Leibniz's Theory of Propositional Terms: A Reply to Massimo Mugnai

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In his contribution to this volume of the *Leibniz Review*, Massimo Mugnai provides a very helpful review of our paper "The Logic of Leibniz's *Generales Inquisitiones de Analysi Notionum et Veritatum*." Mugnai's review, which contains many insightful comments and criticisms, raises a number of issues that are of central importance for a proper understanding of Leibniz's logical theory. We are grateful for the opportunity to explain our position on these issues in greater depth than we did in the paper.

In his review, Mugnai puts forward two main criticisms of our reconstruction of the logical calculus developed by Leibniz in the *Generales Inquisitiones*. Both of these criticisms relate to Leibniz's theory of propositional terms. The first concerns certain extensional features that we ascribe to the relation of conceptual containment as it applies to propositional terms. The second concerns the syntactic distinction we draw between propositions and propositional terms in the language of Leibniz's calculus. In what follows, we address each of these criticisms in turn, beginning with the second.

Propositions and Propositional Terms

One of the most innovative aspects of the *Generales Inquisitiones* is Leibniz's introduction into his logical theory of the device of propositional terms. By means of this device, Leibniz proposed to represent propositions of the form *A is B* by terms such as *A's being B*. Leibniz regarded these propositional terms as abstract terms (*abstracti termini,* A VI 4 987), with the important qualification that they are merely "logical" or "notional" abstracts (A VI 4 740; cf. 992). As Mugnai emphasizes in his review, Leibniz took the theory of propositional terms to be one of his principal contributions to the study of logic, highlighting their philosophical significance in his essay *De Abstracto et Concreto* and again, several years later, in the *Noveaux Essais*. The *Generales Inquisitiones* stands out among Leibniz's writings in that it contains his first systematic treatment of propositional terms and the most detailed exposition of the principles governing their logical operation.

In the *Generales Inquisitiones*, Leibniz first explores the possibility of assimilating propositions to terms in §§61–75. He concludes this preliminary discussion on an optimistic note, proclaiming that, if such an assimilation could be effected, it would be of great value to his logic:

> If, as I hope, I can conceive of all propositions as terms, and of all hypothetical propositions as categorical, and if I can give a universal treatment of them all, this promises a wonderful ease in my symbolism and analysis of concepts, and will be a discovery of the greatest moment. (*Generales Inquisitiones* §75)

In Leibniz's view, when a proposition is "conceived of" as a term, the result is a new, propositional term.[1] Just like other terms, propositional terms can serve as the syntactic constituents *A* and *B* in propositions of the form *A is B* and *A coincides with B*. For example, if *C's being D* and *E's being F* are the propositional terms generated from the propositions *C is D* and *E is F*, respectively, then *C's being D is E's being F* is a well-formed proposition of Leibniz's calculus (§138). By contrast, the propositions *C is D* and *E is F* cannot themselves appear as syntactic constituents in further propositions. Thus, for example, *C is D is E is F* is not a well-formed proposition of Leibniz's calculus.

In his review, Mugnai is skeptical that Leibniz draws such a sharp distinction in the syntax of his calculus between propositions and propositional terms. In particular, he raises doubts that Leibniz consistently enforces the requirement that only propositional terms, and not propositions, can take the place of the schematic letters '*A*' and '*B*' in propositional forms such as '*A is B*'. We agree with Mugnai that, in the *Generales Inquisitiones*, Leibniz does not systematically enforce this requirement prior to §61 of the treatise. In these early portions of the treatise, Leibniz states repeatedly that the schematic letters '*A*' and '*B*' can stand for either terms or propositions (A VI 4 748 n. 6, §4, §13, §35, §55). These statements would not be correct if the above syntactic requirement were in force. Instead, what Leibniz ought to have said is that '*A*' and '*B*' can stand for either non-propositional or propositional terms (or, equivalently, that they can stand for any terms, propositional or otherwise).

In response to this apparent problem with our reconstruction of Leibniz's calculus, it is worth noting that, in the *Generales Inquisitiones*, all of Leibniz's claims to the effect that the schematic letters of the calculus can stand for either terms or propositions occur prior to his discovery (*inventum*, §75) of propositional terms in §§61–75. Thus, it is perhaps not surprising that, prior to this discovery, the distinction between propositions and propositional terms is virtually absent from the

Generales Inquisitiones. Leibniz does mention propositional terms in the opening paragraph of the treatise, remarking that the only abstract terms to be admitted in his calculus are “those which are logical or notional, such as . . . *A’s being B*” (A VI 4 740). However, as can be seen from the apparatus to the text in the *Academy Edition*, this reference to propositional terms is a later addition by Leibniz absent from the original version of the opening paragraph. In this original version, Leibniz had excluded from consideration all abstract terms without qualification, only later adding the exception for propositional terms. It is not clear when Leibniz added this exception, and it may well be the case that he did so only after the “discovery” of propositional terms in §§61–75. Thus, there is no evidence to suggest that, when Leibniz began writing the *Generales Inquisitiones*, he already had in mind the distinction between propositions and propositional terms.[2]

Given this, we should expect the syntactic distinction between propositions and propositional terms to begin to manifest itself in the *Generales Inquisitiones* only after §§61–75. Now, Mugnai observes that, in §75 itself, immediately after introducing propositional terms, “Leibniz writes that with the letter ‘*A*’ he understands *either a term or a proposition*” (p. 134). The relevant portion of §75 reads as follows:

> In general, I call a term false if, in the case of incomplex terms, it is an impossible term … and if, in the case of complex terms, it is an impossible proposition or, at any rate, a proposition which cannot be proved. … Thus, by ‘*A*’ I understand either an incomplex term or a proposition. (*Generales Inquisitiones* §75)

In the final sentence of this passage, Leibniz states that ‘*A*’ can stand for either an incomplex term or a proposition. The added qualification ‘incomplex’ refers to the distinction between complex and incomplex terms adduced in the first sentence of the passage. As Leibniz makes clear elsewhere in the *Generales Inquisitiones*, this distinction corresponds to that between propositional and non-propositional terms.[3] Since the natural complement to the class of incomplex terms is that of complex terms, the most plausible reading of the final sentence is that ‘*A*’ can stand for either an incomplex (non-propositional) term or a complex (propositional) term. This reading is further supported by the fact that Leibniz seems to equate complex terms with propositions in the first sentence of the passage. Indeed, if the label ‘proposition’ in the final sentence was not meant to cover complex terms, Leibniz would be omitting these terms from the scope of the schematic letters of his calculus, which is clearly not his intent in §75.

This reading also helps to explain why Leibniz added the qualification 'incomplex' to the heading 'terms' in the last sentence of the passage just quoted. Had he omitted this qualification, the classes of 'terms' and 'propositions' would fail to be disjoint, given that the latter is here understood to include complex terms. If the class of 'propositions' was not meant to include complex terms, it would be unclear why Leibniz felt the need to add this qualification instead of simply repeating his earlier claims that '*A*' can stand for either a term or a proposition.[4]

Mugnai is, of course, right to point out that the literal reading of the final sentence implies that '*A*' can stand for a proposition. But given the overall context, it is more likely that Leibniz's intention here is to make an oblique reference to propositional terms by citing the propositions from which these terms are generated. In the later sections of the *Generales Inquisitiones*, at any rate, Leibniz emphasizes that the propositions of his calculus express relations between terms rather than propositions. Thus, for example, he writes that "a proposition is that which states what term is or is not contained in another" (§195; similarly, §184). Moreover, in these later sections, Leibniz is careful to ensure that a proposition is first transformed into a propositional term before it is used as a syntactic constituent in the construction of further propositions (§198.7). In our view, such circumspection on Leibniz's part is indicative of his mature conception of propositional terms, according to which a sharp syntactic distinction is to be drawn between these terms and the propositions from which they are generated. This conception, which evolves gradually over the course of the treatise and represents the considered position finally adopted by Leibniz in the *Generales Inquisitiones*, leads him eventually to abandon his earlier claims to the effect that '*A*' can stand for a proposition. Accordingly, such claims do not appear anywhere in the treatise after §75.

In our view, then, the examples adduced by Mugnai do not undermine our claim that, in the mature theory of propositional terms expounded in the later sections of the *Generales Inquisitiones*, Leibniz observes a sharp distinction in the syntax of his calculus between propositions and propositional terms.[5] Of course, acknowledging such a syntactic distinction does not settle the question as to how exactly a propositional term is related to the proposition from which it is generated. It is sometimes maintained that, according to Leibniz's theory of propositional terms, propositions just are terms or concepts.[6] In this case, a propositional term would be identical with the corresponding proposition. Such a view, however, is not borne out by the text of the *Generales Inquisitiones*. While Leibniz states that "all propositions can be conceived of as terms" (*concipere omnes propositiones instar* *terminorum*, §75, §109, §197), this does not mean that propositions simply are terms. For example, Leibniz also claims in the *Generales Inquisitiones* that "we can conceive of a term as a fraction" (*concipiamus terminum instar fractionis*, §187), but he obviously does not think that terms are fractions. Instead, he takes a fraction to "represent" (*repraesentat*) a term (§129). In similar fashion, it is plausible to suppose that Leibniz takes a propositional term to represent the proposition from which it is generated, rather than identifying the two. On such an account, when Leibniz asserts in §75 that '*A*' can stand for a proposition, it would perhaps have been more precise for him to say that '*A*' can stand for something which represents a proposition, namely, a propositional term. Nevertheless, it is understandable why Leibniz might be inclined to adopt the slightly inapt phrasing that he does in §75. For it is a natural semiotic contraction to say that one thing stands for another when, in fact, it stands for something which represents that thing.

If, for Leibniz, propositional terms represent propositions, they do so in a distinctive way. In particular, they do so in such a way that any inferential relation that obtains between the propositions of Leibniz's calculus can be expressed by another proposition of the calculus involving the corresponding propositional terms. For example, if a proposition *E is F* is derivable in Leibniz's calculus from the proposition *C is D*, this fact is expressed by the derivability in the calculus of the complex proposition *C's being D is E's being F*. The device of propositional terms thus allows Leibniz to "reduce consequences to propositions, and propositions to terms" (§198.8). In this way, Leibniz is able to represent complex modes of propositional reasoning in his calculus despite the fact that the language of the calculus includes only simple propositions of the form *A is B* and *A coincides with B*. Crucially, the introduction of propositional terms does not require Leibniz to posit any abstract objects in the intended domain of discourse of his calculus. As Mugnai points out, Leibniz adopts a distinctively nominalistic attitude to propositional terms, regarding them "not as corresponding to things, but as a kind of shorthand for the discourse" (A VI 4 996). This nominalistic attitude comports well with the auto-Boolean semantics for Leibniz's calculus introduced in our paper. In this semantics, the introduction of propositional terms into the calculus does not require any extension of the domain of possible semantic values of terms. This is because, in the auto-Boolean semantics, propositional terms just denote certain elements of the Boolean algebra of semantic values for non-propositional terms (namely, the top and bottom elements). Thus, there is no need to introduce into the domain of this algebra any new abstract objects such as propositional contents or truth-values. Nevertheless, as we show in our paper, the resulting algebra is rich enough to simulate complex modes of propositional reasoning. In this way, the auto-Boolean interpretation of Leibniz's calculus provides a natural model in which to realize Leibniz's ambition to conduct propositional reasoning in a purely nominalistic framework.

Extensional and Intensional Aspects of Propositional Terms

On our reconstruction of Leibniz's calculus, the propositional terms obey all the laws of classical propositional logic. In this respect, our reconstruction agrees with that offered by Castañeda.[7] On the other hand, it differs from the reconstruction offered by Lenzen, according to which the propositional terms of Leibniz's calculus obey only the laws of a sub-classical logic of strict implication.[8] The two reconstructions yield different accounts of the relation of containment between terms, which Leibniz usually expresses by propositions of the form *A is B*.[9] While Lenzen takes this relation of containment to obey the laws of strict implication, we argue that this relation, when applied to propositional terms, is subject to all the laws of material implication. Thus, for example, if the proposition *E is F* is true, it follows that, for any proposition *C is D*, the complex proposition *C's being D is E's being F* is true. In other words, on our view, if *E is F* is true, the propositional term *E's being F* is contained in any propositional term whatsoever. Against this proposal, Mugnai finds it "quite implausible that Leibniz would have" accepted this classical feature of the logic of propositional terms (p. 132). Mugnai is skeptical that Leibniz would have accepted, for example, that the propositional term generated from the true proposition *2+2*=4 is contained in the propositional term generated from *Caesar crossed the Rubicon*. He contends that Leibniz would have rejected such putative containments on the grounds that they fail to satisfy certain constraints of semantic relevance between the antecedent and consequent propositions. More generally, Mugnai is concerned that our reconstruction of the calculus imputes to the relation of containment between propositional terms various extensional features that Leibniz would not accept.

To address these concerns, it is helpful to distinguish between two different senses in which Leibniz's relation of containment might be said to be 'extensional'. In the first sense, extensionality pertains to the semantic values of the terms of Leibniz's calculus and the truth-conditions of propositions of the form *A is B*. In this semantic sense, the relation of containment is extensional if the semantic value of a given term is taken to be the set of individuals falling under this term, and if a proposition of the form *A is B* is true just in case every individual falling under *A* also falls under *B*. By contrast, on an intensional interpretation of containment, the semantic value of a term is the set of concepts that are, in some sense, conceptual parts of this term, and *A is B* is true just in case every conceptual part of *B* is also a conceptual part of *A*.[10] In his logical and philosophical writings, Leibniz tends to prefer the intensional over the extensional interpretation of containment, in part because the former does "not depend on the existence of individuals" (A VI 4 200). Nevertheless, he maintains that both approaches yield valid interpretations of his logical theory (see §§122–3, A VI 4 199–200, 247–8, 838–9). Leibniz thus intends the calculus developed in the *Generales Inquisitiones* to be an abstract calculus admitting of both extensional and intensional interpretations.[11] In our paper, we follow Leibniz in this respect. Our semantics for Leibniz's calculus does not determine what the semantic value of a term is apart from specifying its place in an abstract algebra. Hence, in our reconstruction of Leibniz's calculus, containment is an abstract algebraic relation which is neither extensional nor intensional in the semantic sense just described.

From this semantic sense there is to be distinguished a second sense of extensionality, which pertains to the logical laws and rules of inference that govern the relation of containment in Leibniz's calculus. In this inferential sense, the relation of containment can be said to be extensional if, when applied to propositional terms, it obeys the laws of the material conditional. By contrast, containment can be said to be intensional in the inferential sense if it obeys only the laws of some weaker sub-classical logic, such as a logic of strict implication or some system of relevance logic.

In this inferential sense, we maintain that Leibniz's logic of containment is, in fact, extensional. This means, for example, that any two true propositional terms mutually contain one another. Moreover, it follows that containment between propositional terms is subject to the law of explosion, according to which a false propositional term contains any propositional term whatsoever. As Mugnai rightly points out, Leibniz never states this law of explosion. Mugnai does identify three passages where, in his view, Leibniz seems to move in the direction of asserting this law (A VI 4 121, 276, A VI 6 450). Even these passages, however, provide no evidence that Leibniz endorsed explosion, since they only imply the possibility of inferring a true conclusion from false premises, not that of inferring any conclusion whatsoever from such premises. Still, despite the fact that Leibniz never explicitly endorses the law of explosion, Mugnai concedes that this law "is a straightforward consequence of claims explicitly endorsed in the *General Inquiries*" (p. 133). In particular, the law follows immediately from the following three principles:

If A is false and C contains A, then C is false. (§58)

A *non*-B contains A. (§77, A VI 4 274, 280, 292, 813)

If A *non*-B is false, then A contains B. (§169, §199, §200)

These principles imply that, if A is false, then A contains B.[12] Since 'A' and 'B' stand for any terms including propositional terms, this implies the law of explosion, that a false propositional term contains any propositional term.

It is worth emphasizing how simple this derivation of explosion is, and that it proceeds solely from principles which Leibniz explicitly endorses in the *Generales Inquisitiones*. The most substantive of these principles is the third one, which asserts that, if A is false, then A contains B. This is one direction of the principle that we refer to in our paper as 'Leibniz's Principle'. It is ultimately this direction of Leibniz's Principle that is responsible for the extensional features of the calculus developed in the *Generales Inquisitiones*.[13]

Leibniz's Principle appears only in the later sections of the treatise, where it plays a crucial role in solving the recalcitrant problem of establishing the law of contraposition in Leibniz's calculus. Moreover, as far as we can see, the principle does not appear in any of Leibniz's earlier logical writings. This opens the possibility that Leibniz did not have this principle in mind when he wrote the earlier sections of the *Generales Inquisitiones*, and hence that the tentative versions of the calculus explored by Leibniz in these sections lack the extensional features introduced by the relevant direction of Leibniz's Principle. There can be no doubt, however, that in the mature version of the calculus axiomatized by Leibniz in the concluding sections of the treatise (§§198–200), this principle is firmly in place. Consequently, the final theory of containment arrived at in the *Generales Inquisitiones* is extensional in the inferential sense just described.

Mugnai takes the fact that Leibniz does not ever state the law of explosion to indicate a reluctance on Leibniz's part to endorse this law. In support of this view, Mugnai contends that the prevailing orthodoxy among logicians since antiquity was to reject explosion and to adopt some form of relevance logic which requires there to be a relevant connection between the premises and the conclusion of a valid argument. Mugnai suggests that such a requirement of relevancy also "applies to Leibniz's logical calculi" (p. 134). As a first response to Mugnai's contention, it may be noted that the commitment to relevancy and the rejection of explosion were by no means unanimous in the history of logic prior to Leibniz. It is of course correct that, dating back to Aristotle, many logicians adopted some version of relevance logic. There were, however, a number of prominent dissenters. For example, Philo of Megara proposed a truth-functional account of the conditional which closely resembles the semantics for the material conditional in modern propositional logic.[14] In the medieval period, explicit arguments for explosion date back at least to the 12th century, appearing, for example, in the writings of the Paris school of logicians known as the *Parvipontani*, the followers of Adam of the Petit-Pont.[15] Subsequently, the law of explosion was endorsed by Buridan in the 14th century.[16] Thus, on the basis of historical precedent alone, it cannot be taken for granted that Leibniz's silence on explosion expresses his adherence to a default relevantist position.

Indeed, there is no passage in which Leibniz asserts that the relation of containment between propositional terms is subject to constraints of semantic relevancy. On the contrary, Leibniz endorses a number of principles which directly contradict central tenets of relevance logic. For example, one of Leibniz's most fundamental principles of containment is the law that, for any term B, the composite term AB contains A (§38).[17] Thus, if A is the propositional term generated from the proposition *2+2*=4 and B the propositional term generated from *Caesar crossed the Rubicon*, then the composite of these two terms contains the former, even though there is no obvious relevant connection between A and B. Similarly, as Mugnai concedes, Leibniz endorses a principle of monotonicity to the effect that any proposition φ follows from the pair of propositions φ and ψ (A VI 4 149). This principle of monotonicity runs counter to the spirit of relevance logic since it permits valid arguments in which some premises have no relevant connection to the conclusion. Moreover, this principle straightforwardly entails the law of explosion, given Leibniz's commitment to the following rule of *reductio ad absurdum*:

> In the method of *reductio*, we use the following principle: if the conclusion is false (i.e., if its contradictory is true), and one of the premises is true, then the other premise must necessarily be false, or, its contradictory must necessarily be true. (A VI 4 499)

According to Leibniz's principle of monotonicity, φ follows from φ and the contradictory of ψ. Hence, by the rule of *reductio* just quoted, we immediately obtain the law of explosion, that any proposition ψ follows from φ and the contradictory of φ.

In sum, then, while Leibniz neither asserts nor rejects explosion in his writings, the preponderance of textual evidence suggests that he would have been inclined to accept it. Thus, in our view there is little reason to maintain that Leibniz would have resisted a theory of containment that is extensional in the inferential sense described above. Crucially, however, this does not mean that Leibniz's theory of containment is extensional in the semantic sense. Taking containment to be extensional in the inferential sense is fully compatible with adopting an intensional semantics, in which the truth-conditions of *A is B* make reference to the conceptual parts of *A* and *B* rather than the individuals that fall under them. Doing so, of course, has a number of substantive consequences for the theory of conceptual containment, since it means that this relation, when applied to propositional terms, is subject to the laws of classical propositional logic. As Mugnai points out, one such consequence is that any two true propositional terms mutually contain one another (e.g., the propositional terms generated from the propositions *2+2*=4 and *Caesar crossed the Rubicon*). Another, related consequence is that any true propositional term is contained in a term such as *Peter*, i.e., the complete concept corresponding to the concrete individual Peter. Thus, for example, in our reconstruction of Leibniz's calculus, the term *Peter* contains *Caesar's having crossed the Rubicon* (i.e., the propositional term generated from the true proposition *Caesar crossed the Rubicon*).[18] This containment obtains despite the fact that there is no obvious relation of semantic relevancy between *Peter* and *Caesar's having crossed the Rubicon*. As we have argued, however, the apparent lack of a semantically relevant connection between these two terms does not mean that Leibniz would reject a containment between them. On the contrary, there is reason to think that Leibniz would accept that *Peter* contains *Caesar's having crossed the Rubicon*.

For Leibniz, the complete concept of a concrete individual is the term which contains every term that can be truly predicated of that individual.[19] Moreover, Leibniz holds that, for any individual and any term *A*, either *A* or *non-A* can be truly predicated of that individual.[20] Consequently, for any term *A*, the complete concept *Peter* either contains *A* or contains *non-A*.[21] Given that '*A*' stands for any term including propositional terms, it follows that, for any propositional term, *Peter* contains either this propositional term or its privative.[22] Since exactly one of these terms is true and the other is false, and since the complete concept of an individual cannot contain a false term, it follows that *Peter* contains every true propositional term. In other words, the complete concept of Peter contains every truth, regardless of whether this truth stands in any obvious relevant connection to Peter. More generally, for Leibniz, the complete concept of any individual contains every truth and no falsehood, and, in this sense, is a reflection of the entire universe.[23]

This fact about the logic of containment comports well with certain central tenets of Leibniz's metaphysics. For example, in the *Discours de métaphysique,* written in the same year as the *Generales Inquisitiones*, Leibniz characterizes the complete concept of Alexander as containing "traces of everything that happens in the universe":

> God, seeing Alexander's individual notion or haecceity, sees in it at the same time the basis and reason for all the predicates which can be said truly of him, for example, that he vanquished Darius and Porus . . . . Thus, when we consider carefully the connection of things, we can say that from all time in Alexander's soul there are vestiges of everything that has happened to him and marks of everything that will happen to him and even traces of everything that happens in the universe, even though God alone could recognize them all. (A VI 4 1540–1)

In similar fashion, Leibniz goes on to assert in the *Discours*:[24]

> Every substance is like a complete world and like a mirror of God or of the whole universe . . . . For it expresses, however confusedly, everything that happens in the universe, whether past, present, or future. (A VI 4 1542)

It is clear from these passages that Leibniz is committed to a view of conceptual containment unencumbered by any limiting constraints of semantic relevancy. In our reconstruction of Leibniz's calculus, this commitment is captured by Leibniz's Principle and the way in which this principle imparts extensional features to the logic governing the intensional relation of conceptual containment. These extensional features of the logic of containment serve, in effect, to do away with any partitioning of truths into separate domains of relevance. They thus ensure that all truths are contained in the complete concept of any individual, even if such containments cannot be grasped by finite minds like ours:

> Each thing is so connected to the whole universe, and one mode of each thing contains such order and consideration with respect to the individual modes of other things, that in any given thing, indeed in each and every mode of any given thing, God clearly and distinctly sees the universe as implied and inscribed. As a result, when I perceive one thing or one mode of a thing, I always perceive the whole universe confusedly; and the more perfectly I perceive one thing, the better I come to know many properties of other things from it. (A VI 4 1668)

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Notes

[1] When a proposition is conceived of as a term, Leibniz describes the proposition as giving rise to a "new term" (*terminus novus*, §197 and §198.7; *contra* Parkinson 1966: 86 n. 2). Leibniz does not use the phrase 'propositional term', but instead uses 'complex term' to refer to propositional terms and compounds involving such terms (§61, §65, §75; cf. A VI 4 528–9). In a similar fashion, he uses 'incomplex term' to refer to simple terms and non-propositional compounds thereof. The label 'propositional term' is borrowed from Barnes 1983: 537 and Swoyer 1995: 332–33.
[2] It is true that the early sections of the *Generales Inquisitiones* include a number of references to 'complex' and 'incomplex' terms, which is Leibniz's way of distinguishing between propositional and non-propositional terms (A VI 4 749 n. 11, 753–4 n. 18, §32b). However, all of these references appear in later additions to the original text, either in marginal notes or in passages inserted by Leibniz into the main text. The first reference to complex and incomplex terms that is not part of such a later addition appears in §61.
[3] See n. 1 above.
[4] The addition of the qualification 'incomplex' in the final sentence of the passage from §75 just quoted was clearly the result of a deliberate decision on Leibniz's part. Leibniz originally wrote "Itaque per *A* intelligo vel terminum, vel . . .," which would have been in keeping with his earlier claims that '*A*' can stand for either a term or a proposition. He then corrected himself by striking out the second occurrence of 'vel' and completing the sentence as follows: "Itaque per *A* intelligo vel terminum incomplexum, vel propositionem" (see the apparatus on this passage at A VI 4 764).
[5] As a further piece of evidence against our view, Mugnai points to a passage from an essay titled *Specimina Calculi Rationalis*. In this passage, Leibniz writes that "by '*A*' or '*B*' here I understand either a term or a statement" (A VI 4 808). This passage appears in §13 of the *Specimina Calculi Rationalis*. It is only in §16 of this essay, however, that Leibniz officially introduces the device of propositional terms and the principles that govern their operation. From this point on in the *Specimina*, Leibniz is careful to distinguish between expressions signifying propositions and those signifying propositional terms. Thus, for example, in §16 of the *Specimina* he writes: 'If *A is B* is called *C*, then *C* is the same as *A's being B*. Therefore, when

we say *E is F* follows from *A is B*, this is the same as if we were to say *A's being B is E's being F*' (A VI 4 809). Here, Leibniz uses the phrase '*A est B*' to express the proposition *A is B*, and the phrase '*A esse B*' to express the propositional term *A's being B* generated from this proposition.

[6] Ishiguro 1972: 19–20, Castañeda 1974: 384 and 395, Mates 1986: 54, 125–6, and 176.

[7] Castañeda 1990: 24–5.

[8] Lenzen 1987, 2004: 35–9.

[9] Throughout the *Generales Inquisitiones*, Leibniz uses the phrases '*A is B*' and '*A contains B*' interchangeably.

[10] For such an intensional semantics of containment in Leibniz's calculus, see, e.g., Lenzen 1983: 145 and van Rooij 2014: 188–92.

[11] Swoyer 1995: 104–6, Bassler 1998: 132–6; cf. also Lenzen 1983.

[12] Kauppi (1960: 259) objects to this proof of explosion on the grounds that, in her view, the third principle listed above only applies when both *A* and *non-B* are not themselves false terms. Kauppi is right that, in the *Generales Inquisitiones*, Leibniz sometimes reasons under the assumption that no term appearing as a constituent of a true proposition is false (e.g., §§43–6, §91). For example, in §153 of the *Generales Inquisitiones* Leibniz provisionally adopts the view that any proposition in which a false term appears as a constituent is itself false. In §155, however, he rejects this view and instead adopts the position that any proposition of the form *A* = *A* is true even if the term *A* is false. Leibniz clearly prefers this latter approach, remarking that 'all things considered, [this approach] will perhaps be better' (§155; see Schupp 1993: 161–2 and 204). This is the default position adopted by Leibniz in the later sections of the *Generales Inquisitiones* (see, e.g., §156 and §171). Accordingly, when Leibniz states the principles of the final axiomatization of his calculus in §§198–200, he imposes no restriction on these principles to terms which are not false. In particular, contrary to what Kauppi suggests, no such restriction is imposed on the third principle listed above, which states that *A* contains *B* if *A non-B* is false (§200). Given that this principle is endorsed by Leibniz in its unrestricted form, the proof of explosion based on this principle is correct.

[13] This direction of Leibniz's Principle parallels the mereological principle of (strong) supplementation, according to which, if *B* is not a part of *A*, then some part of *B* is disjoint from *A*. When this principle of supplementation is added to

the minimal system of mereology, the result is called 'extensional' mereology (Varzi 1996: 262).

[14] See Bobzien 1999: 84–5. Diodorus Cronus proposed an alternative semantics of the conditional which more closely resembles that of strict implication. Even the Diodorean conditional, however, is not relevant since it is true whenever either the antecedent is impossible or the consequent is necessary, "regardless of whether there is any relevant connection between the two constituent propositions" (Bobzien 1999: 85–6).

[15] See Martin 1987: 398–400, 2012: 302; cf. Iwakuma 1993. In the 13th century, various arguments in favor of explosion were put forward by authors such as Nicholas of Paris, Matthew of Orléans, and John le Page; see Spruyt 1993: 174–80.

[16] See Buridan, *Treatise on Logical Consequences*, Book 1, chapter 8, Theorem 1 (transl. by King 1985: 196); cf. d'Ors 1993: 205–211.

[17] By contrast, the propositional schema *If* ϕ *and* ψ*, then* ϕ is invalid in all standard systems of relevance logic.

[18] More generally, it is a theorem of our reconstruction of Leibniz's calculus that every true propositional term is contained in any term whatsoever. This follows straightforwardly from the fact that Leibniz's calculus is complete with respect to the class of auto-Boolean algebras.

[19] A VI 4 553, 559, 568, 572, 575, 1646; cf. also A VI 4 306, 389, 672, 940.

[20] A VI 4 292; see Lenzen 1983: 23.

[21] See Adams 1994: 72.

[22] This presupposes that the language of Leibniz's calculus allows for mixed propositions of the form *A is B* in which *B* is a propositional term and *A* is a non-propositional term. While Leibniz's calculus can in principle be developed without countenancing such mixed propositions, there is no evidence to suggest that Leibniz meant to exclude them from the language of his calculus. On the contrary, in §109 of the *Generales Inquisitiones* Leibniz countenances propositions such as *Man's being animal is a reason* and *Man's being animal is a cause*. He makes it clear that the subject term *man's being animal* is a propositional term, describing it as "a proposition conceived of as a term" (§109). At the same time, the predicate terms *reason* and *cause* are presumably non-propositional terms. This suggests that Leibniz intends to include in the language of his calculus mixed propositions relating propositional and non-propositional terms.

[23] This is in accordance with Adams's suggestion that, for Leibniz, a complete concept "contains all truths" (Adams 1984: 14).
[24] See also A VI 4 1663 and 2770.