Fairer Chess: A Reversal of Two Opening Moves in Chess Creates Balance Between White and Black

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Abstract

Unlike tic-tac-toe or checkers, in which optimal play leads to a draw, it is not known whether optimal play in chess ends in a win for White, a win for Black, or a draw. But after White moves first in chess, if Black has a double move followed by a double move of White and then alternating play, play is more balanced because White does not always tie or lead in moves. Symbolically, *balanced alternation* gives the following move sequence:

> After White's (W) initial move, first Black (B) and then White each have two moves in a row (BBWW), followed by the alternating sequence, beginning with W, which altogether can be written as WB/<u>BW</u>/WB/WB/WB… (the slashes separate alternating pairs of moves).

Except for reversal of the 3rd and 4th moves from WB to BW (underscored), this is the standard chess sequence. Because balanced alternation lies between the standard sequence, which favors White, and a comparable sequence that favors Black, it is highly likely to produce a draw with optimal play, rendering it fair.

1. Introduction

The rules of chess have evolved over the past 1,500 years, but beginning in the 19th century they were standardized to facilitate national and international competition. While the most sophisticated chess-playing computer programs are now able to defeat the best human players, we still seem no closer to answering the question of whether chess is *fair*: When the players make optimal choices, neither White, who moves first, nor Black, who moves second, can force a win, rendering the outcome a draw.

Although most chess experts believe that a draw is the product of optimal play (for different views, see https://en.wikipedia.org/wiki/First-move_advantage_in_chess), there is no proof of this. Because of the astronomical number of choices in chess, no brute-force check of all possible moves, even with the fastest computers, can verify this. Accordingly, we take an indirect approach by showing that a small variation in the standard rules of chess—giving Black and White double moves after White's initial move, which reverses the order of play of the 3rd and 4th moves from White-Black to Black-White—balances the opportunities of White and Black to win with optimal play. If neither player has a decisive advantage, it is highly unlikely that either player can force a win for itself (and loss for its opponent), making the outcome of optimal play a draw.

By *optimal play*, we mean that

- if one player defeats its opponent, its opponent could not have done better—drawn or won—by making different moves;
- if the outcome is a draw, neither player could have won by making different moves.

If both these conditions are met, the optimal strategies of the players constitute a Nash equilibrium in a 2-person zero-sum game of perfect information, so neither player would depart from its optimal strategy (if known). Zermelo (1913) showed in chess that either (i) one player can force a win or (ii) both players can force a draw with optimal play.

In principle, backward induction, by working backwards from a final move when checkmate or a rule that forces a draw (e.g., the three-fold repetition of moves) occurs, can be used to find optimal moves of the players in a finite game such as chess. In checkers, this approach was applied by Schaeffer et al. (2007) to prove, using multiple computers making calculations over almost two decades, that optimal play always ends in a draw. But because chess is a far more complex game than checkers, a comparable calculation to ascertain the outcome of optimal play of chess appears beyond the capability of computers for the foreseeable future.

Accordingly, we take a different approach by making a sequencing argument for the variation in chess we mentioned earlier—that it gives Black more opportunity to win and thereby creates more of a balance between Black and White, almost surely rendering the product of optimal play a draw. It also has a major practical advantage, obviating the need of contestants to play both Black and White in a tournament, for reasons we give in section 3.

2. Statistics on Chess

In tournament games that have a winner in chess (*decisive games*), White on average beats Black in 55 percent of them (Elo ratings of 2100 or above), but for elite players (Elo ratings of 2700 or above), the winning percentage is 64 percent. The proportion of draws also increases with skill from 35 percent for nonelite players to 58 percent for elite players (Adorján, 2004, p. 68).[1] These statistics have not changed much in recent years: White on average enjoys about a 2:1 advantage over Black at the highest (human) level of play, but at this level most games end in a draw.

Statistics from computer play of chess by the strongest programs amplify the advantage of White at the same time that they increase the proportion of draws. In 2020,

[1] These statistics are based on chess games played under classical time controls. For more information, see https://www.chessgames.com/chessstats.html.

when the reputedly most powerful chess engine in the world, AlphaZero, played against itself in 10,000 games, taking one minute per move, White won in 86 percent of the decisive games, but these games constituted only 2 percent of the total—98 percent were draws (Tomašev et al., 2020). Other expert programs, including Leela Chess Zero and Stockfish, when pitted against each other in the superfinal of the unofficial world computer chess championship (TCEC), give White even greater odds of winning, but the outcome is still a draw in the large majority of games (see https://tcec-chess.com). Despite the fact that computer programs start the play from 50 preselected opening positions in the TCEC superfinal (once as White and once as Black), it is remarkable that Black has not won a single game in the last two TCEC superfinals. All 49 decisive games were won by White, which was either StockFish or Leela Chess Zero.

Although the forgoing statistics indicate that chess is biased against Black in decisive games, Black is usually able to survive by drawing. It is, nevertheless, surprising that White, by making the first move, is able to achieve almost a 6:1 advantage of winning in decisive games, based on the aforementioned AlphaZero statistics. Whether White has an *inherent* advantage—can force a win when both players make optimal choices—remains an open question.

More light would be shed on this question if the two leading machine-learning chess programs, AlphaZero and Leela Chess Zero, were taught to play with our proposed change in the order of the 3rd and 4th moves from White-Black to Black-White. Still, even if this change enabled Black to win a greater proportion of decisive games, it would not prove that a draw is the inevitable product of optimal play, just as White's advantage in standard chess does not prove that it can always win. As advanced as machine-learning programs are today, they are not able to mimic perfectly all the moves prescribed by backward induction from every possible endpoint—a draw or a win for one player—in chess.

In this paper, we offer reasons in section 3 why we think our proposed reversal of two opening moves would tend to equalize the probability that either Black or White can win and, consequently, make the game fairer. Other proposed rule changes that might render chess more balanced or provide other desirable changes (e.g., speed up play) are extensively analyzed in Tomašev et al. (2020). None of this study's rule changes, however, such as the elimination of castling, is as simple as our reversal of two opening moves in chess or as likely to create balance between the two players and force a draw with optimal play.

3. Making Chess Fairer

Our argument for changing the order of two opening moves is theoretical: The new order lies between the present one that favors White and a comparable sequence, which we discuss below, that favors Black. The favoritism each player obtains from a sequence, we postulate, is mainly a function of being able to move earlier than its opponent or later, after observing its move.

Moving earlier gives a player a greater opportunity to "set the stage," whereas moving later enables a player to observe the move of its opponent and respond to it. These two factors are in conflict, creating a tension between moving earlier or later.

There are occasions in chess in which responding to a move can put a player in a more advantageous position than moving first, which is called *Zugzwang* (see https://en.wikipedia.org/wiki/Zugzwang). However, these occasions almost always arise late in a game, when a player's king is in peril; we know of no instances in which Zugzwang can or has happened in the opening moves of chess, which is the kind of change we focus on here.

We assume, consistent with the statistics in section 2, that White has an advantage over Black with the standard chess sequence,

$$\text{WB/WB/WB....} \tag{1}$$

Black can counter this advantage if it has two moves in a row after White's initial move (WBB), followed by the alternating sequence beginning with W, WB/WB/WB…. The resulting sequence,

WBB/WB/WB/WB…,

can be written as

WB/BW/BW/BW…, (2)

where the slashes separate pairs of moves.

Observe that both (1) and (2) start with WB, but the alternation from the 3rd move on of (1) is WB/WB/WB…, whereas that for (2) is BW/BW/BW…. This makes (1) *White favorable*, because W precedes B for every pair from the 3rd move on. By comparison, with (2), B precedes W from the 3rd move on, so this sequence is *Black favorable.*

In effect, giving Black a double move after White's initial move swings the pendulum from favoring White with the standard chess sequence to favoring Black because of the potency of two moves in a row for Black. This raises the question of whether there is an intermediate sequence that creates a balance—between the White favorable sequence of (1) (standard chess) and the Black favorable sequence of (2)—that does not favor either side?

If we add a double move by White (WW) immediately following Black's double move (BB) in the Black favorable sequence of (2), then Black (B) and White each have two moves in a row (BBWW), followed by the alternating sequence beginning with B on the 6th move (underscored). This gives

W/BB/WW/<u>B</u>W/BW/BW…,

which can be written as

WB/<u>BW</u>/WB/WB/WB…, (3)

which we call the *balanced sequence*, or *balanced alternation*.

Except for reversal of the 3rd and 4th moves from WB to BW (underscored), the balanced sequence is the standard chess sequence.[2] The balanced sequence is bracketed by the White favorable (standard) sequence of (1) and the Black favorable sequence of (2), making (3) a neutral alterative that favors neither White nor Black.

Notice that there is no difference in the first four moves of (2) and (3), which both start with WB/BW. The difference lies in the alternating sequences, beginning on the 6th move, with the alternation of (2), BW/BW/BW…, favoring Black and the alternation of (3), WB/WB/WB…, favoring White. The alternation of (3) offsets Black's early double move of (2) and provides—with White's later double move of (3)—the balance we seek.

This is not to say that we can prove that the balanced alternation of (3), which translates into a switch of the 3rd and 4th moves of standard chess, always leads to a draw with optimal play. But because balanced alternation neutralizes (i) the apparent advantage that White derives from (1)—moving first in standard chess—and (ii) the apparent advantage that Black derives from (2) with a single double move, it seems more likely to force a draw with optimal play than either (1) or (2). This does not rule out the possibility that (1), (2) or both, though biased, also force a draw with optimal play.

It is worth pointing out that the first four moves of (2) and (3), WBBW, are identical. They are also the same as those given by the Prouhet-Thue-Morse sequence (see https://en.wikipedia.org/wiki/Thue–Morse_sequence), which can be written as

$$\text{WBBW/BWWB/BWWB/WBBW}\ldots, \tag{4}$$

and is what Brams and Taylor (1999) also call "balanced alternation," as opposed to the "strict alternation" of WB/WB/WB… that is the standard chess sequence. Although the

[2] We assume that a player may give a check only on the second move of a double move under balanced alternation, which ensures that its opponent is given the opportunity to respond to a check.

Prouhet-Thue-Morse sequence is arguably fair, it has the disadvantage of allowing more than two double moves, as in the first 16 moves of (4) which contain four double moves.

Another well-known sequence, known as Marseillais chess (see https://en.wikipedia.org/wiki/Marseillais_chess), uses only double moves,

WW/BB/WW/BB/WW/BB…. (5)

Like the Prouhet-Thue-Morse sequence, (5) critically alters the strategy and the tactics in chess.

To make a rule change acceptable to chess players, we think it should not have more than two double moves, as does (3). The fact that (3) requires only a reversal of the 3rd and 4th moves from WB to BW makes it even more palatable—it's only a minor revision in the order of play at the beginning of a game, not, as with (4) and (5), throughout. Besides being impartial, (3) seems to be the sequence most likely to force a draw with optimal play, which would make it fair as well.

4. Conclusions

The balanced alternation of (3) is relatively easy to implement, reversing only the 3rd and 4th moves of standard chess. It is certainly more likely to be fairer to Black than (1), and fairer to White than (2), and more likely than either to force a draw with optimal play.

Balanced alternation would make tournament play more efficient. Presently, because of the bias that favors White in standard chess, contestants in most tournaments must play White and Black an equal number of times to neutralize this bias.

But this would probably not be necessary with balanced alternation. In knockout tournaments, in particular, immediate elimination could occur without the need to play White and Black an equal number of times. Thereby tournaments could accommodate twice as many contestants, each playing as few as one game before elimination.

True, chess without strictly alternating moves in the beginning will require an adjustment in players' thinking about optimal openings. White, for example, might make a different first move under balanced alternation than it would under strict alternation. And Black, able to make two moves in a row (BB) after White's initial move, might use each move for a different purpose—or use both moves to launch a broader attack or defense—as might White with its subsequent double move (WW).

We believe these changes are compelling for two reasons: (i) they require only a switch in the order of the 3rd and 4th moves; and (ii) they would not be difficult for chess players to learn and adapt to, after which the familiar alternating moves of WB/WB/WB… occur. Perhaps the main benefit of balanced alternation is that it almost surely will make chess fairer, putting Black on a par with White, even if we cannot guarantee that it forces a draw with optimal play.

This is not to say that there may not be other sequences, such as giving only Black a double move, but later than we assumed in (2). What especially appeals to us about (3) is that the reversal occurs early and strikes an even balance between the White favorableness of (1) and the Black favorableness of (2).

Finally, we wish to make clear that we have not proved that balanced alternation equalizes the chances of White and Black winning, much less that it forces a draw with optimal play. Our argument is one of plausibility, based on the comparison of move sequences, including that of standard chess, that are biased in favour of White or Black. Because balanced alternation falls in between them, it seems as impartial, and simple to implement, as any rule change that one might hope for to render chess fair.

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