and β_i is the marginal productivity of the agent's labour in technology B. Each agent knows his or her own type, and beliefs about the opponent's type are $\Pr(\alpha = 2) = \frac{3}{4} \equiv p$ and $\Pr(\beta = 1) = \frac{1}{5} \equiv q$. If an agent devotes t units of time to conflict, then he or she has $(4 - t)$ units of time to devote to production, and the amounts of the consumption good that can be produced with technologies A and B are, respectively, $\alpha l(4 - t)$ and $\beta(4 - t)$. Because technology A requires land, only the winner (for whom $l = 1$) can use it successfully.[4]

In the absence of any institutions to enforce the claim of one or the other to the land, the agents must determine the possession of it through open conflict, modelled here as a war of attrition, or second-price all-pay auction. Each agent chooses his or her strategy s, the maximum amount of time that he or she is willing to commit to an effort to obtain land, that is, the bid. When one agent quits, the other agent is able to observe his or the opponent's exit and stops devoting time to the conflict. To keep this discrete example simple, assume that if both players quit at the same time, neither can use the land. (Such an assumption will not be necessary in the continuous game.) After the possession of land is settled, agents devote their remaining time to production. Note that only agents for whom $\alpha > \beta$ have incentives to engage in contests for land; agents with $\alpha \leq \beta$ concede defeat immediately.

At each point in time t, an agent i chooses to commit that unit of time to conflict if his or her expected benefit from doing so—the probability that opponent j will quit, given that the agent has not yet done so, times the benefit of having land from the remaining time exceeds the opportunity cost of the unit of time:

$$\frac{\Pr(s_j = t)}{\Pr(s_j \geq t)}(\alpha - \beta)(4 - t) \geq \beta. \tag{1}$$

Dividing both sides of the inequality by β, the condition is expressed in terms of the ratio of the agent's value of land to his or her opportunity cost of conflict. It is evident that if it is satisfied for some value $\frac{\alpha-\beta}{\beta}$, then it must be satisfied for all higher values of that ratio as well, and hence the players' strategies are monotonically increasing in $\frac{\alpha-\beta}{\beta}$. It follows that, in the symmetric game, the winner will be the player with a higher value of this ratio.

4. The assumption that the parcel of land is of a fixed size, that is, is indivisible, should be understood in conjunction with the form of the production functions. The implicit idea embedded in the form of these assumptions is that having more land reduces the amount of labour per unit of land, producing the same outcome. A single individual with a limited amount of labour farms his or her plot; increasing the size of the plot reduces the amount of labour that can be devoted to each unit of land, reducing the per-unit productivity of land. These assumptions are consistent with the economic activities described by Umbeck (1977), De Soto (1989), and others.

Note that, an agent of type $(2,3)$ being less productive using land than not, chooses to quit immediately. Given that $s(2,3)=0$ and $\Pr(2,3)=\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)$, from (1) $\Pr(2,3)\left(\frac{5-3}{3}\right)(4-1)=\frac{6}{5}>1$ implies that an agent of type $(5,3)$ prefers to devote a unit of time to conflict in the hope that his or her opponent is of type $(2,3)$. One can readily verify that if type $(5,3)$ believes that the opponent's strategy is such that $s(5,3)=1$, he or she will not be willing to devote a second unit of time to the conflict: $\frac{\Pr(5,3)}{1-\Pr(2,3)}\left(\frac{5-3}{3}\right)(4-2)=\frac{2}{3}<1$. One can similiarly verify that $s(2,1)=2$ and $s(5,1)=3$.

Note that, in addition to wasting potentially productive resources on conflict, allocating land through conflict is inefficient because it awards possession to a player of type $(2,1)$ over one of type $(5,3)$. The total production per unit of time of the two players would be $5+1=6$ if the land were used by $(5,3)$, but it is only $2+3=5$ when $(2,1)$ uses the land.

2.1.2. The continuous game. Suppose now, that time is continuous and that marginal productivities α and β are independently distributed over the intervals $[a,b]$ and $[c,d]$, respectively, with constant probability densities

$$p(\alpha)=\frac{1}{b-a} \quad \text{and}$$
$$p(\beta)=\frac{1}{d-c}. \tag{2}$$

The lowest possible marginal productivity with technology A is no greater than the lowest possible marginal productivity with B, $a\le c$, and the highest possible marginal productivity with A is greater than that with B, $b>d$.[5] Each agent is endowed with amount T of time per period, t of which can be allocated to appropriating or securing land and $T-t$ of which to producing the consumption good.

$$\mathrm{A}(\alpha,t,l)=\alpha l(T-t) \quad \text{and}$$
$$\mathrm{B}(\beta,t)=\beta(T-t). \tag{3}$$

The pay-off for agent i when matched against agent j is

$$u_i=\begin{cases}\beta_i(T-t_i), & \text{if } t_i<t_j\\ \alpha_i l_i(T-t_j), & \text{if } t_i>t_j\end{cases}. \tag{4}$$

Agent i chooses the maximum amount of time that he or she is willing to commit to an attempt to obtain land, t_i. Letting $p(t)$ represent the expected probability that his or her randomly selected opponent, agent j, will quit at time t, the optimal strategy, given the respective type (α_i,β_i) and the strategies played by other agents in the economy, satisfies

$$t_i\in\arg\max\left(\Pr(t_j>t_i)[\beta_i(T-t_i)]+\int_0^{t_i}p(t)[\alpha_i l_i(T-t)]dt\right).$$

Let $t(\alpha,\beta)$ denote the symmetric Bayesian equilibrium of this game, that is, the solution to this optimization problem for every agent in the economy, and let $p(\alpha,\beta)$ denote the expected

5. The substantive results are robust to relaxing this assumption, which is made purely for the sake of mathematical tractability.

probability density of type (α, β) in the population of agents, which is the probability density of type (α, β) in the distribution from which the population was drawn. Then agent i's optimization problem can be rewritten as

$$\max_{t_i} \left(\Pr((\alpha_j, \beta_j) : t(\alpha_j, \beta_j) > t_i)[\beta_i (T - t_i)] \right.$$
$$\left. + \int_0^{t_i} p((\alpha_j, \beta_j) : t(\alpha_j, \beta_j) = t)[\alpha_i l_i (T - t)]dt \right).$$

The first-order condition is

$$p((\alpha_j, \beta_j) : t(\alpha_j, \beta_j) = t_i)[\alpha_i l_i (T - t_i) - \beta_i (T - t_i)]$$
$$- \beta_i \Pr((\alpha_j, \beta_j) : t(\alpha_j, \beta_j) > t_i) = 0.$$

The symmetric strategy $t(\alpha, \beta)$ cannot be solved from this expression because multiple—in fact, infinitely many—vectors (α, β) correspond to a given t. A key step in solving this problem is finding the expression that relates α to β along level curves of t. Essentially, for two individuals pursuing the same strategy t in equilibrium, we must understand how their differences in β compensate for their differences in α. Agents are willing to commit resources to obtaining land if the product of that land is great enough to compensate them for the resources expended, the value of which is measured in terms of their next-best use. Thus, as in the example above, the relevant relationship between α and β is the ratio of the additional product per unit of labour with land to the opportunity cost of obtaining land. Denote this ratio k.

$$k_i = k(\alpha_i, \beta_i) = \frac{\alpha_i - \beta_i}{\beta_i}. \tag{5}$$

As shown in Proposition 1 below, agents with the same ratio of the marginal product of land to the opportunity cost of conflict allocate the same amount of time to attempts to obtain land, even though they are of different underlying marginal productivities. Agents with higher ratios of the marginal product of land to the opportunity cost of conflict will allocate more time to attempts to obtain land. These properties ultimately allow the derivation of a unique solution for the symmetric equilibrium strategy.

Agents' strategies as functions of this ratio are represented by $s(k)$; from $s(k)$ we can later recover the agents' strategies in terms of (α, β) through the substitution $s(k(\alpha, \beta))$. For notational convenience, henceforth $s(k)$ refers to the equilibrium strategies with respect to k. The expected probability density of k in the population of agents is denoted $p(k)$ and is derived in the section labelled "Derivation of $P(k)$ and $p(k)$" in the Appendix from the distributions of α and β and (5).

The necessary condition for Bayesian equilibrium in terms of k is

$$s_i \in \arg\max \Pr \left((s(k_j) > s_i)(T - s_i) + \int_{\{k_j : s(k_j) < s_i\}} p(k)[(k_i + 1)(T - s(k))]dk \right). \tag{6}$$

Define $\bar{k}$ as the minimum k such that $s(\bar{k}) = T$ or the maximum value of k, whichever is lower; likewise, $\bar{s} = s(\bar{k})$. We can now state the following proposition:

Proposition 1. *$s(k)$ is strictly increasing on the interval $(0, \bar{k})$, that is,*

(i) *if $k(\alpha', \beta') = k(\alpha'', \beta'')$, then $s(k(\alpha', \beta')) = s(k(\alpha'', \beta''))$ even though $(\alpha', \beta') \neq (\alpha'', \beta'')$;*
(ii) *for $k \in (0, \bar{k})$, if $k(\alpha', \beta') > k(\alpha'', \beta'')$, then $s(k(\alpha', \beta')) > s(k(\alpha'', \beta''))$.*

Proof. See Appendix. ||

Because the player who is willing to commit more time to conflict wins, an immediate consequence of this result is that the player with higher k will be the one to use the land. Since the total production of the two players is maximized when the one with a higher difference in productivity $(\alpha - \beta)$ uses the land (and the other produces using technology B), the allocation of land that results from conflict may or may not be classically efficient, depending on whether the player with higher k is also the one with higher $(\alpha - \beta)$.

The following lemma is instrumental in proving the uniqueness of the equilibrium (Proposition 2 below).

Lemma 1. *If an equilibrium exists, then the equilibrium strategy $s(k)$ has the following properties:*

(i) $s(0) = 0$;
(ii) $s(k)$ *is continuous on* $(0, \bar{k})$;
(iii) *The inverse of* $s(k)$, $K(s)$, *exists and is differentiable over* $(0, \bar{s})$.

Proof. See Appendix. ||

This lemma allows us to rewrite agent i's optimization problem, (6), in the form

$$s_i \in \arg\max \left((1 - P(K(s_i)))(T - s_i) + \int_0^{s_i} p(K(s))K'(s)[(k_i + 1)(T - s)]ds \right).$$

The revised first-order condition

$$-(1 - P(K(s_i))) + p(K(s_i))K'(s_i)(k_i(T - s_i)) = 0,$$

yields the non-homogeneous first-order linear differential equation

$$s(k) + \frac{1 - P(k)}{kp(k)} s'(k) = T, \tag{7}$$

the solution of which takes the following form on $k \in (0, \bar{k})$:

$$s(k) = T - T \exp\left(-\int_0^k \frac{xp(x)}{1 - P(x)} dx \right).$$

Proposition 2. *The equilibrium is unique, and the equilibrium strategy is characterized by*

$$s(\alpha, \beta) = \begin{cases} T\left(1 - \left(1 - \frac{c+d}{2b-c-d}\frac{\alpha-\beta}{\beta}\right)^{(2b/c+d)-1} e^{(\alpha-\beta)/\beta}\right), & \textit{if } \alpha \in \left[\frac{a}{c}\beta, \frac{b}{d}\beta\right] \\ T\left(1 - \frac{\alpha}{\beta}\left(1 - \frac{c}{b-c}\frac{\alpha-\beta}{\beta}\right)^{2((b/c)-1)} e^{(\alpha-\beta)/\beta}\right), & \textit{if } \alpha > \frac{b}{d}\beta \end{cases}.$$

Proof. See Appendix. ||

Note that agents' equilibrium strategies are pure strategies. Recall that agents who differ markedly in productivity may play the same strategy and thus have the same probability of winning.

Proposition 3. *The equilibrium strategy $s(\alpha, \beta)$ is increasing in α and decreasing in β.*

Proof. See Appendix. ||

Ceteris paribus, increasing an agent's marginal productivity in the production process that requires land increases the amount of time allocated to conflict (and thus decreases the amount of time allocated to productive activities); and increasing an agent's marginal productivity in the process that requires only labour decreases the amount of time allocated to conflict (and thus increases the amount of time allocated to production).

In aggregate, a population of agents that is more adept with respect to technology A relative to technology B devotes more time to conflict overall. Similarly, an improvement in technology A, which is equivalent to a global increase in skill α, results in a greater amount of time being devoted to conflict, both because conflicts between agents last longer and because more conflicts occur. Fewer agents have $\alpha \leq \beta$, so fewer agents choose to concede possession of the land to their opponents without a fight. Thus, in the absence of property rights, an improvement in the technology that requires the capital input decreases the amount of labour allocated to both forms of production.

2.2. *The infinite dynamic game*

Randomly matching members of a large population of agents in each period to play a game like the one discussed above produces a dynamic infinite sequential game, where agents' pay-offs, and thus their optimal strategies, change from period to period in response to the evolution of the expected distribution of agent types among those who possess land and those who do not. The evolution of the distribution of agent types is, however, itself a consequence of the strategies agents choose. Modelling this dynamic element explicitly permits the direct analysis of the long-term effects of the absence or failure of rights-enforcing institutions on the level of conflict and on the distributions of both capital goods and income.

The initial conditions of the infinite sequential game are essentially the conditions described in the one-period game. Suppose that, before play begins, Nature selects a fixed and finite population of agents from a two-dimensional continuum of types distributed with constant density. Nature then endows some agents with land. Henceforth, agents who possess land will be said to be members of set L, the remaining agents will be called members of set N, and we will assume $|L| \leq |N|$. Once chosen, the population remains fixed throughout. Agents' skill types do not change, but agents can move from set L to set N and vice versa, keeping the number of agents in each set constant. Once again, it is assumed that parcels of land cannot be divided among agents.[6] The skill dimensions (2) and production technologies (3), and hence the per-period pay-offs (4), are the same ones described earlier; however, agents now consider the sums of their discounted expected future pay-offs when allocating their time between production and conflict and not just their single-period pay-offs.

At the beginning of each period, Nature randomly pairs agents in set L with agents in set N. Note that $|N| - |L|$ agents will not be matched with opponents; these agents have no opportunity to acquire land that period and allocate the entire period to producing the consumption good with technology B.[7] Conflicts over the possession of land are modelled as incomplete information wars of attrition, as described in the one-shot game. Initially, the expected distributions of agent

6. For a model in which the size of each agent's parcel of land is determined endogenously, see Baker (2003).

7. If Nature endows an agent for whom $\alpha < \beta$ with land and then matches that agent with another individual for whom $\alpha < \beta$, the land lies fallow for the first period. Both agents use technology B(.). The final distribution of the land between them is irrelevant.

types are identical in N and L, but after the first round of contests both the expected and the actual distributions of types are different among agents who possess land (the winners), L, from what they are among those who do not, N. Consequently, agents of the same type k in different groups may have different optimal strategies after the first round of play, since they assign different probabilities to their opponents being of any given type. An agent without land who challenges a landholder knows that his or her opponent won in the previous period; similarly, the landholder knows that, if the opponent fought in the previous period, he or she must have lost. To avoid the proliferation of unnecessary notation, I first assume that agents remember neither the identities nor the actions of past opponents and later argue that relaxing this assumption to allow perfect recall does not affect the main substantive results. After the possession of a parcel of land has been determined, the agents who contested it use the remainder of the period to produce using whatever technology is available to them.[8] If both agents commit the entire period to conflict, the landholder retains possession of the parcel.[9]

Theorem 1. *There exists a stationary perfect Bayesian equilibrium in non-decreasing strategies.*

Proof. See Appendix. ‖

Before proceeding with the analysis of the equilibrium strategies, it is useful to confirm that some of the basic properties of the optimal strategies in the one-period war of attrition are also true of the optimal strategies in the infinite sequential game and to define formally agents' expectations about the evolving distributions of agent types among landholding and landless agents. The strategy of an agent of type k in group L in period t is denoted $s_t^L(k)$. Once again, $\bar{k}$ is defined as the minimum k such that $s(\bar{k}) = T$ or the maximum value of k, whichever is lower; conversely, $\bar{s} = s(\bar{k})$.

Proposition 4. *The equilibrium strategies $s_t^L(k)$ and $s_t^N(k)$ have the following properties:*

(i) $s_t^L(0) = s_t^N(0) = 0$;
(ii) *Within each group, for all t, strategy s is strictly monotonically increasing in k on* $[0, \bar{k}]$;
(iii) *Within each group, for all t, strategy s is continuous on the interval* $[0, \bar{k}]$;
(iv) *The inverses of $s_t^L(k)$ and $s_t^N(k)$, $K_t^L(s)$ and $K_t^N(s)$, exist and are differentiable over* $(0, \bar{s})$.

Proof. See Appendix. ‖

As in the one-shot game, the classically efficient outcome is for agents with higher $(\alpha - \beta)$ to use the land; thus, the most efficient allocation of resources in each period would be for the $|L|$ agents with the highest values of $(\alpha - \beta)$ to produce with technology A using the land, for the remaining agents to use technology B, and for all agents to devote all of their time to production.

8. Recall that any agent may use technology B; possessing land does not bar an agent from choosing technology B over technology A. Only agents with $\beta \geq \alpha$ would make such a choice, however, and they are not likely to possess land after the first period, since it is not in their interest to engage in conflict over it.

9. This assumption captures the notion that the capital good is already in some agent's possession: a challenger must actually wrest control of the resource from its current holder in order to make use of it. The common assumption of a probabilistic tie-breaker (flipping a coin) is less appropriate in this context because it treats the resource as if it were entirely in the public domain. The substantive results in this paper are robust to allowing conflicts to continue into subsequent periods. The continuation of conflict is made possible by matching contestants who fight until the end of the period with each other again in the next period, rather than with new opponents. Because this extension complicates the recursive definitions of the probability distributions of agent types considerably, while yielding the same substantive results as the basic model, the simpler assumption is used here.

In period $t+1$, the probability that a landless agent j assigns to the event that a landholding agent is of type k can be defined recursively. Recall that the expected probability density of k in the first period is given (Appendix section labelled "Derivation of $P(k)$ and $p(k)$"). The probability that a landholding agent is of type k, $p_{t+1}^L(k)$, is the sum of the probability that a landholding agent was of type k in t and retained possession of the land and the probability that a landless agent was of type k in t and acquired land. Consider a single landholder of type k who possesses land. The landholder retains possession of that land if the opponent concedes before he or she does; the expected probability that the landless opponent is of a type that concedes first is $P_t^N(K_t^N(s_t^L(k)))$. An agent of type k who does not have land wins it if he or she is matched with a landholding agent of a type that concedes first; this occurs with probability $P_t^L(K_t^L(s_t^N(k)))$. The expression for $p_{t+1}^N(k)$ is derived similarly but is complicated slightly by the fact that $|L| < |N|$.

$$
\begin{aligned}
p_{t+1}^L(k) &= p_t^L(k)P_t^N(K_t^N(s_t^L(k))) + p_t^N(k)P_t^L(K_t^L(s_t^N(k))) \\
p_{t+1}^N(k) &= \frac{|L|}{|N|}p_t^L(k)[1 - P_t^N(K_t^N(s_t^L(k)))] \\
&\quad + \frac{|L|}{|N|}p_t^N(k)[1 - P_t^L(K_t^L(s_t^N(k)))] + \frac{|N|-|L|}{|N|}p_t^N(k). \qquad (8)
\end{aligned}
$$

Let $E[U_t^L(k)]$ represent the expected sum of discounted future pay-offs for agent k evaluated in period t, conditional on him or her beginning period $t+1$ in group L; similarly, $E[U_t^N(k)]$ represents expected pay-offs conditional on beginning period $t+1$ in N. The agents' optimization problems are

$$
s_t^N \in \arg\max \left((1 - P_t^L(s_t^N))((T - s_t^N) + E[U_t^N(k_i)]) + \int_0^{s_t^N} p_t^L(s)[(k_i+1)(T-s) + E[U_t^L(k_i)]]ds \right) \qquad (9)
$$

and

$$
s_t^L \in \arg\max \left((1 - P_t^N(s_t^L))((T - s_t^L) + E[U_t^N(k_i)]) + \int_0^{s_t^L} p_t^N(s)[(k_i+1)(T-s) + E[U_t^L(k_i)]]ds. \right) \qquad (10)
$$

Having established that the inverse functions $K_t^L(s)$ and $K_t^N(s)$ exist and that the agents' beliefs about the distributions of types they face evolve according to (8), the optimization problem can be rewritten to produce first-order conditions that result in the following system of non-homogeneous differential equations:

$$
s_t^N(k) = T + \frac{\Delta_t(k)}{k} - \frac{1 - P_t^L(K_t^L(s_t^N(k)))}{kp_t^L(K_t^L(s_t^N(k)))K_t^{L\prime}(s_t^N(k))} \quad \text{and}
$$

$$
s_t^L(k) = T + \frac{\Delta_t(k)}{k} - \frac{1 - P_t^N(K_t^N(s_t^L(k)))}{kp_t^N(K_t^N(s_t^L(k)))K_t^{N\prime}(s_t^L(k))}, \qquad (11)
$$

where $\Delta_t(k) = E[U_t^L(k)] - E[U_t^N(k)]$.

Proposition 5. *For $k \in (0, \bar{k}_t)$ and $t > 1$, $s_t^L(k) > s_t^N(k)$.*

Proof. See Appendix. ||

After the first round of play, a wedge is driven between the strategies of agents of the same type in different groups. Let $\bar{k}_t$ represent the smallest k such that $s_t^N(k) = T$ or the largest possible value of k, $\frac{b-c}{c}$, whichever is smaller. A player of a given type $(0, \bar{k}_t)$ is willing to commit more time to retaining possession of land than to obtaining it.

Thus, the expected (and actual) distributions of types that result from the first round of play further advantage those agents who won in the first period. Over time, only agents of progressively higher types can expect to acquire land if they do not already possess it, but agents of progressively lower types can expect to retain land once they acquire it. Thus, a structural advantage in favour of those who already possess land arises. After a sufficient number of periods, no agents in N choose to challenge landholders.

Theorem 2. *In a finite number of periods,*

(i) *L and N converge to some L^* and N^* such that, for every i in L^* and every j in N^*, $s^L(k_i) > s^N(k_j)$;*
(ii) *for every type k in N^*, $s^N(k) = 0$.*

Proof. See Appendix. ||

Thus, the allocation of land across agents becomes fixed, and in the steady-state equilibria there is virtually no conflict in the state of nature. Active conflict dies away after the allocation of land becomes fixed because agents' strategies are a function of their beliefs about the populations of types rather than of the actual populations of types. Landholders have secure possession of their land, and agents who do not possess land appear to "respect" the security of the landholders' property.

It is important to note that the sorting of agents across the two groups (N and L) need not be complete: It is quite possible that in the steady state there are two agents, i and j, such that $i \in L$, $j \in N$, and $k_i < k_j$. The trick, as Proposition 2 implies, is that such situations are sufficiently uncommon to not make it worthwhile for $j \in N$ to challenge a randomly matched member of group L who is unknown to him. The exact composition of L^* and N^* depends not only on the primitives of the economy, which determine the strategies the agents choose, but also on the initial allocation of land across agents and the realizations of the random matches. All the equilibria, however, share the qualitative characteristics described in Propositions 2 and 3.

The following proposition establishes the relationship between the extent of the steady-state sorting and the speed of convergence to the steady state. Before formulating this proposition, however, we need to introduce a measure of agent sorting in this economy. Define $k^*(a, b, c, d, N, L)$ such that the cumulative probability function $P(k^*) = \frac{|N|}{|N|+|L|}$. Because k^* is a function of primitives and not agent type, I suppress its arguments. When agents sort completely in accordance with k, then all agents with types between k^* and the highest value of k, $\frac{b-c}{c}$, are in group L; likewise, all agents with types between k^* and the lowest value of k, $\frac{a-d}{d}$, are in group N. A measure of the incompleteness of sorting, then, is the number of agents in N whose k-types are between $\frac{b-c}{c}$ and k^*, $\left|N^* \cap \left(k^*, \frac{b-c}{c}\right)\right|$.

Theorem 3. *The more rapidly $s^L(k)$ and $s^N(k)$ diverge,*

(i) *the sooner actual conflict ceases entirely, that is, the sooner $s^N(k) = 0$ for every type k in N;*

(ii) *the less complete the sorting of agents by k into L and N, that is, the larger is* $|N^* \cap \left(k^*, \frac{b-c}{c}\right)|$.

Proof. See Appendix. ||

An economy that converges more slowly devotes more time to conflict and hence less time to productive activities. It also sorts more completely, so that the allocation of land across agents is more efficient in the steady state, in the sense that the agents with greater differences in productivity with and without land are more likely to possess land. Thus, there is an apparent trade-off between improving efficiency on the path of convergence and in the steady state.

2.3. *Robustness*

Recall now, that the foregoing has assumed that agents do not have perfect recall—in particular, that they do not retain memory of the identities and actions of their own past opponents. The assumption of perfect recall complicates the analysis because an agent learns something about the opponent's type from the outcome of the war of attrition, altering not only the beliefs about the type of the opponent when he or she is matched with someone he or she has encountered before, but also the agent's expected future utility. To see that the substantive results embodied in Theorems 2 and 3 are robust to assuming perfect recall consider the following. The probability that j assigns to the event that some agent with whom he or she has been matched before is of type k depends on the information obtained from observing that opponent's strategy (if j won) or from observing that that opponent's strategy was greater than his or her own (if j lost). Thus, updated beliefs about the type, and hence the current strategy, of an opponent he or she has fought before are specific to that opponent. Because every agent is an independent draw from the underlying distribution of types and because agents do not observe conflicts (or their outcomes) other than those in which they are participants, information about one agent's type does not alter beliefs about the type of any other agent. Thus, we can consider the effects of the possibility of encountering known agents separately.

Denote the cumulative distribution function that represents j's beliefs about agent i's type in period t as $P_t^i(k, h_j)$, where h_j contains j's information about past play. Let $\Delta_t(k, h) = E[U_t^L(k, h)] - E[U_t^N(k, h)]$, where $E[U_t^L(k, h)]$ is the expected utility of an agent of type k with history h who starts the next period in group L, and, similarly, $E[U_t^N(k, h)]$ is the expected utility of an agent of type k and history h who starts the next period in group N. Agent j's strategy when facing a past opponent is the solution to a differential equation of the same form as (11), but using the opponent-specific $P_t^i(k, h_j)$ and $p_t^i(k, h_j)$ and a difference in expected utilities $\Delta_t(k, h_j)$ that accounts for the possibility of being matched with past opponents, with whom the strategies played in the war of attrition, may be different.

Consider first, the effects of j's beliefs about i's type on his or her strategy, bearing in mind that i's beliefs about j's type are also a function of the outcome of their previous encounter. If j lost to i before, then he or she assigns greater probability to i being of higher types than to someone whom he or she had not met before, and consequently the agent chooses a lower strategy. The parallel conclusion that i, having beaten j before and hence having lower expectations of j's type, chooses a higher strategy than he or she would have chosen had he or she been matched with someone whom he or she had not met before, reinforces j's choice to concede sooner. Thus, the direct effects of the contestants' updated beliefs about each other's type is consistent with the sorting by k type that occurs when agents do not remember the identities of past opponents.

Information about other agents garnered through past play also affects the difference between the agent's expected utility if the agent wins in the current period and the expected future utility if he or she loses, $\Delta_t(k)$. The substantive content of Proposition 5 and Theorems 2 and 3 is robust to these effects because the altered difference in expected utility is still increasing in k. Consider separately, the agent's beliefs about expected future interactions with unknown opponents and with known opponents. (Recall that each agent is an independent draw from the underlying distribution of types, and hence that information about one opponent's type does not alter the agent's beliefs about any other potential opponent's type.) If the difference in the expected future utility is increasing in k for both unfamiliar potential opponents and for familiar (known) potential opponents, then the difference in the expected future utility when he or she has acquired knowledge of some but not all potential opponents' types is also increasing in k. It has already been demonstrated that the difference in expected future utility is increasing in k when the agent's beliefs about the opponents are based solely on knowledge of the underlying distribution of types and the equilibrium of the game, that is, when all potential opponents are unfamiliar. It remains to be seen that it is increasing in k when the potential opponents are familiar. When two contestants in a war of attrition know each other's types, the lower type conceding immediately (and the higher type fighting as long as is necessary to win) is an equilibrium. It follows that, when the pool of known potential opponents is known, the difference in expected utility is $\left(P^N_{t+1}(k) - \frac{|L|}{|N|}P^L_{t+1}(k)\right)(kT + \Delta_{t+1}(k))$, which is increasing in k and ranges from $\left(1 - \frac{|L|}{|N|}\right)(kT + \Delta_2(k))$, achieved in $t = 1$, to $kT + \frac{\delta kT}{1-\delta}$.

Indeed, the fact that the complete information war of attrition has an equilibrium, in which the strategy of the higher type is to fight forever and that of the lower type is to quit immediately, guarantees that the higher type always wins without any actual conflict occurring.[10] Thus, in the extreme case in which every player knows every other player's type, the main results of the incomplete information game—convergence to a steady state in which no conflict occurs but distributional inefficiencies persist—continue to hold. Furthermore, there would be no actual conflict at any point in the game, and in the steady state the group of landholders would consist of the $|L|$ agents with the highest values of k.

3. DISCUSSION

3.1. *The allocation of factor goods*

Note first, that agents with the same ratios of net marginal productivity with land to opportunity cost of conflict, $k = \frac{\alpha-\beta}{\beta}$, earn different pay-offs from production because of the differences in their underlying productivities (α and β). Because agents' success in conflict, and hence the possession and use of land depends on $\frac{\alpha-\beta}{\beta}$ rather than $\alpha - \beta$, there is a discrepancy between the steady-state equilibrium allocation of land and labour and the efficient allocation of land and labour. Although the form of k is a result of this particular model, for example, of the particular forms of the production and utility functions, this substantive conclusion is robust.

For example, consider using a CSF rather than the war of attrition in the above model (Hirshleifer, 1991). Suppose that each agent has an endowment w of some resource that is necessary for both conflict and production and that they simultaneously choose an amount r to devote to conflict. Let the probability that i wins the conflict be $\frac{r_i}{r_i+r_j}$, and suppose that the agents' types are common knowledge.[11] Thus,

10. In the complete information game, the reverse—for the lower type to fight forever and the higher to quit—is also an equilibrium, as well as a mixed-strategy profile in which the higher type is more likely to quit earlier. For obvious reasons, the applied literature focuses on the pure-strategy equilibrium discussed in the main text.

11. The CSF is unwieldy in incomplete information games like the one presented above, since it is not clear that equilibrium strategies would be monotonic.

$$E[u_i(r_i, r_j)] = \frac{r_i}{r_i + r_j}\alpha_i(w - r_i) + \frac{r_j}{r_i + r_j}\beta_i(w - r_i).$$

The best response of i is readily obtained by solving the first-order condition. It can be expressed in terms of k and is strictly increasing in k for all $k > 0$. Hence, although the probabilistic nature of the CSF ensures that the distribution of land across agents never becomes completely fixed, the allocation of land will be biased in favour of agents with higher values of k.

Indeed, for any means of determining who will use a resource that does not depend *solely* on the agents' valuations of that resource, allocational inefficiencies will result. It is intuitive that agents' optimal strategies in any contest in which their participation is costly will depend on both their valuation of the prize and the costs of their participation. It is the costliness of the conflict over the resource that causes the discrepancy between the ultimate distribution of the resource that results from conflict and the efficient distribution of the resource. Note that the addition of a third dimension to the agents' primitive types to capture any differences in their ability to use the conflict technology, for example, differences in their primitive abilities to fight would not affect the conclusion that inefficiencies are the natural result of allocating goods through conflict. It follows that allocational inefficiencies of the sort that result from the model presented here are robust to a wide range of conflict and production technologies and to the heterogeneity of the population in aspects relevant to them.

These allocational inefficiencies are exacerbated by the fact that, ultimately, agents do not sort completely even with respect to the composite type, k (Theorem 2). Incomplete sorting exacerbates inefficiency in expectation because, as discussed above, agents' strategies in conflict do depend (positively) on their valuation of the prize. Thus, the allocation of the resource through conflict does correlate, however imperfectly, with the efficient allocation, whereas the assumed initial allocation was completely uncorrelated with it. The extent to which agents are sorted according to the composite type (k) in the steady state depends on how quickly the gap between the conflict strategies of identical types in L and N, $s^L(k)$ and $s^N(k)$, widens and hence on how quickly the system converges (Theorem 3). The more quickly the gap widens, the more quickly the system converges and the more incomplete the sorting; thus, the less severe are the losses due to the allocation of time to conflict and the more severe are the losses associated with the misallocation of land across agents.

3.2. *Security vs. transferability*

Although this *de facto* security of property has some of the desirable characteristics of a socially legitimated and enforced right to the security of property, such as providing incentives to engage in productive activities and even to invest, it has a critical shortcoming: The widespread voluntary (market) exchange of land undermines it. Because the ownership pattern that arises from conflict is inefficient, there are potential gains from trade in the steady state. Agents cannot exploit these gains from trade, however, without developing either a more sophisticated means of enforcing ownership or a more sophisticated type of exchange than the outright sale of land, because they cannot credibly commit to refrain from expropriating the land they have just "sold". When an agent loses land through conflict, there is no incentive to return to challenge the victor again; he or she has just discovered that that particular agent is willing to commit more time than to securing the land. However, when an agent agrees to cede possession of land to another agent without conflict, he or she may find it in his or her interest to renege by re-expropriating the land.

Because security depends on the reputed personal attributes of the landholders—their willingness to fight—anything that weakens the group reputation also degrades the security of land. Because voluntary exchange would be a means for agents who are less successful in conflict to become landholders, sufficiently widespread exchange would undermine the validity of the inference that a landholder is "tough" from the fact of being a landholder. To see this more clearly, consider the steady-state equilibrium described in Theorem 2. A landholder with low $\alpha - \beta$ and a landless agent with high $\alpha - \beta$ could mutually benefit from trading the land for payments of the consumption good. Landless agents with low k would initially be willing to make such deals, because, given the commonly shared beliefs about the distributions of types in L and in N in the steady state, they would anticipate that their possession of the land would not be challenged. However, after many such exchanges, the (perceived) distributions of types could be sufficiently altered to make challenges from members of the landless group profitable again, initiating a new stage of pervasive conflict. Note that all members of the landholding group would ultimately experience losses in utility as a result of these voluntary exchanges, and not only the "new" members or the members of lower types, because all landholders would be forced to commit time to conflict in order to defeat challengers. Thus, agents in group L have some incentive to *prevent* the voluntary exchange of land altogether. The basis of the *de facto* security of property in the steady-state equilibrium may actually produce incentives to inhibit the transfer of land.

One possible means of alleviating the inefficiencies that persist in the steady-state allocation of land is to exchange the temporary use of land for some amount of the consumption good produced with it. Such an arrangement might resemble feudalism or share cropping (*e.g.* Hafer, 2003).

4. CONCLUSION

This paper provides a dynamic model of conflict and production in a "state of nature". The analysis demonstrates that, over time, a systematic bias in favour of landholders develops even when offensive and defensive technologies are the same, no saving or long-term investment in conflict is possible, and the initial conditions are symmetric. The advantage that accrues to incumbent landholders is a direct result of the fact that the distribution of land across agents is determined by the outcomes of past conflicts, and so an agent's possession of land credibly indicates to the potential challengers that the agent is of a type that is difficult to defeat. With each successive round of conflict, landholding agents seem more intimidating to their adversaries, who are eventually completely dissuaded from challenging landholders.

In spite of the absence of any actual conflict in the steady state, the model economy does not attain the highest possible level of production because of the inefficient allocation of land. This inefficiency occurs because allocating land through conflict systematically favours the agents with higher ratios of the marginal product of land to the opportunity cost of conflict rather than those with the higher marginal product of land.

The results obtained above suggest several appealing directions for future research. As already suggested in the preceding section, a systematic analysis of the mechanisms that would enable the exploitation of the potential gains from trade in the steady-state environment may shed light on the development or improvement of market institutions, especially customary systems of land tenure and exchange. Extending the model to permit conflict or alliances between landholders, and hence the effective consolidation of land and economic power, might also yield insights into the development of political power and state formation.

APPENDIX

Proof of Proposition 1. For notational convenience, let $s' = s(k(\alpha', \beta'))$ and $s'' = s(k(\alpha'', \beta''))$. By the definition of $s(k)$ as an equilibrium strategy, an agent of type $k(\alpha', \beta')$ must prefer s' to s''; similarly an agent of type $k(\alpha'', \beta'')$ must prefer s'' to s'. From (6), the following conditions must be true in equilibrium:

$$\Pr(s(k_j) \geq s')(T-s') + \int_{\{k_j : s(k_j) < s'\}} p(k_j)[(k'+1)(T-s(k_j))]dk_j$$
$$\geq \Pr(s(k_j) \geq s'')(T-s'') + \int_{\{k_j : s(k_j) < s''\}} p(k_j)[(k'+1)(T-s(k_j))]dk_j$$

and

$$\Pr(s(k_j) \geq s'')(T-s'') + \int_{\{k_j : s(k_j) < s''\}} p(k_j)[(k''+1)(T-s(k_j))]dk_j$$
$$\geq \Pr(s(k_j) \geq s')(T-s') + \int_{\{k_j : s(k_j) < s'\}} p(k_j)[(k''+1)(T-s(k_j))]dk_j.$$

Subtracting the smaller side of the second expression from the larger side of the first, and subtracting the larger side of the second expression from the smaller side of the first, I obtain

$$\int_{\{k_j : s'' \leq s(k_j) < s'\}} p(k_j)[(k'-k'')(T-s(k_j))]dk_j \geq 0.$$

Thus, if $k(\alpha', \beta') = k(\alpha'', \beta'')$, then $s' = s''$; and if $k(\alpha', \beta') > k(\alpha'', \beta'')$, then $s' > s''$. ‖

Proof of Lemma 1 .

(i) For $k = 0, \alpha = \beta$. If the agent commits $s > 0$, the pay-off is $\alpha l(T-s)$, which is less than βT, the pay-off for $s = 0$.

(ii) An agent's expected utility, as is clear from (6), is continuous in k.
Suppose $s(k)$ is not continuous in k. Then, for some $\tilde{k}$, $\lim_{k+\to\tilde{k}} s(k) = s' < s'' = \lim_{k-\to\tilde{k}} s(k)$. Because an agent i knows with certainty that the opponent j will not concede defeat in the interval $(s', s'']$ and conflict is costly, i does better conceding immediately after s' than at any $s \in (s', s'']$. Furthermore, i does not concede immediately after s'', because either $p(s'')$ is negligible, in which case i is better off conceding at s', or $p(s'')$ is non-negligible, in which case i is better off waiting until $s > s''$, and hence, following this logic, no agent concedes until T.

(iii) The existence, continuity, and monotonicity of $K(s)$, defined on $s \in (0, \bar{s})$ to take values in the interval $(0, \bar{k})$, follow from the monotonicity and continuity of $s(k)$. For $K(s)$ continuous and monotonic and $p(K(s)) > 0$, the differentiability of $K(s)$ follows by lemma 1, part 4 of Fudenberg and Tirole (1986). ‖

Proof of Proposition 2 . The proof of uniqueness is by contradiction. Suppose that there exist at least two distinct equilibria, $s(k)$ and $\tilde{s}(k)$. For some $k > 0$, $s(k) \neq \tilde{s}(k)$. If $s(k) > \tilde{s}(k)$, (7) implies that $s'(k) < \tilde{s}'(k)$; likewise, if $s(k) < \tilde{s}(k)$, then $s'(k) > \tilde{s}'(k)$. But, from Lemma 1, $s(0) = \tilde{s}(0) = 0$, and from Proposition 1, $s(k)$ and $\tilde{s}(k)$ are strictly increasing. Thus, $s(k) > \tilde{s}(k)$ implies $s'(k) > \tilde{s}'(k)$, a contradiction.

Recovering the equilibrium strategy in terms of α and β, $s(k(\alpha, \beta))$, is complicated by the fact that infinitely many pairs of (α, β) correspond to the same value k. To derive the distribution of k as a function of the underlying parameters α and β, note that the probability of a given k is the probability of all possible combinations of (α, β) that produce that value k, that is, $p(k) = p\left((\alpha, \beta) : \frac{\alpha-\beta}{\beta} = k\right)$. The explicit formulae for the cumulative distribution of k, $P(k)$, and the probability density of k, $p(k)$, are derived in the Appendix section labelled "Derivation of $P(k)$ and $p(k)$". They are

$$P(k) = \begin{cases} \dfrac{(a-d(1+k))^2}{2(b-a)(d-c)(1+k)}, & \text{if } k < \frac{a}{c} - 1 \\ \dfrac{(d+c)(k+1)-2a}{2(b-a)}, & \text{if } k \in \left[\frac{a}{c}-1, \frac{b}{d}-1\right] \\ \dfrac{2bd(1+k)-b^2-(k+1)(c^2(1+k)+2a(d-c))}{2(b-a)(d-c)(1+k)}, & \text{if } k > \frac{b}{d} - 1 \end{cases}$$

$$p(k)=\frac{dP(k)}{dk}=\begin{cases}\dfrac{d^2(1+k)^2-a^2}{2(b-a)(d-c)(1+k)^2}, & \text{if } k<\frac{a}{c}-1\\ \dfrac{c+d}{2(b-a)}, & \text{if } k\in\left[\frac{a}{c}-1,\frac{b}{d}-1\right].\\ \dfrac{b^2-c^2(1+k)^2}{2(b-a)(d-c)(1+k)^2}, & \text{if } k>\frac{b}{d}-1\end{cases}$$

Substituting $P(k)$, $p(k)$ and (5) into (7) yields the expression for $s(\alpha,\beta)$ in the statement of the proposition. ||

Derivation of $P(k)$ *and* $p(k)$. Let $P(k)$ denote the cumulative probability function, that is, $P(k)=\Pr(\kappa\le k)$, and $p(k)$ denote the probability density function, $p(k)=\frac{\partial P(k)}{\partial k}$, where $k=\frac{\alpha-\beta}{\beta}$.

From the definitions of $P(k)$ and k, $P(k)=\Pr\left((\alpha,\beta):\frac{\alpha-\beta}{\beta}\le k\right)$, which can be expressed in terms of the known distributions of α and β, as follows:

$$\begin{aligned}P(k)&=\int_c^d \Pr\left(\frac{\alpha-\beta}{\beta}\le k|\beta\right)p(\beta)\partial\beta=\int_c^d \Pr(\alpha\le\beta(k+1)|\beta)\,p(\beta)\partial\beta\\ &=\int_c^d\left(\int_a^{\beta(k+1)}p(\alpha)\partial\alpha\right)p(\beta)\partial\beta. \qquad (A.1)\end{aligned}$$

Recall that α and β are distributed with constant density over the intervals $[a,b]$ and $[c,d]$, respectively:

$$p(\alpha)=\begin{cases}\frac{1}{b-a}, & \text{if } \alpha\in[a,b]\\ 0, & \text{if } \alpha\notin[a,b]\end{cases}$$

$$p(\beta)=\begin{cases}\frac{1}{d-c}, & \text{if } \beta\in[c,d]\\ 0 & \text{if } \beta\notin[c,d]\end{cases}.$$

Substituting these values in (A.1),

$$P(k)=\begin{cases}\int_{\frac{a}{k+1}}^{d}\left(\int_a^{\beta(k+1)}\frac{1}{b-a}\partial\alpha\right)\frac{1}{d-c}\partial\beta, & \text{if } k<\frac{a}{c}-1\\ \int_c^d\left(\int_a^{\beta(k+1)}\frac{1}{b-a}\partial\alpha\right)\frac{1}{d-c}\partial\beta, & \text{if } k\in\left[\frac{a}{c}-1,\frac{b}{d}-1\right]\\ \int_c^{\frac{b}{k+1}}\left(\int_a^{\beta(k+1)}\frac{1}{b-a}\partial\alpha\right)\frac{1}{d-c}\partial\beta, & \text{if } k>\frac{b}{d}-1\end{cases}$$

$$=\begin{cases}\dfrac{(a-d(1+k))^2}{2(b-a)(d-c)(1+k)}, & \text{if } k<\frac{a}{c}-1\\ \frac{(d+c)(k+1)-2a}{2(b-a)}, & \text{if } k\in\left[\frac{a}{c}-1,\frac{b}{d}-1\right].\\ \dfrac{2bd(1+k)-b^2-(k+1)\left(c^2(1+k)+2a(d-c)\right)}{2(b-a)(d-c)(1+k)}, & \text{if } k>\frac{b}{d}-1\end{cases} \qquad (A.2)$$

Recalling that the probability density $p(k)=\frac{\partial P(k)}{\partial k}$, it follows from (A.2) that

$$p(k)=\begin{cases}\dfrac{d^2(1+k)^2-a^2}{2(b-a)(d-c)(1+k)^2}, & \text{if } k<\frac{a}{c}-1\\ \dfrac{c+d}{2(b-a)}, & \text{if } k\in\left[\frac{a}{c}-1,\frac{b}{d}-1\right].\\ \dfrac{b^2-c^2(1+k)^2}{2(b-a)(d-c)(1+k)^2}, & \text{if } k>\frac{b}{d}-1\end{cases} \qquad (A.3)$$

The conditional probability density of β, $p(\beta|k)$, is the inverse of the length of the projection of the k-isoquant onto the β-coordinate axis. The conditional probability of α, $p(\alpha|k)$ can be found applying the same logic.

Suppose $b-a > d-c$; then

$$p(\beta|k) = \begin{cases} \frac{k+1}{dk+d-a}, & \text{if } k < \frac{a}{c}-1 \\ \frac{1}{d-c}, & \text{if } k \in \left[\frac{a}{c}-1, \frac{b}{d}-1\right] \\ \frac{k+1}{b-ck-c}, & \text{if } k > \frac{b}{d}-1 \end{cases}$$

$$p(\alpha|k) = \begin{cases} \frac{1}{dk+d-a}, & \text{if } k < \frac{a}{c}-1 \\ \frac{1}{(d-c)(k+1)}, & \text{if } k \in \left[\frac{a}{c}-1, \frac{b}{d}-1\right] \\ \frac{1}{b-ck-c}, & \text{if } k > \frac{b}{d}-1 \end{cases}.$$

Proof of Proposition 3. We proceed by signing the derivatives of equilibrium $s(\alpha,\beta)$ derived from Proposition 2:

$$\frac{\partial s(\alpha,\beta)}{\partial\alpha} = \begin{cases} \frac{\partial}{\partial\alpha}\left[T\left(1-\left(1-\frac{c+d}{2b-c-d}\frac{\alpha-\beta}{\beta}\right)^{(2b/c+d)-1}e^{(\alpha-\beta)/\beta}\right)\right], & \text{if } \alpha\in\left[\frac{a}{c}\beta, \frac{b}{d}\beta\right] \\ \frac{\partial}{\partial\alpha}\left[T\left(1-\frac{\alpha}{\beta}\left(1-\frac{c}{b-c}\frac{\alpha-\beta}{\beta}\right)^{2(b/c-1)}e^{(\alpha-\beta)/\beta}\right)\right], & \text{if } \alpha > \frac{b}{d}\beta \end{cases}$$

$$= \begin{cases} -\dfrac{(c+d)(-2b+c+d)e^{(\alpha-\beta)/\beta}T\left(1+\frac{(c+d)(\alpha-\beta)}{(-2b+c+d)}\right)^{2b/c+d}(\alpha-\beta)}{(c\alpha+d\alpha-2b\beta)^2} & \text{if } \alpha\in\left[\frac{a}{c}\beta, \frac{b}{d}\beta\right] \\ \dfrac{(b-c)^2e^{(\alpha-\beta)/\beta}T(\alpha-\beta)(c\alpha+b\beta)\left(\frac{c\alpha-b\beta}{c\beta-b\beta}\right)^{2b/c}}{(-c\alpha+b\beta)^3} & \text{if } \alpha > \frac{b}{d}\beta \end{cases}.$$

The top expression is positive if and only if $-2b+c+d<0$ and therefore if and only if $b>\frac{c+d}{2}$, which is always true since $b>d>c$. The bottom expression is positive if and only if $-c\alpha+b\beta>0$, that is, iff $\frac{b}{c}>\frac{\alpha}{\beta}$, which is always true since $\alpha\le b$ and $\beta\ge c$.

Similarly,

$$\frac{\partial s(\alpha,\beta)}{\partial\beta} = \begin{cases} \dfrac{(c+d)(-2b+c+d)e^{(\alpha-\beta)/\beta}T\alpha\left(1+\frac{(c+d)(\alpha-\beta)}{(-2b+c+d)}\right)^{2b/c+d}(\alpha-\beta)}{\beta(c\alpha+d\alpha-2b\beta)^2}, & \text{if } \alpha\in\left[\frac{a}{c}\beta, \frac{b}{d}\beta\right] \\ -\dfrac{(b-c)^2e^{(\alpha-\beta)/\beta}T\alpha(\alpha-\beta)(c\alpha+b\beta)\left(\frac{c\alpha-b\beta}{c\beta-b\beta}\right)^{2b/c}}{\beta(-c\alpha+b\beta)^3}, & \text{if } \alpha > \frac{b}{d}\beta \end{cases}.$$

As both expressions for $\frac{\partial s(\alpha,\beta)}{\partial\beta}$ are negative if and only if the expressions in the corresponding intervals for $\frac{\partial s(\alpha,\beta)}{\partial\alpha}$ are positive, the statement of the proposition follows. ‖

Proof of Theorem 1. Because agents remember neither identities nor actions of their past opponents, all agents have common beliefs about the distribution of types in each group in each period. The only impact of an agent's play in the current period t on expected pay-offs in subsequent periods is through the determination of his or her group membership in $t+1$. Expected future pay-offs are a function of k_i, t, and membership in N or L but not $s_i^t(k_i)$ directly. Thus, $E[U^L(k_i,t)]$ and $E[U^N(k_i,t)]$ may be treated as constraints in period t, given future sequentially rational behaviour. Therefore, equilibrium strategy must satisfy (9) and (10).

I first prove the existence of a fixed point in the best-response correspondence for period t, given $P_t^L(.)$, $P_t^N(.)$, $E[U_t^L(.)]$, $E[U_t^N(.)]$. The stationary perfect Bayesian equilibrium consists of that behavioural strategy profile in each period, with beliefs updated after each period according to Bayes' Rule. As in Athey (2001), I prove the existence of the equilibrium behavioural strategy profile using a limiting argument by first establishing equilibrium existence in a finite action game and then establishing that the limit of the equilibria of a sequence of finite-action games that converge to the continuous game is the equilibrium of the continuous action game.[12]

12. Equilibrium existence in the finite-action game is readily established using results from Athey (2001), but establishing that the limit of the sequence of equilibria is itself an equilibrium requires extending the arguments made in that paper. The chief complication here is the dependence of the value of winning on the opponent's action. Notation introduced in the proof follows that of Athey (2001) to facilitate comparison.

Consider a sequence of games $\{\Gamma^n\}$ with successively finer action spaces and which are otherwise identical to the interaction in period t. For each game in the sequence, the distribution of types k is bounded and has no mass points on $\left[\frac{a-d}{d}, \frac{b-c}{c}\right]$. The expected utility of i is well defined and finite $\forall i$, for all subsets of i's opponent's possible types, and for all possible strategies of i's opponent. The game satisfies the Single Crossing Condition for games of incomplete information, and therefore each game in the sequence of finite-action games has a Pure Strategy Nash Equilibrium, in which each player's equilibrium strategy $s_{i,n}(k_i)$ is non-decreasing (Athey, 2001, theorem 1). From the Nash condition, $\forall n$, $\forall k_i \leq 0, s_{i,n}(k_i) = 0$.

Let $\tau_i^{W*}(s')$ be the event that agent i ties and wins the tie playing s', and let $\tau_i^{L*}(s')$ be the event that agent i ties and loses playing s'.

Lemma 2. *Construct $s^*(k)$ as the limit of the sequence of non-decreasing equilibrium strategies in finite games, $s_n(k)$. Then $s^*(k)$ has no mass points on $k > 0$, that is,* $\forall i \in L$, $\forall s' \in [0, T]$, $\Pr(s_i^*(k_i) = s')\Pr(\tau_i^{L*}(s')) = 0$ *and* $\forall j \in N$, $\forall s' \in [0, T]$, $\Pr(s_j^*(k_j) = s')\Pr(\tau_j^{L*}(s')) = 0$.

Proof. We prove by contradiction. Suppose there exists player j and action $s' \in [0, T]$, such that $\Pr(s_j^*(k_j) = s')\Pr(\tau_j^{L*}(s')) > 0$. By assumption, $\Pr(\tau_i^{L*}(.)) = 0$ $\forall i \in L$. Therefore, $j \in N$ and $\Pr(\tau_j^{L*}(b)) = \Pr(s_i^*(k_i) = s')$, $\Pr(s_j^*(k_j) = s')\Pr(s_i^*(k_i) = s') > 0$.

$s_{i,n}(k_i)$ and $s_{j,n}(k_j)$ are measurable and converge almost everywhere to $s_i^*(k_i)$ and $s_j^*(k_j)$. Therefore, the sequences converge uniformly to $s_i^*(k_i)$ and $s_j^*(k_j)$ except on a set of arbitrarily small measure. Thus, $s_j^*(k_j) = s'$ on an open interval $S_j = \{k_j : s_j^*(k_i) = s'\}$. It follows that $\forall \eta > 0$, $\exists N_d$ such that $\forall n > N_d$, $\forall k_j \in S_j \backslash E_j$, $|s_{j,n}(k_j) - s'| < d$. The same argument follows for $s_i^*(k_i) = s'$ on an open interval S_i.

Choose d. Choose N_d such that $\forall n > N_d$, $\forall k \notin E$, $|s_{i,n}(k_i) - s_i^*(k_i)| < d$ and $|s_{j,n}(k_j) - s_j^*(k_j)| < d$. Therefore, $\forall n > N_d$, $\forall k_i \in S_i \backslash E_i$, $s_{i,n}(k_i) \in (s' - d, s' + d)$, and $\forall k_j \in S_j \backslash E_j$, $s_{j,n}(k_j) \in (s' - d, s' + d)$. Suppose now that each player's action space in the n-th finite game is $\left\{0 + \frac{m}{10^n} T | m = 0, \dots, 10^n\right\}$, that is, the increment between actions is $\frac{T}{10^n}$. Given $s_n(k)$ is an equilibrium, the first-order conditions for j implies

$$\begin{aligned}
&\frac{\Pr\left(\left\{k_i : s_{j,n}(k_j) - \frac{T}{10^n} \leq s_{i,n}(k_i) < s_{j,n}(k_j)\right\}\right)}{1 - \Pr\left(\left\{k_i : s_{i,n}(k_i) < s_{j,n}(k_j) - \frac{T}{10^n}\right\}\right)} (k_j(T - s_{j,n}(k_j)) + \Delta(k_j, t)) \\
&\geq \frac{T}{10^n} \\
&\geq \frac{\Pr\left(\left\{k_i : s_{j,n}(k_j) \leq s_{i,n}(k_i) < s_{j,n}(k_j) + \frac{T}{10^n}\right\}\right)}{1 - \Pr(\{k_i : s_{i,n}(k_i) < s_{j,n}(k_j)\})} \left(k_j\left(T - s_{j,n}(k_j) - \frac{T}{10^n}\right) + \Delta(k_j, t)\right).
\end{aligned}$$

$s' > 0$ and $s_{j,n}(k_j)$ a best response implies $(k_j(T - s_{j,n}(k_j)) + \Delta(k_j, t)) > 0$. If it were negative, k_j would be better off choosing lower action, since he or she must pay the bid even when there is 0 probability of winning. As $d \to 0$ and $n \to \infty$,

$$\Pr\left(\left\{k_i : s_{j,n}(k_j) \leq s_{i,n}(k_i) < s_{j,n}(k_j) + \frac{T}{10^n}\right\}\right) \to \Pr(k_i \in S_i \backslash E_i),$$

which is positive and $\frac{T}{10^n} \to 0$. This establishes a contradiction with the Nash conditions. Therefore, for all players j, $\forall s' \in (0, T)$, $\Pr(s_j^*(k_j) = s')\Pr(\tau_j^{L*}(s')) = 0$. ‖

Lemma 3. $\forall i$ *and almost all k_i, such that $s_{i,n}(k_i)$ converges to $s_i^*(k_i)$,*

(i) $E[u_i(s_i, s_k^*(.), k_i)]$ *is continuous at* $s_i = s_i^*(k_i)$;
(ii) $E[u_i(s_{i,n}(k_i), s_{j,n}(.), k_i)]$ *converges to* $E[u_i(s_i^*(k_i), s_j^*(.), k_i)]$.

Proof.

(i) The pay-offs from winning, $((k_i + 1)(T - s_j) + E[U^L(k_i, t)])$, are continuous in s_i. The pay-offs from losing, $(T - s_i + E[U^N(k_i, t)])$ are continuous in s_i. Since $s_j^*(k)$ has no mass points for $k_j > 0$, it follows that $\forall j \in N$, $\Pr(\tau_j^{L*}(s_j^*(k_j))) = 0$. Since, by assumption, $\Pr(\tau_j^{w*}(.)) = 0$, it must be that the probability of winning is continuous at $s_j^*(k_j)$ $\forall k_j > 0, \forall j \in N$.

$\forall j \in N$, $\Pr(s_j^*(k_j) = s')\Pr(\tau_j^{L*}(s')) = 0$. Therefore, $\Pr(\tau_i^{w*}(s')) = 0$ $\forall s'$, $\forall i \in L$. By assumption, $\Pr(\tau_i^{L*}(s')) = 0$ $\forall s'$, $\forall i \in L$. Hence, the probability of winning is continuous $\forall k_i > 0, \forall i \in L$. $s_{i,n}(k_i)$ converges to $s_i^*(k_i)$ $\forall i$ and almost all k_i. Hence part (i) of the lemma follows.

(ii) Consider k_i such that $s_{i,n}(k_i) \to s_i^*(k_i)$ and the probability of winning with action s_i is continuous at $s_i = s_i^*(k_i)$ (from part (i)).

$$\begin{aligned} &E[u_i(s_i^*(k_i), s_j^*(k_j), k_i)] - E[u_i(s_{i,n}(k_i), s_{j,n}(k_j), k_i)] \\ &= (E[u_i(s_i^*(k_i), s_j^*(k_j), k_i)] - E[u_i(s_{i,n}(k_i), s_j^*(k_j), k_i)]) \\ &+ (E[u_i(s_{i,n}(k_i), s_j^*(k_j), k_i)] - E[u_i(s_{i,n}(k_i), s_{j,n}(k_j), k_i)]). \end{aligned}$$

The first difference on the R.H.S. converges to 0 as $n \to \infty$ (by part (i)). It remains to prove that $E[u_i(s_i', s_{j,n}(k_j), k_i)]$ converges uniformly to $E[u_i(s_i', s_j^*(k_j), k_i)]$ in the neighbourhood of $s_i' = s_i^*(k_i)$. Pick $\eta > 0$, $d > 0$. There exists E with measure $< \eta$ and N_d such that $\forall k \notin E$, $\forall n > N_d$, and for all players i, $|s_i^*(k_i) - s_{i,n}(k_i)| < d$. Then, $\forall n > N_d$,

$$\begin{aligned} &\max_{s_i' \in (s_i^*(k_i)-d, s_i^*(k_i)+d)} |E[u_i(s_i', s_{j,n}(k_j), k_i)] - E[u_i(s_i', s_j^*(k_j), k_i)]| \\ &< \max_{s_i' \in (s_i^*(k_i)-d, s_i^*(k_i)+d)} \Bigg| \Pr(s_i' \in (s_{j,n}(k_j), s_j^*(k_j)|k_j \notin E)\Pr(k_j \notin E)(k_i(T-(s_j^*(k_j)-s)) + \Delta(k_i,t)) \\ &\quad + \Pr(s_i' \in (s_{j,n}(k_j), s_j^*(k_j)|k_j \in E)\Pr(k_j \in E)(k_i(T-(s_j^*(k_j)-s)) + \Delta(k_i,t)) \\ &\quad + \int_{k_j \notin E} |s_{j,n}(k_j) - s_j^*(k_j)|p(k_j)dk_j + \int_{k_j \in E} |s_{j,n}(k_j) - s_j^*(k_j)|p(k_j)dk_j \Bigg|. \end{aligned}$$

As $d \to 0$, $\Pr(s_i' \in (s_{j,n}(k_j), s_j^*(k_j))|k_j \notin E) \to \Pr(s_i^*(k_i) = s_j^*(k_j)) = 0$, and $|s_{j,n}(k_j) - s_j^*(k_j)| \to 0$. Since d can be chosen so that the first two terms are arbitrarily close to 0, and η can be chosen so that the last two terms are arbitrarily close to 0, it follows that $|E[u_i(s_i', s_{j,n}(k_j), k_i)] - E[u_i(s_i', s_j^*(k_j), k_i)]|$ is bounded above by a value arbitrarily close to 0 for s_i' near $s_i^*(k_i)$, and part (ii) of the lemma follows. ||

By Lemma 2, $s_i^*(k_i)$ is a best response to $s_j^*(k_j)$ for almost all k_i. $\forall n$ and almost all k_i,

$$E[u_i(s_{i,n}(k_i), s_{j,n}(.), k_i)] \geq E[u_i(s_i', s_{j,n}(.), k_i)]\ \forall s' \in \left\{0 + \frac{m}{10^n}T | m = 0, \ldots, 10^n\right\}.$$

Let D^i be set of all actions s' such that for large enough N, $s' \in \left\{0 + \frac{m}{10^n}T | m = 0, \ldots, 10^n\right\}\ \forall n > N$. From Lemma 3, if $s' \in D^i$, $E[u_i(s_i^*(k_i), s_j^*(.), k_i)] \geq E[u_i(s', s_j^*(.), k_i)]$. Suppose $s' \notin D^i$. If $E[u_i(s', s_j^*(.), k_i)]$ is continuous at s', then $\exists\{s^k\}$, $s^k \in D^i$, that converges to s'. Hence $E[u_i(s^k, s_j^*(.), k_i)] \to E[u_i(s', s_j^*(.), k_i)]$. If $\Pr(s_j^*(k_j) = s') > 0$ and $k_i > 0$, then $\exists\delta > 0$ such that $s' + \delta$ is used on a set of opponent's types of measure 0 in the limit and such that $s' + \delta \in D^i$. Since $s' + \delta$ wins against j at s', it follows that for sufficiently small δ, $E[u_i(s' + \delta, s_j^*, k_i)] > E[u_i(s_i', s_j^*(.), k_i)]$. Since $s' + \delta \in D^i$, it must be that $E[u_i(s_i^*(k_i), s_j^*(.), k_i)] \geq E[u_i(s_i + \delta, s_j^*(.), k_i)]$, which completes the proof. ||

Proof of Proposition 4 .

(i) This follows from the fact that an agent of type $k \leq 0$ is at least as productive using the B technology as the A technology and thus only reduces the productive output by allocating a positive amount of time to conflict.

(ii) The proof proceeds in the same manner as the proof of Proposition 1. The conditions for the optimal strategies are:

$$\begin{aligned} s_t^N(k_i) \in \arg\max \Bigg(&(1 - P_t^L(s_t^N))((T - s_t^N) + E[U_t^N(k_i)]) \\ &+ \int_{\{k_j : s_t^L(k_j) < s_t^N\}} p_t^L(k_j)[(k_i + 1)(T - s_t^L(k_j)) + E[U_t^L(k_i)]]dk_j \Bigg) \end{aligned}$$

and

$$s_t^L(k_i) \in \arg\max \Bigg((1 - P_t^N(s^L))((T - s_t^L) + E[U_t^N(k_i)])$$

$$+ \int_{\{k_j : s_t^N(k_j) < s_t^L\}} p_t^N(k_j)[(k_i + 1)(T - s_t^N(k_j)) + E[U_t^L(k_i)]]dk_j \Bigg),$$

where $E[U_t^N(k_i)]$ is the expected sum of discounted future pay-offs for agent k_i evaluated in period t, conditional on him or her beginning period $t+1$ in group N, and $E[U_t^L(k_i)]$ is his or her expected sum of discounted future pay-offs if period begins at $t+1$ in group L. Let $\Delta_t(k)$ represent the difference $E[U_t^L(k)] - E[U_t^N(k)]$. As in the proof of Proposition 1, the conditions for the optimal strategies can be manipulated to produce expressions of the form

$$\int_{\{k_j : s'' \le s(k_j) < s'\}} p(k_j)(k' - k'')(T - s(k_j))dk_j$$

$$+ [\Pr(s(k_j) < s') - \Pr(s(k_j) < s'')](\Delta(k') - \Delta(k'')) \ge 0$$

for each group, N and L. Using the additional fact that $\Delta(k)$ is increasing in k, if $k' > k''$, it must be that $s' > s''$ and vice versa.

(iii) From (10) and (9), the expected utility of agent i is continuous in k_i. Given that pay-offs are continuous with respect to agent type, the equilibrium strategy is continuous with respect to agent type by the same argument as in the proof of Lemma 1, part 2.

(iv) The existence, continuity, and monotonicity of $K_t^L(s)$, defined on $s \in (0, \bar{s}_t^L)$ to take values in the interval $(0, \bar{k}_t^L)$, follow from the monotonicity and continuity of $s_t^L(k)$. For $K_t^L(s)$ continuous and monotonic and $p(K_t^L(s)) > 0$, the differentiability of $K_t^L(s)$ follows from lemma 1, part 4 of Fudenberg and Tirole (1986). The same argument holds for $K_t^N(s)$. ‖

Proof of Proposition 5 . It is useful to establish the following lemma before proceeding with the proof of the proposition. Let $k^* = k$ such that $P_1(k^*) = \frac{1}{2}$.

Lemma 4. *In the second period,*

(i) $P_2^N(k) = \frac{|N|+|L|}{|N|} P_1(k) - \frac{|L|}{|N|}(P_1(k))^2$ *and* $P_2^L(k) = (P_1(k))^2$;

(ii) $p_2^N(k) = \frac{|N|+|L|}{|N|} p_1(k) - 2\frac{|L|}{|N|} p_1(k)P_1(k)$ *and* $p_2^L(k) = 2p_1(k)P_1(k)$;

(iii) $\frac{\partial p_2^N(k)}{\partial k} < 0$;

(iv) $\frac{\partial}{\partial k}\left(\frac{1 - P_2^N(k)}{p_2^N(k)}\right) > 0$.

Proof. Because the actual population of types is fixed, $\forall t$, $\forall k$,

$$(|N| + |L|)p_1(k) = |N|p_t^N(k) + |L|p_t^L(k)$$

$$(|N| + |L|)P_1(k) = |N|P_t^N(k) + |L|P_t^L(k).$$

Solving for $P_2^N(k)$, $P_2^L(k)$, $p_2^N(k)$, and $p_2^L(k)$ yields the expressions given in the statement of the lemma.

Substituting the known expressions for $P_1(k)$ and $p_1(k)$ derived in the Appendix section labelled "Derivation of $P(k)$ and $p(k)$" and taking the appropriate derivatives, it is evident that $\frac{\partial p_2^N(k)}{\partial k} < 0$ and $\frac{\partial}{\partial k}\left(\frac{1-P_2^N(k)}{p_2^N(k)}\right) > 0$. ‖

To prove Proposition 5, note first that from Proposition 4, $s_2^N(0) = s_2^L(0) = 0$ and $s_2^N(k)$ and $s_2^L(k)$ are each monotonically increasing in k; therefore it must be that either $\exists k \in (0, \bar{k})$ such that $s_2^N(k) = s_2^L(k)$ or either $s_2^N(k) > s_2^L(k)$ or $s_2^N(k) < s_2^L(k)$ for all $k \in (0, \bar{k})$.

Suppose that $s_2^N(k) = s_2^L(k)$ for some $k \in (0, \bar{k})$ and, therefore, that $K_2^L(s) = K_2^N(s)$ for the corresponding value of s. From (11), it follows that

$$\frac{1 - P_t^L(k)}{p_t^L(k)} = \frac{1 - P_t^N(k)}{p_t^N(k)}. \tag{A.4}$$

From parts (i) and (ii) of Lemma 4,

$$\frac{1 - P_2^L(k)}{p_2^L(k)} = \frac{1 - (P_1(k))^2}{2p_1(k)P_1(k)} \quad \text{and}$$

$$\frac{1 - P_2^N(k)}{p_2^N(k)} = \frac{|N| - (|N| + |L|)P_1(k) - |L|(P_1(k))^2}{(|N| + |L|)p_1(k) - 2|L|p_1(k)P_1(k)}.$$

Substituting into (A.4) and reducing yields $P_1(k) = 1$, which is false $\forall k \in (0, \bar{k})$. Therefore, $s_2^N(k) \neq s_2^L(k)$ and either $s_2^N(k) > s_2^L(k)$ or $s_2^N(k) < s_2^L(k)$ $\forall k \in (0, \bar{k})$.

Given that $s_2^N(0) = s_2^L(0)$, both $s_2^N(k)$ and $s_2^L(k)$ are monotonically increasing in k, and there does not exist a $k \in (0, \bar{k}_2)$ such that $s_2^N(k) = s_2^L(k)$, it is enough to show that $s_2^L(k) > s_2^N(k)$ for some $k \in (0, \bar{k}_2)$ to demonstrate that $s_2^L(k) > s_2^N(k)$ for any $k \in (0, \bar{k}_2)$. I prove, by contradiction, that $s_2^L(k) > s_1(k) > s_2^N(k)$. I then prove that $s_{t+1}^L(k) > s_t^L(k) > s_{t+1}^N(k)$.

Suppose $s_2^L(k) < s_1(k)$. Consider $\tilde{k} > 0$ such that $s_1^L(\tilde{k}) = \tilde{s}$, where $\tilde{s}$ is sufficiently close to 0 that $P_2^N(\tilde{s}) > P_1(\tilde{s})$, and such that $\tilde{k}$ is sufficiently close to 0 that an agent of that type is more likely to lose than to win in the first period; thus $p_2^N(\tilde{k}) > p_1(\tilde{k})$. (From Lemma 4 and the continuity of $P_2^N(k)$ and $s_2^L(k)$, $\exists \tilde{k} > 0$ such that $P_2^N(\tilde{s}) > P_1(\tilde{s})$.) From the first-order conditions (11), it must be that

$$s_1^L(\tilde{k}) = \tilde{s} = -\frac{1 - P_1^N(\tilde{s})}{\tilde{k} p_1^N(\tilde{s})} + T + \frac{\Delta(\tilde{k})}{\tilde{k}}.$$

$P_2^N(\tilde{s}) > P_1(\tilde{s})$ and $s_2^L(k) < s_1(k)$ imply $p_2^N(\tilde{s}) < p_1(\tilde{s})$. Combining inequalities, $p_2^N(\tilde{s}) < p_1(\tilde{s}) = p_1(\tilde{k}) < p_2^N(\tilde{k})$. Given part (iii) of Lemma 4, $p_2^N(\tilde{s}) < p_2^N(\tilde{k})$ implies $s_2^N(\tilde{k}) < \tilde{s} = s_1^N(\tilde{k})$.

Given the assumption that $s_2^L(k) < s_1(k)$, from the first-order conditions (11), it must be that

$$\frac{1 - P_t^N(K_t^N(s_t^L(k)))}{p_t^N(K_t^N(s_t^L(k)))K_t^{N\prime}(s_t^L(k))} > \frac{1 - P_1(k)}{p_1(k)}.$$

From Lemma 4, $P_2^N(k) > P_1(k)$ $\forall k < \bar{k}$ and $p_2^N(k) > p_1(k)$ $\forall k < k^*$, where k^* is such that $P_1(k^*) = \frac{1}{2}$. Hence

$$\frac{1 - P_2^N(k)}{p_2^N(k)} < \frac{1 - P_1(k)}{p_1(k)}.$$

Combined with the fact that $\frac{\partial}{\partial k}\left(\frac{1 - P_2^N(k)}{p_2^N(k)}\right) > 0$ from Lemma 4, this implies that $K_t^N(s_t^L(k)) > k$, that is, that $s_2^N(k) > s_1(k)$, a contradiction.

Therefore, $s_1(k) < s_2^L(k)$. It is readily shown, using the same reasoning as in the preceding paragraph, that $s_2^N(k) < s_1(k)$ follows from $s_1(k) < s_2^L(k)$.

Because $s_2^N(k) < s_1(k) < s_2^L(k)$, $P_3^N(0) > P_2^N(0)$, $P_3^L(0) < P_2^L(0)$, and $p_3^L(k) < p_3^N(k)$ for $k < k^*$; hence, it can be shown that $s_t^L(k) > s_t^N(k)$ for all $k \in (0, \bar{k}_t)$ for $t \geq 3$ using the same reasoning. ||

Proof of Theorem 2 .

(i) Because the population of agents is fixed and finite, there is a finite number of combinations of L agents, that is, there is a finite number of divisions of agents between sets N and L. Because, from Proposition 4, $\forall t$, $\frac{\partial s^L}{\partial k} > 0$ and $\frac{\partial s^N}{\partial k} > 0$, and from Proposition 5, $\forall t > 1$, $s_t^L(k) > s_t^N(k)$, agents in L can be replaced only by agents in N of higher type k. Therefore, in a finite number of periods, L and N converge to L^* and N^* such that, $\forall i \in L^*$ and $\forall j \in N^*$, $s^L(k_i) > s^N(k_j)$.[13]

13. I thank David Austen-Smith for drawing my attention to the fact that the time required for convergence is finite.

(ii) As $t \to \infty$, $\forall k$ such that $\lim_{t\to\infty} p_t^N(k) > 0$, $P_t^L(K_t^L(s_t^N(k))) \to 0$. Therefore, $\forall k$ such that $\lim_{t\to\infty} p_t^N(k) > 0$, $\frac{1-P_t^L(K_t^L(s_t^N(k)))}{p_t^L(K_t^L(s_t^N(k)))} \to \infty$.

Let $\hat{k}$ be the highest type in N^*. Because $\frac{\partial s^N}{\partial k} \geq 0$ (from Proposition 4), if $s^N(\hat{k}) = 0$, then $s^N(k) \to 0\ \forall k \in N^*$. From (11),

$$s_t^N(k) = T + \frac{\Delta(k)}{k} - \frac{1-P_t^L(K_t^L(s_t^N(k)))}{kp_t^L(K_t^L(s_t^N(k)))K_t^{L\prime}(s_t^N(k))}.$$

Therefore, $s^N(\hat{k}) = 0$ when

$$(\hat{k}T + \Delta(\hat{k})) \leq \frac{1-P_t^L(K_t^L(s_t^N(\hat{k})))}{p_t^L(K_t^L(s_t^N(\hat{k})))K_t^{L\prime}(s_t^N(\hat{k}))}.$$

Given that $\hat{k}T + \Delta(\hat{k}) \leq \hat{k}T + \frac{\delta}{1-\delta}\hat{k}T = \frac{\hat{k}T}{1-\delta}$, it is sufficient that

$$\frac{\hat{k}T}{1-\delta} \leq \frac{1-P_t^L(K_t^L(s_t^N(\hat{k})))}{p_t^L(K_t^L(s_t^N(\hat{k})))K_t^{L\prime}(s_t^N(\hat{k}))}.$$

Given that $\lim_{t\to\infty} \frac{1-P_t^L(K_t^L(s_t^N(\hat{k})))}{p_t^L(K_t^L(s_t^N(\hat{k})))K_t^{L\prime}(s_t^N(\hat{k}))} = \infty$ and $\frac{\hat{k}T}{1-\delta}$ is finite and constant with respect to time, there must exist some t^* such that

$$\frac{1-P_{t*}^L(K_{t*}^L(s_{t*}^N(\hat{k})))}{p_{t*}^L(K_{t*}^L(s_{t*}^N(\hat{k})))K_{t*}^{L\prime}(s_{t*}^N(\hat{k}))} \geq \frac{\hat{k}T}{1-\delta} > \frac{1-P_{t*-1}^L(K_{t*-1}^L(s_{t*-1}^N(\hat{k})))}{p_{t*-1}^L(K_{t*-1}^L(s_{t*-1}^N(\hat{k})))K_{t*-1}^{L\prime}(s_{t*-1}^N(\hat{k}))}. \quad \|$$

Proof of Theorem 3. Consider two distinct economies such that

$$\tilde{s}_t^L(k) - \tilde{s}_t^N(k) > \hat{s}_t^L(k) - \hat{s}_t^N(k) \quad \forall t > 1, \forall k > 0,$$

where (suppressing subscripts and superscripts) $\tilde{s}(k)$ and $\hat{s}(k)$ denote the respective equilibrium strategies played in these economies. By Proposition 5, this implies $\tilde{s}_t^L(k) > \hat{s}_t^L(k)$ and $\tilde{s}_t^N(k) < \hat{s}_t^N(k)$. Therefore,

$$\tilde{K}_t^L(\tilde{s}_t^N(k)) < \hat{K}_t^L(\hat{s}_t^N(k)) < \hat{K}_t^N(\hat{s}_t^L(k)) < \tilde{K}_t^N(\tilde{s}_t^L(k)).$$

Hence,

$$\tilde{P}_t^L(\tilde{K}_t^L(\tilde{s}_t^N(k))) < \hat{P}_t^L(\hat{K}_t^L(\hat{s}_t^N(k))) \quad \forall t > 1, \tag{A.5}$$

that is, the proportion of agents in $\tilde{L}_t$ that $j \in \tilde{N}_t$ of type k would defeat is less than the proportion of agents in $\hat{L}_t$ that $j \in \hat{N}_t$ of type k would defeat.

From (A.5), it follows that, $\forall k$, such that $\lim p_t^N(k) > 0$, as $t \to \infty$, $\tilde{P}_t^L(\tilde{K}_t^L(\tilde{s}_t^N(k)))$ goes to 0 faster than $\hat{P}_t^L(\hat{K}_t^L(\hat{s}_t^N(k)))$. It follows that, as $t \to \infty$, $\frac{1-\tilde{P}_t^L(\tilde{K}_t^L(\tilde{s}_t^N(k)))}{\tilde{p}_t^L(\tilde{K}_t^L(\tilde{s}_t^N(k)))\tilde{K}_t^{L\prime}(\tilde{s}_t^N(k))}$ goes to ∞ faster than $\frac{1-\hat{P}_t^L(\hat{K}_t^L(\hat{s}_t^N(k)))}{\hat{p}_t^L(\hat{K}_t^L(\hat{s}_t^N(k)))\hat{K}_t^{L\prime}(\hat{s}_t^N(k))}$. Following the proof of part 2 of Theorem 2, $\tilde{t}^*$ such that

$$\frac{1-\tilde{P}_{t*}^L(\tilde{K}_{t*}^L(\tilde{s}_{t*}^N(k)))}{\tilde{p}_{t*}^L(\tilde{K}_{t*}^L(\tilde{s}_{t*}^N(k)))\tilde{K}_{t*}^{L\prime}(\tilde{s}_{t*}^N(k))} \geq \frac{kT}{1-\delta} > \frac{1-\tilde{P}_{t*-1}^L(\tilde{K}_{t*-1}^L(\tilde{s}_{t*-1}^N(k)))}{\tilde{p}_{t*-1}^L(\tilde{K}_{t*-1}^L(\tilde{s}_{t*-1}^N(k)))\tilde{K}_{t*-1}^{L\prime}(\tilde{s}_{t*-1}^N(k))}$$

is less than $\hat{t}^*$ such that

$$\frac{1-\hat{P}_{t*}^L(\hat{K}_{t*}^L(\hat{s}_{t*}^N(k)))}{\hat{p}_{t*}^L(\hat{K}_{t*}^L(\hat{s}_{t*}^N(k)))\hat{K}_{t*}^{L\prime}(\hat{s}_{t*}^N(k))} \geq \frac{kT}{1-\delta} > \frac{1-\hat{P}_{t*-1}^L(\hat{K}_{t*-1}^L(\hat{s}_{t*-1}^N(k)))}{\hat{p}_{t*-1}^L(\hat{K}_{t*-1}^L(\hat{s}_{t*-1}^N(k)))\hat{K}_{t*-1}^{L\prime}(\hat{s}_{t*-1}^N(k))}.$$

From (A.5), the probability that j would move from $\tilde{N}_t$ into $\tilde{L}_{t+1}$ is less than the probability that $j \in \hat{N}_t$ of type k would move from $\hat{N}_t$ into $\hat{L}_{t+1}$. Likewise, $1 - \tilde{P}_t^N(\tilde{K}_t^N(\tilde{s}_t^L(k))) < 1 - \hat{P}_t^N(\hat{K}_t^N(\hat{s}_t^L(k)))$, that is, the proportion of agents in $\tilde{N}_t$ who defeat $i \in \tilde{L}$ of type k is less than the proportion of agents in $\hat{N}_t$ who could defeat $i \in \hat{L}$ of type k. Hence the probability of an agent of type k moving from $\tilde{L}_t$ into $\tilde{N}_{t+1}$ is less than the probability of an agent of type k moving from $\hat{L}_t$ into $\hat{N}_{t+1}$. Therefore, $\left|\tilde{N}^* \cap \left(k^*, \frac{b-c}{c}\right)\right| > \left|\hat{N}^* \cap \left(k^*, \frac{b-c}{c}\right)\right|$. $\|$

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REFERENCES

ATHEY, S. (2001), "Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information", *Econometrica*, **69**, 861–890.

BAKER, M. J. (2003), "An Equilibrium Conflict Model of Land Tenure in Hunter-Gatherer Societies", *Journal of Political Economy*, **111**, 124–173.

BERMAN, H. (1983) *Law and Revolution: The Formation of Western Legal Institutions* (Cambridge, MA: Harvard University Press).

DE MEZA, D. and GOULD, J. R. (1992), "The Social Efficiency of Private Decisions to Enforce Property Rights", *Journal of Political Economy*, **100**, 561–580.

DE ROOVER, R. A. (1965), "The Organization of Trade", in M. M. Postan, E. E. Rich and E. Miller (eds.) *Cambridge Economic History of Europe*, Vol. III (Cambridge, MA: Cambridge University Press) 112–118.

DE SOTO, H. (1989) *The Other Path: The Invisible Revolution in the Third World* (New York: Harper and Row).

ELLICKSON, R. C. (1991) *Order Without Law: How Neighbors Settle Disputes* (Cambridge, MA: Harvard University Press).

ENSMINGER, J. (1992) *Making a Market: The Institutional Transformation of an African Society* (Cambridge, MA: Cambridge University Press).

FIELD, E. (2002), "Entitled to Work: Urban Property Rights and Labor Supply in Peru" (Princeton University Research Program in Development Studies Working Paper # 220).

FIRMIN-SELLERS, K. (1996) *The Transformation of Property Rights in the Gold Coast: An Empirical Analysis Applying Rational Choice Theory* (Cambridge, MA: Cambridge University Press).

FRYE, T. and SHLEIFER, A. (1997), "The Invisible Hand and the Grabbing Hand: Private Shops in Moscow and Warsaw", *American Economic Review Papers and Proceedings*, **87**, 354–358.

FUDENBERG, D. and TIROLE, J. (1986), "Exit in Duopoly", *Econometrica*, **54**, 943–960.

GROSSMAN, H. I. (2001), "The Creation of Effective Property Rights", *American Economic Review Papers and Proceedings*, **91**, 347–352.

GROSSMAN, H. I. and KIM, M. (1995), "Swords or Plowshares? A Theory of the Security of Claims to Property", *Journal of Political Economy*, **103**, 1275–1288.

GROSSMAN, H. I. and KIM, M. (1996), "Predation and Accumulation", *Journal of Economic Growth*, **1**, 333–351.

HAFER, C. (2003), "Contracting with the Powerful: The Politico-Economic Foundations of Feudalism" (New York University Department of Politics Mimeo).

HIRSHLEIFER, J. (1991), "The Technology of Conflict as an Economic Activity", *American Economic Review*, **81**, 130–134.

HIRSHLEIFER, J. (1995), "Anarchy and Its Breakdown", *Journal of Political Economy*, **103**, 26–52.

HOTTE, L. (2001), "Conflicts Over Property Rights and Natural-Resource Exploitation at the Frontier", *Journal of Development Economics*, **66**, 1–21.

KAPLOW, L. and SHAVELL, S. (2002), "Economic Analysis of Law", in A. J. Auerbach and M. Feldstein (eds.) *Handbook of Public Economics* (Amsterdam: North-Holland) 1661–1784.

KRISHNA, V. and MORGAN, J. (1997), "An Analysis of the War of Attrition and the All-Pay Auction", *Journal of Economic Theory*, **72**, 343–362.

LIBECAP, G. D. (1978), "Economic Variables and the Development of the Law: The Case of Western Mineral Rights", *Journal of Economic History*, **38**, 338–362.

MUTHOO, A. (2003), "On the Foundations of Basic Property Rights, Part I: A Model of the State-of-Nature with Two Players" (University of Essex Department of Economics Mimeo).

OSTROM, E. (1990) *Governing the Commons: The Evolution of Institutions for Collective Action* (Cambridge, MA: Cambridge University Press).

SHLEIFER, A. (1994), "Establishing Property Rights" (Institute for Economic Research Discussion Paper 1683, Harvard University).

SKAPERDAS, S. (1992), "Cooperation, Conflict, and Power in the Absence of Property Rights", *American Economic Review*, **82**, 720–739.

SKAPERDAS, S. and SYROPOULOS, C. (1995), "Gangs as Primitive States", in G. Fiorentini and S. Peltzman (eds.) *The Economics of Organised Crime* (Cambridge, MA: Cambridge University Press).

SKAPERDAS, S. and SYROPOULOS, C. (1996*a*), "Can the Shadow of the Future Harm Cooperation?", *Journal of Economic Behavior and Organization*, **29**, 355–372.

SKAPERDAS, S. and SYROPOULOS, C. (1996*b*), "Competitive Trade with Conflict", in M. R. Garfinkel and S. Skaperdas (eds.) *The Political Economy of Conflict and Appropriation* (Cambridge, MA: Cambridge University Press).

SUGDEN, R. (1986) *The Economics of Rights, Co-operation and Welfare* (Oxford: Basil Blackwell).

UMBECK, J. (1977), "The California Gold Rush: A Study of Emerging Property Rights", *Explorations in Economic History*, **14**, 197–226.

UMBECK, J. (1981) *A Theory of Property Rights* (Ames, IA: Iowa State University Press).