where $\mathcal{Y}$ denotes the support of the distribution for η_H.

The supply of efficiency units in occupation H is

$$\tilde{X}_{jt} = \int_{y \in \mathcal{Y}} y F_H \left(y, \frac{\tilde{w}_H}{\tilde{w}_M} y, \ldots, \frac{\tilde{w}_H}{\tilde{w}_L} y \right) dy \tag{38}$$

and the wage bill accrued by workers in occupation H is

$$\tilde{w}_H \tilde{X}_{jt} = \tilde{w}_H \int_{y \in \mathcal{Y}} y F_H \left(y, \frac{\tilde{w}_H}{\tilde{w}_M} y, \ldots, \frac{\tilde{w}_H}{\tilde{w}_L} y \right) dy \tag{39}$$

Analogous expressions to (37), (38) and (39) hold for the other occupations.

To evaluate quantitatively the model, we consider the particular case where the $\eta_j^{\prime}s$ are independent and distributed log-Normal with the mean and standard deviation of $\log \eta_j$ being μ_j and σ_j.[28]

3.2 Competitive Equilibrium

We focus our analysis on the competitive equilibrium of this economy. A competitive equilibrium is defined by a sequence of prices $\{\{p_{st}\}_{s\in\mathcal{S}}, \tilde{w}_{lt}, \tilde{w}_{mt}, \tilde{w}_{ht}\}_{t=0}^{t=T}$, allocations $\{\{c_{hst}\}_{s\in\mathcal{S},h\in\mathcal{H}}\}$ and household occupational choices such that

1. Each household chooses the occupation that maximizes labor income, (35).
2. Household income equals household expenditure E_{ht} period by period. Income is equal to labor income plus the return to capital (which is assumed to be uniform across households).
3. Each household maximizes utility (8) subject to the budget constraint $\sum_{s\in\mathcal{S}} p_{st} c_{hst} = E_{ht}$.
4. Firms maximize profits taking prices as given, $\max p_{st} A_{st} K_{st}^{1-\beta_{st}} \left(\prod_{j\in\{H,M,L\}} \tilde{X}_{jst}^{\alpha_{jst}} \right)^{\beta_{st}} - \tilde{w}_{lt}\tilde{X}_{lst} - \tilde{w}_{mt}\tilde{X}_{mst} - \tilde{w}_{ht}\tilde{X}_{hst}$.
5. Aggregate effective labor supply in an occupation (equation 38) equals aggregate demand (equation 26).
6. All goods markets clear at any point in time, $\int c_{hst} dh = Y_{st}$.

[28] We note that assuming a Fréchet distribution (or a multi-variate Fréchet in the max-stable familiy as described in Lind and Ramondo, 2018) in this setting would have the counterfactual prediction that average wage per worker is equalized across occupations. Authors that have used the Fréchet distribution in this setting need to resort to unobserved costs or worker attributes.

3.3 Calibration

To study the drivers of the relative evolution of employment and wages across occupations we must calibrate two types of parameters: those that are fixed, and those that vary exogenously. To calibrate the first, we use data moments from 1980, while for the latter, we use data from 1980 and 2016 to determine their initial and final values, respectively.

Let's start by discussing the calibration of the distributions of the productivity parameters $\{\eta_j\}$. First note that the definition of an efficiency unit is arbitrary, therefore, without loss of generality, we can set $\{\mu_j\}_{j=\{l,m,h\}}$ to 1. To calibrate $\{\sigma_j\}_{j=\{l,m,h\}}$, we take advantage of the fact that our model is modular in the sense that conditional on the relative wage bill across occupations, the distribution of productivities determines the relative wages per worker across occupations, independently of the rest of parameters in the model. To better understand this property, note that equations (38) and (39) determine the wage bill for occupation j supplied by workers. Additionally, equation (37) determines the share of workers employed in each occupation. These three equations depend only on the distribution of productivities and on the equilibrium efficiency wages ($\{\tilde{w}_j\}_{j=\{l,mh\}}$). Therefore, we can proceed as follows. For any given $\{\sigma_j\}_{j=\{l,m,h\}}$, the requirement that the relative wage bill supplied at each occupation matches the distribution of wage bills in 1980 pins down the equilibrium efficiency wages ($\{\tilde{w}_j\}_{j=\{l,mh\}}$). The dispersion parameters $\{\sigma_j\}_{j=\{m,h\}}$ can be calibrated by additionally requiring that the average wage per capita for h and l occupations relative to the average wage per capita for m occupations match the equivalent ratios observed in 1980.

There are two relevant observations worth noting. The first is that because we match the 1980 relative wage bill and average wage per workers across occupations, we will also match the employment shares across occupations in 1980. The second is that because we only use information on relative wage bills and relative wage per worker across occupations, without loss of generality, we can normalize σ_l to 1.

We calibrate the occupation intensities, $\left\{\{\alpha_{jst}\}_{j=\{l,m,h\}}\right\}_{s=1}^{S}$, by matching the relative wage bill in each sector (equation 3) computed with wage information from the CPS and occupation information from the ACS. We calibrate $\{\beta_{st}\}_{s=1}^{S}$ by matching the sectoral labor share (equation 25) computed from the CPS, ACS and KLEMS.[29] The preference parameters $\{\varepsilon_s\}_{s=1}^{S}$ and σ are estimated from the CEX as described in the Appendix. Given those, we calibrate the taste parameters, $\{\zeta_s\}_{s=1}^{S}$, to match the aggregate value added share of each sector in 1980.

To study the drivers of the evolution of relative wages and employment shares, we consider exogenous changes in three parameters. The changes in the factor intensities, $\{\alpha_{jst}\}$, and

[29]We compute labor shares by computing wage bills using hours worked from the ACS and wages from the CPS. We calibrate the relative values of $\{\beta_s\}$ from the ratio of total wage bill taken from the ACS and CPS to nominal VA in each sector. While these values correctly capture the relative share of employment in each sector, the resulting level is too small. We then adjust the level of $\{\beta_s\}$ by multiplying all values of by min(labor compensation share in KLEMS/β). To calibrate r we use the private sector lending rate from the IMF International Financial Statistics. For our baseline model we use the average value between 1980-2016. We then run robustness checks were we let it vary over time.

labor shares, $\{\beta_{st}\}$, are directly measured in the data. Most importantly, we would like to implement the equivalent in this full-blown model to the neutral increase in aggregate nominal expenditure we have introduced in section 2. This requires the use of two of the parameters to induce the increases in aggregate expenditures and in the price level we have observed in the data.[30] Since the model pins down the equilibrium value of relative wages, the level of wages is a free parameter. This allow us to exogenously induce changes in nominal variables by altering directly one of the wage rates, (e.g., $\tilde{w}_{lt}$). The second instrument we use to induce the neutral increase in expenditure is aggregate TFP. Consequently, we set the increases in TFP and $\tilde{w}_{lt}$ to simultaneously match the increase in nominal personal consumption expenditure per capita and in the pce deflator observed in the US from 1980 to 2016. Note that these two variables have different effects on real and nominal expenditure because for given inputs, TFP only affects real income while $\tilde{w}_{lt}$ affects both. In our baseline calibration, we set the rental rate of capital equal to the average private sector lending rate from 1980-2016 as reported in the IMF International Financial Statistics. [31]

3.4 Results

The results from our baseline analysis are reported in Table 6. The first two rows report the actual values of the key variables in the data. As before, we study the wage bills of the three occupations but also the relative wages and the share of hours worked in each of the three occupations. Row 3 reports the model simulations for 1980 which, by design, match the data. Rows 4 to 6 report the values of the variables of interest for 2016 produced by the model in different simulations. We consider three exercises. The neutral increase in expenditure (row 4), a simultaneous increase in the sectoral factor intensities and in the sectoral labor shares as the one observed in the data (row 5), and the effect of conducting simultaneously both exercises (row 6). Row 7 reports the fraction of the observed change in each of the variables from 1980 to 2016 that the model produces. Rows 8 and 9 report the contributions of the neutral increase in expenditure and the change in factor intensities and labor shares in the evolution of each variable using the approach detailed in section (D) in the appendix.[32]

There are two key findings of the quantitative evaluation of our model. The first is that the model does a good job in generating the polarization of the labor markets both in terms of the evolution of the relative wages of high vs. medium and low vs. medium skill occupations,

[30]To construct the Fisher price index we use as a target for our model we use nominal value added and price deïňĆators for each sector provided by the BEA. We use the values from the baseline year 1980 and each target year to first construct the Laspeyres and Paasche index between the two periods. Then we construct the Fisher index as the geometric mean of the two. The key difference between this index and the one reported by the BEA is that we do not chain it over the years. We construct this alternative price index since our model simulation does not run for each of the years between the two periods we are interested in.

[31]We conduct robustenss checks using the initial and final values as well as allowing the rental rate to vary over time.

[32]Note that these contributions are relative to the change induced by the model when all the exogenous variables change.

Table 6: Full Quantitative Model

	Year	$\frac{W_L}{W_M}$	$\frac{W_H}{W_M}$	L_s	M_s	H_s	$\frac{W_L L}{\sum_k W_k K}$	$\frac{W_M M}{\sum_k W_k K}$	$\frac{W_H H}{\sum_k W_k K}$	Exercise
Data	1980	0.74	1.24	0.095	0.653	0.252	0.068	0.630	0.302	
	2016	0.80	1.49	0.129	0.488	0.383	0.088	0.421	0.491	
Model	1980	0.74	1.24	0.095	0.653	0.252	0.068	0.630	0.302	
	2016	0.87	1.47	0.137	0.531	0.332	0.105	0.467	0.428	E
	2016	0.77	1.41	0.095	0.580	0.324	0.066	0.522	0.413	α+β
	2016	0.88	1.58	0.128	0.494	0.378	0.093	0.410	0.497	E+$\beta + \alpha$
Fraction of observed change[1]		2.33	1.36	0.97	0.96	0.96	1.25	1.05	1.03	
Contribution of E		0.86	0.59	1.14	0.65	0.53	1.28	0.63	0.54	
Contribution of $\alpha + \beta$		0.14	0.41	-0.14	0.35	0.47	-0.28	0.38	0.46	

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

and in terms of the hollowing out of medium skill hours worked increasing the share of hours worked in high and low- skill occupations. Specifically, the model slightly over-predicts the 2016 relative wages of low vs. medium (0.87 vs. 0.8 in the data) and high vs. medium (1.57 vs. 1.49 in the data). It virtually nails the increase in the share of hours worked by each occupation (0.125 vs. 0.129 for low, 0.499 vs. 0.488 for medium and 0.376 vs. 0.383 for high-skill occupations). Consequently, the model virtually nails the 2016 wage bill distribution across occupations (0.091 vs. 0.088 for low, 0.416 vs. 0.421 for medium and 0.493 vs. 0.491 for high-skill occupations).

The second key finding from Table 6 is that the neutral increase in aggregate expenditures accounts for a remarkable share of the polarization in labor markets generated by our model both in terms of the evolution of relative wages and hours worked across all occupations. In particular, the neutral increase in expenditure accounts, respectively, for 85% and 55% of the increases in the low vs. medium and high vs. medium relative wages generated by the model. The neutral increase in expenditure is responsible for a share of the change in the share of hours worked generated by the model that ranges from 50% for high-skill occupations to 113% for low-skill occupations. Finally, the neutral increase in expenditures is responsible for a share of the change in the wage bill shares generated by the model that ranges from 51% for the high-skill occupation to 126% for the low skill occupation.

The relevance of the neutral increase in aggregate expenditure for labor market polariza-

generates a slightly smaller polarization in labor markets because by assigning all capital income to the workers in high-skill occupations, their share in expenditure increases, and we need a smaller increase in $\tilde{w}_{lt}$ to match the observed increase in the pce deflator from 1980 to 2016. Despite this, the neutral increase in expenditures accounts, respectively, for 89% and 50% of the increases in the low vs. medium and high vs. medium relative wages generated by the model and between 45% to 133% of the change in the share of hours worked. Furthermore, the neutral increase in expenditures is responsible for a fraction of the change in the wage bill shares generated by the model that ranges from 46% for the high-skill occupation to 164% for the low skill occupation, which is quite close to the results from our closed economy model.

4 Additional Exercises

The forces introduced by our model – non-homotheticities that induce changes in the sectoral composition of the economy and labor demand – have surely been relevant in other periods of history and in other geographies. In this section we explore this possibility. Specifically, we study the role of non-homotheticities in explaining labor market dynamics in the US during the period 1950-1980 and the polarization of labor markets in other advanced economies during the period 1980-2016. We conclude our analysis by using our framework to look into the evolution of distributional labor market outcomes over the next 15-20 years.

4.1 Back-tracking 1950-1980

Now that we have used our model to better understand the drivers of US labor market polarization from 1980 to 2016, a natural question to ask is what drove the the evolution of wages and hours worked across occupations during period 1950-1980. This exercise is particularly interesting because much of the job polarization debate concerns the post-1980 period with the implicit assumption that some new developments triggered the polarization in labor markets. Yet, as documented by Siegel and Barany (), the labor market polarization dates back, at least to 1950 (See rows 1 and 2 in Table 11). Therefore, by conducting the back-tracking simulation we can study whether the same forces that have been important in the polarization wave from 1980 to 2016 may be responsible for the rise of the middle class we observed from 1950 to 1980 in the US.

To conduct our simulation, we use the same values for the preference parameters that we have used in section 3.4. As in our baseline calculation, we measure directly the sectoral intensity of each occupation and the labor shares. We calibrate the neutral increase in expenditure so that the model's level of aggregate expenditure per capita and the Fisher price index match the levels observed in 1950.

Rows 3-5 of Table 11 contain the labor market outcomes generated by the model for 1950 in response to the neutral decline in expenditure (row 3), the change in factor intensities and

Table 11: Model Simulation 1950-1980

	Year	$\frac{W_L}{W_M}$	$\frac{W_H}{W_M}$	L_s	M_s	H_s	$\frac{W_L L}{\sum_k W_k K}$	$\frac{W_M M}{\sum_k W_k K}$	$\frac{W_H H}{\sum_k W_k K}$	Exercise
Data	1950	0.70	1.15	0.106	0.731	0.163	0.075	0.736	0.189	
	1980	0.74	1.24	0.095	0.653	0.252	0.068	0.630	0.302	
Model	1950	0.68	1.17	0.075	0.699	0.226	0.050	0.689	0.261	E
	1950	0.79	1.18	0.122	0.660	0.218	0.094	0.651	0.254	α+β
	1950	0.72	1.06	0.102	0.728	0.170	0.075	0.742	0.183	E+$\beta + \alpha$
	1980	0.74	1.24	0.095	0.653	0.252	0.068	0.630	0.302	
Fraction of observed change[1]		0.50	2.00	0.64	0.96	0.92	1.00	1.06	1.05	
Contribution of E		3.25	0.53	-2.86	0.76	0.45	-2.64	0.67	0.47	
Contribution of $\alpha + \beta$		-2.25	0.47	3.86	0.24	0.55	3.64	0.33	0.53	

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

labor shares (row 4), and simultaneously conducting both types of changes in the exogenous variables (row 5). The first observation we can draw is that the model accounts quite well for the change in labor market outcomes from 1950 to 1980 (row 7). In particular, it over predicts the change in the relative wage of high-skill occupations but under predicts the relative wage of low-skill occupations. It accounts for a significant share of the change in the share of hours worked in middle and high-skill occupation with values of 107% and 104% respectively, and somewhat less for low-skill occupations with a value of 45%. The model also provides a good account of the changes in the occupational wage bills which represent from 71% of the observed change for low-skill to 107% for medium skill occupations. Looking specifically at the contribution to the model performance of the neutral change in aggregate expenditure, we find that it plays an important role in all the dimensions other than for the evolution of the share of hours worked and wage bill accrued by low-skill workers. In particular, for high-skill occupations, it accounts for approximately 50% of the model's increase in the relative wage, share of hours worked and wage bill from 1950 to 1980. For medium skill occupations, the neutral increase in expenditure accounts for 77% of the model generated change in hours worked and 69% of the wage bill. For the low-skill occupations, the model accounts for all the increase in relative wages, but it does not account for any of the decline in the share of hours worked and in the wage bill.

Intuitively, between 1950 and 1980 there was significant sectoral variation in value added growth. As in our baseline period, there is a strong correlation between sectoral growth and the sectoral share in employment for high and low-skill occupations. Additionally, the model does a great job in predicting sector level growth from 1950 to 1980 (See panel A in Figure x). This is largely due to the differential effect of the neutral increase in expenditure on value added across sectors. In particular, the covariance between actual nominal sectoral growth and sectoral growth produced by the model in resposne to the neutral increase in expenditure represents 97% of the variance of actual sectoral growth between 1950 and 1980. Hence, the model's ability to capture the evolution of the wage bill distribution across occupations. As in the post 1980- period, our model would split the evolution of the wage bill between relative wages and the distribution of hours worked. This works for medium and high-skill occupations but not for low-skill occupations that experience an increase in relative wages but a decline in the share of hours worked. Given the simplicity of our job assignment model, no single driving force can explain this negative co-movement between hours worked and wages of an occupation. So, how is it that the model does a great job in explaining both? The explanation is that while the neutral expenditure force induces an increase in the wage bill of low-skill occupations, during the 1950-80 period, the intensity in production of low skill workers declined causing a reduction in their demand. This second force tends be dominant for hours worked and the wage bill but not for the relative wages of low-skill occupations. Hence the model ability to explain the 1950-80 changes in labor outcomes for all occupational groups.

4.2 Polarization in Other Economies

European countries have experienced labor market polarization processes similar to the U.S. Goos et al., 2014 have documented the polarization of hours worked from middle-skilled to low- and high-skilled occupations in various European countries during the period 1990-2010. In this subsection we explore the relevance of non-homotheticities in demand in the evolution of distributional outcomes in European labor markets. Prior to studying the model implications, we extend Goos et al., 2014 by documenting the polarization of hourly wages and in the wage bill. Due to data consistency considerations, we focus our analysis in five large countries (Germany, France, UK, Italy and the Netherlands) and the period 2005 to 2015.[35] Our data analysis shows a strong polarization in hourly wages in Netherlands, Germany and France and to a smaller extent in Italy and teh UK (See Tables 14-18). As in Goos et al., 2014, we find a polarization of hours worked for our sample of countries and period with the exception of high-skilled occupations in Italy whose share in the total hours worked declined slightly from 38.8To study the importance of non-homotheticities, we calibrate the occupation intensities $\left\{\left\{\{\alpha_{jstc}\}_{j=\{l,m,h\}}\right\}_{s=1}^{S}\right\}_{c}$ by matching the relative wage bill in each country computed with wages taken from the SILC and hours worked from the LFS microdata. We calibrate

[35]See Appendix B.3 for details on the data construction and sources.

$\left\{\{\beta_{stc}\}_{s=1}^{S}\right\}_c$ by matching the sectoral labor share computed from the SILC, LFS and KLEMS as before. For each country we re-calibrate the taste parameters $\left\{\{\zeta_s\}_{s=1}^{S}\right\}_c$ to match the aggregate value added shares of each sector in 2005, where value added shares are taken from KLEMS. Similarly, we re-calibrate the productivity dispersion in the job assignment model for each country to match the 2005 relative hourly wages .

Findings

4.3 Looking into the future of US labor markets: 2016-2035

We conclude our analysis by looking at the future and using the model to forecast the state of labor markets around 2035. To this end, we simulate our model economy if starting in 2016 there was an neutral increase in aggregate expenditures similar to half of that observed from 1980 to 2016. Note that in our simulations we assume that factor intensities and labor shares remain constant at the 2016 levels. Our model predicts a continuation of the labor market polarization. The relative of low-skill occupations increases from 0.8 to 0.93, while the relative wage of high-skill occupations increases from 1.49 to 1.7. The share of hours worked for low and high-skill occupations increases, respectively from 0.12 to 0.142 and from 0.384 to 0.414, while the share of hours worked in medium skill occupations declines by 5 percentage points. These changes in wages and hours worked imply a reallocation of the wage bill from medium skill to low- and especially to high-skill occupations. Specifically, the wage bill of medium skill occupations declines by 8 percentage points, while the wage bill of low- and high-skill occupations increases, respectively, by 2 and almost 6 percentage points.

The magnitude of these changes are quite significant both in absolute terms and compared to the polarization observed during the 1980-2016 period. This is especially the case for relative wages which our model predicts will increase by proportionately more than in the 1980-2016 period. The share of hours worked and the wage bill for low-skill occupations will also increase by proportionately more from 2016-2035 than in 1980-2016. However, the shares of hours worked and the wage bills for medium and high-skill occupations will increase by between 22% and 38% of the overall increase in 1980-2016. Though these are very significant increases, they suggest that the pace at which the wage bill has been reallocated from medium to high-skill occupations will slow down slightly.

5 Conclusion

This paper makes two contributions. First, it documents a positive correlation between sectoral income elasticity and low- and high-skill occupation intensity. Based on this fact, we propose a demand-driven labor market polarization mechanism: as income grows and demand shifts to high-income-elastic sectors, the relative demand of high- and low-skilled workers increases.

Table 12: Model Prediction for 2035

	Year	$\frac{W_L}{W_M}$	$\frac{W_H}{W_M}$	L_s	M_s	H_s	$\frac{W_L L}{\sum_k W_k K}$	$\frac{W_M M}{\sum_k W_k K}$	$\frac{W_H H}{\sum_k W_k K}$	Exercise
Data	2016	0.80	1.49	0.129	0.488	0.383	0.088	0.421	0.491	
Model, E	2035	0.95	1.73	0.148	0.429	0.429	0.108	0.330	0.562	
% increase of 1980-2016	2035	2.50	0.96	0.85	0.41	0.34	1.30	0.46	0.38	

We quantify this mechanism in a multi-sector general equilibrium framework and find that it accounts for a substantial part of the US labor market polarization.

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A Tables and Figures

Table 13: Wage Bill Decomposition by Decades

		H-M	L-M
1980-2000:	Overall	3.26	1.10
	Incr Sect Shares	0.79	0.73
	Incr alpha	0.38	0.09
	Cov	2.09	0.28
2000-2016:	Overall	3.75	2.33
	Incr Sect Shares	1.60	1.69
	Incr alpha	0.29	0.13
	Cov	1.86	0.51

B Data Description

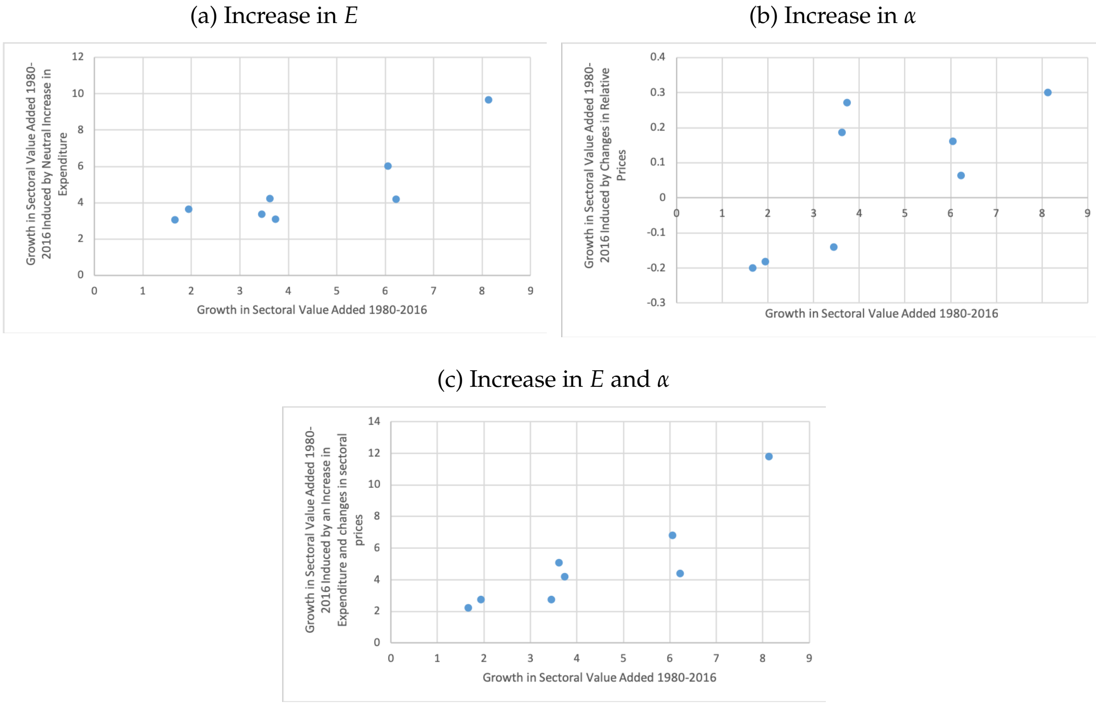
In this section we briefly detail our data sources. We take it from previous work and, thus, we provide a relatively brief description and relegate the interested reader to the original papers for further details.

B.1 Labor-Market Outcomes

We follow Acemoglu and Autor (2011) in the construction of the baseline data on occupations, wages and employment shares. Here we provide a brief overview and refer the reader to the original work by Acemoglu and Autor for the details. The data for employment comes from IPUMS USA and it includes the decennial censuses between 1980-2000 (with 10 years intervals) and annual data from the American Community Survey (ACS) between 2000-2007. The sample is restricted to individuals aged 16-64 who were employed in the previous year and are assigned to a known occupation (i.e., not n/a or unemployed). We further restrict the sample to exclude the top and bottom 5% of the hourly wage distribution. Wage data comes from the Current Population Survey (CPS) to compute wages per occupation. We follow Acemlogu and Autor on this choice because the data in the ACS is not consistent across years. [36] Occupations and industries are classified based on the 1990 Census Bureau classification scheme that is consistent for all the sample years. These industries are mapped to the BEA industry classification through mutual mapping to the NAICS codes. For each BEA industry, we compute the share of individuals within each occupation.

[36]Specifically, weeks worked last year has only intervals starting in 2007.

Figure 6: Predicted vs. Actual Value-Added Growth



Simulation Results on the $y-$axis, actual data in the $x-$axis.

Our occupation classification is also taken from Acemoglu and Autor (2011). They divide the 382 original occupations into 4 broader categories that are characterized by their skill level: (1) managerial, professional and technical occupations; (2) sales, clerical and administrative support occupations; (3) production, craft, repair and operative occupations; and (4) service occupations. The first group is characterized by high skill occupations, the second and third groups are characterized by middle skill occupations and the last group is characterized by low skill occupations. They measure skill by the average hourly wage of individuals in the occupation in 1980 where the mean wage in each occupation is calculated using workers' hours of annual labor supply times the Census sampling weights..[37]

B.2 Household Expenditure Data

We use cross-sectional data on household expenditure from the Consumer Expenditure Survey (CEX) to estimate the elasticity parameters of our nonhomothetic CES demand system, $\{\sigma, \varepsilon_s\}_{s \in \mathcal{S}}$. We use data from the 2000-2007 period.[38] We follow the procedure described in

[37] Our results on the negative correlation between occupation shares in middle-skill workers and income elasticity parameters are robust to decomposing middle-skill between groups (2) and (3). We report these correlations in Table **??** in the appendix.

[38] We have experimented with different time frame periods, between 1999 and 2007. The estimates are very stable across subsamples. Aguiar and Bils (2015) also report a similar finding in their estimated income elasticities.

Aguiar and Bils (2015) to clean the data. In particular, we restrict our sample to urban households with ages of the reference person between 25 and 64. We drop households if they report spending less than 100 dollars in food in 3 months per individual in the household, they have negative total or food consumption expenditure, total income is reported incomplete, they have not responded all (four quarterly) interviews , income is below 50% of minimum wage or if they earn money but do not work. To mitigate measurement error concerns, we drop the top and bottom 5% richest households according to their total income (after taxes) and we winsorize top and bottom 5% sectoral expenditures.[39] We then follow the procedure described in Buera et al. (2015) and convert the final good expenditures reported in the CEX into value added expenditures using the BEA's 2000 input-output tables. We do so by matching the finest level of expenditure categories in the CEX (called UCCs) to each sector in the BEA table.[40] Following Comin et al. (2015), we also use sectoral, regional urban price series provided by the BLS for our estimation of price elasticities.[41]

B.3 European Countries Data

We use microdata from the LFS and SILC in order to estimate the wages, hours worked and implied $\left\{\left\{\left\{\left(\alpha_{jstc} \quad {}_{j=\{l,m,h\}}\right\}_{s=1}^{S}\right\}_{c}\right($. Specifically we use the LFS data to calculate the hours worked by each skill level in each sector. In order to do so we map the NACE Rev 1.1 and 2 to our sector classification using NACE- NAICS correspondence tables provided by Eurostat. For occupation classification We use the one provided by Goos et al., 2014. This classification differs from the US classification by including Laborers in mining, construction, manufacturing, and transport as well as Models, salespersons, demonstrators and elementary sales occupations in the low skill employees definition. This classification also leaves out the following occupations: legislators and senior officials, teaching professionals and teaching associate professionals, skilled agricultural and fishery workers and agricultural, fishery and related laborers. We focus in individuals that are employed, are not family workers between the ages 16-64. Next, we use the SILC data to calculate the mean wage for each skill type across all industries. We use the same sample restrictions and skill classification. We calculate the wage per hour by dividing the monthly or annual labor income, depending on data availability, by the hours worked in the relevant period. For this purpose we use usual weekly hours worked, multiplied by number of months worked in a year and assuming individuals worked 4 weeks in each month since this data is not directly provided. To make sure the annual labor income was

[39] Total income after taxes is computed as in Aguiar and Bils (2015).

[40] We use the correspondence in Buera et al. (2015). We extend the correspondence for the few cases in which there are UCCs from 2000-2007 missing from their original list.

[41] When possible, we create a household-specific Stone price index for each sector from more disaggregated possible price series categories that belong to each sector. We then also convert final expenditure prices to value added prices by assuming a Cobb-Douglas production function and perfect competition, such that the log price of a sector is the input-share weighed mean of log-prices.

earned while working in the current occupation we further restrict the sample to individuals that did not switch work since the previous year. For total labor compensation share and value added share in each sector we use EU KLEMS data.

C Estimation of Demand Elasticities

C.1 Cross-Country Regressions

We also estimate expenditure elasticities using a cross-country panel for OECD countries from KLEMS. In particular, we run an analogous version to the Aguiar-Bils specification,

$$\ln \frac{x_{st}^n}{\bar{x}_{st}} = \alpha_{st}^n + \eta_s \ln E_t^n + \Gamma_s Z_t^n + \delta_n + \delta_s + u_{st}^n \tag{41}$$

where x_{st}^n denotes value-added in sector s, country c and year t, $\bar{x}_{st}$ denotes the average across n, E_t^n is total value added in n and year t, Z_t^n are country controls (log employment compensation and log total number of employees in the sector) and u_{st}^n is an error term. Finally, α_{st}^n absorbs prices in the first order approximation in Aguiar and Bils. We use the log-price deflator of sector s, and the CES price index with time-varying expenditure shares in the sector and country as weights and price elasticity of 0.5 (which corresponds to the micro elasticity).[42]

We merge the KLEMS data with a time series for oil price shocks from Ramey and Vine (2011). The resulting dataset spans 1995-2012. We construct a price and total value-added instruments by using the time series of oil prices, oil prices squared and interacting the oil price time series with sectoral factor intensity in energy and sectoral factor intensity squared.[43] We instrument total value added in a country with the oil price shocks from Ramey and Vine. Table 14 reports our estimated expenditure elasticities. We find the same broad ranking and a similar range of the expenditure elasticities to our household estimates. The correlation with the household estimates is 0.73. The only outlier is FIRE and other services which have a negative point estimate but appear to be very imprecisely estimated.

D Calculation of contribution of different exogenous factors to the evolution of an endogenous variable

In this section we briefly describe how we calculate the contribution to the evolution of a given variable when there are multiple factors that impact it. For simplicity, consider the case where there are two exogenous drivers (a and b) for the endogenous variable v. Our model provides a

[42] That is, the price index we use is $\left(\sum_s x_{st}^n (p_{st}^n)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$. We have also expermiented with using other price indices, such as the Stone or log of a linear aggregator and obtain very similar results.

[43] We use the US input-output table for this and use the share of sector 211 (oil and gas extraction) in 1997.

Table 14: Expenditure Elasticity Estimates from Cross-Country Data

Education and Health Care	2.45
	(0.80)
Arts, Entertainment,	1.45
Recreation and Food Services	(0.50)
Government[1]	1.26
	(0.19)
Finance, Professional, Information,	-0.59
other services (excl. gov't)	(1.10)
Manufacturing	0.35
	(0.18)
Retail, Wholesale Trade and	0.71
Transportation	(0.14)
Construction	0.95
	(0.38)
Agriculture, Mining and	0.14
Utilities	(0.08)

mapping between the exogenous vector (a, b) and v. In a slight abouse of notation, we denote this mapping also by $v(.)$. The change in v between times 0 and F can be expressed as

$$\triangle v = v(a_F, b_F) - v(a_0, b_0), \tag{42}$$

where we have used subscripts F and 0 to denote the times.

We are interested in decomposing the change in v between the contribution from a and b. Note that $\triangle v$ can be decomposed in two different ways:

$$\begin{aligned} v(a_F, b_F) - v(a_0, b_0) &= v(a_F, b_F) - v(a_0, b_F) + v(a_0, b_F) - v(a_0, b_0) = \frac{\triangle v}{\triangle a}|_{b_F} + \frac{\triangle v}{\triangle b}|_{a_0} \\ v(a_F, b_F) - v(a_0, b_0) &= v(a_F, b_F) - v(a_F, b_0) + v(a_F, b_0) - v(a_0, b_0) = \frac{\triangle v}{\triangle b}|_{a_F} + \frac{\triangle v}{\triangle a}|_{b_0} \end{aligned}$$

where we have used the notation $\frac{\triangle v}{\triangle a}|_{b_F}$ to denote the change in v when a changes keeping b at its value in the final period, for example. Taking the average of both expressions, it follows that

$$v(a_F, b_F) - v(a_0, b_0) = \frac{1}{2}\left[\left(\frac{\triangle v}{\triangle a}|_{b_F} + \frac{\triangle v}{\triangle b}|_{a_0}\right] + \frac{1}{2}\left[\left(\frac{\triangle v}{\triangle b}|_{a_F} + \frac{\triangle v}{\triangle a}|_{b_0}\right]\right(\tag{43}$$

Dividing throughout by the LHS we obtain the following expression for the contributions of changes in a and b to v:

$$1 = \overbrace{\frac{\frac{1}{2}\left[\frac{\triangle v}{\triangle a}|_{b_F} + \frac{\triangle v}{\triangle a}|_{b_0}\right]}{v(a_F, b_F) - v(a_0, b_0)}}^{\text{Contribution by } a} + \overbrace{\frac{\frac{1}{2}\left[\frac{\triangle v}{\triangle b}|_{a_F} + \frac{\triangle v}{\triangle b}|_{a_0}\right]}{v(a_F, b_F) - v(a_0, b_0)}}^{\text{Contribution by } b} \tag{44}$$

By construction, the sum of the contributions adds up to 1. Implicitly, this computation evenly assigns the effect of jointly changing a and b on v between the two drivers.

E Construction of the Value-Added Trade Data

We use the consolidated Input-Output table for the US from the World Input Output Database (available at http://www.wiod.org) to compute the share of value-added relative to total gross inputs by sector, α_s, $j = 1, \ldots, S$. We compute the average across all years available for the 2013 WIOD release (1995-2011).[44] Armed with the sectoral value-added shares $\{\alpha_s\}$, we compute the value-added content of net exports by sector and year. We use COMTRADE data on sectoral trade flows for 1980 and 2016 (since the WIOD input output table does not span a sufficiently long horizon). We also map sectoral trade flows and value-added shares into our eight sectors. The only sectors with positive trade flows are: Agriculture, Mining and Utilities and Manufacturing.

Note that we are imputing the US value-added shares to US imports (in addition to exports). The reason is that we are interested in understanding the effects of trade diversion on the US economy. Thus, a reduction in demand to US producers due to increased imports translates into a decline in labor demand of US producers. In order to capture this effect appropriately we need to use US value-added shares for imports.

Calibration Details To account for international trade we calibrate $\{\zeta_s\}$ and a sector specific TFP terms. We calibrate $\{\zeta_s\}$ so that the domestic aggregate demand in the model matches the domestic VA shares in each sector observed in 1980. We calibrate the sector specific TFP terms that so that the domestic demand augmented by the factor $(1 - \tau_{s,1980})^{-1}$ as discussed in equation (40) matches the total VA share in each sector observed in 1980. The calibration of the distribution parameters of effective units are done to match relative average wages and employment shares. They are done as in the baseline calibration since this part is independent from the trade module. In our main exercise for 2016 we augment each sector specific TFP term by a factor of $(1 - \tau_{s,2016})$ as well as adjust factor and labor intensity parameters α_{st}, β_{st}

[44] We have checked that there are no significant trends in value-added shares for agriculture and manufacturing. If we regress value-added shares on year and a constant we find a non-significant coefficient on time for agriculture and a significant but economically very small coefficient of 0.18% for manufacturing (this coefficient implies an increase over 16 years of 2.9% over a base of 34.6%).

Table 15: Model Simulation with Time Varying Interest Rate

	Year	$\frac{W_L}{W_M}$	$\frac{W_H}{W_M}$	L_s	M_s	H_s	$\frac{W_L L}{\sum_k W_k K}$	$\frac{W_M M}{\sum_k W_k K}$	$\frac{W_H H}{\sum_k W_k K}$	Exercise
Data	1980	0.74	1.24	0.095	0.653	0.252	0.068	0.630	0.302	
	2016	0.80	1.49	0.129	0.488	0.383	0.088	0.421	0.491	
Model	1980	0.74	1.24	0.095	0.653	0.252	0.068	0.630	0.302	
Time varying r	2016	0.90	1.53	0.144	0.504	0.352	0.111	0.431	0.459	E
	2016	0.85	1.53	0.119	0.519	0.362	0.086	0.442	0.472	α+β
	2016	0.90	1.63	0.133	0.474	0.393	0.097	0.384	0.519	E+$\beta+\alpha$
Fraction of observed change[1]		2.67	1.56	1.12	1.08	1.08	1.45	1.18	1.15	
Contribution of E		0.66	0.50	0.83	0.54	0.46	0.93	0.52	0.47	
Contribution of $\alpha+\beta$		0.34	0.50	0.17	0.46	0.54	0.07	0.48	0.53	

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

and then re-calibrate the change in $\tilde{w}_{lt}$ and a TFP shock that is common to all sectors to match the increase in nominal personal consumption expenditures per capita and the price index.

F Further robustness checks

F.1 Time-varying interest rates

sec:time-var int

G Discussion on the use of Log-Normal

Consider low types first l. The terms appearing in the integrals to the three moments $m_{l,i}$ for $i = 0, 1, 2$ are

$$m_{l,i} = \int_0^\infty z^i \frac{1}{z\sigma_l\sqrt{2\pi}} \exp\left(-\frac{(\ln z-\mu_l)^2}{2\sigma_l^2}\right)\cdot F\left(\frac{(\ln(w_l z/w_m)-\mu_m)^2}{2\sigma_m^2}\right)\left(F\left(\frac{(\ln(w_l z/w_h)-\mu_h)^2}{2\sigma_h^2}\right)\right)dz \tag{45}$$

where F is a function related to the CDF of the log-normal distribution. From here we see that changing μ or σ is equivalent if we do not pin down wages separately. Thus, since we target relative wages, we can normalize all means and one variance.

Table 16: Model Simulation - Germany

	Year	$\frac{W_L}{W_M}$	$\frac{W_H}{W_M}$	L_s	M_s	H_s	$\frac{W_L L}{\sum_k W_k K}$	$\frac{W_M M}{\sum_k W_k K}$	$\frac{W_H H}{\sum_k W_k K}$	Exercise
Data	2005	0.79	1.29	0.166	0.379	0.455	0.12	0.345	0.535	
	2015	0.85	1.43	0.189	0.339	0.472	0.137	0.289	0.574	
Model	2005	0.79	1.29	0.166	0.379	0.455	0.12	0.345	0.535	
	2015	0.9	1.47	0.197	0.295	0.508	0.146	0.242	0.613	E
	2015	0.81	1.26	0.185	0.382	0.432	0.14	0.354	0.506	α+β
	2015	0.86	1.41	0.186	0.321	0.493	0.136	0.272	0.592	E+$\beta + \alpha$
Fraction of observed change[1]		1.17	0.86	0.87	1.45	2.24	0.94	1.30	1.46	
Contribution of E		1.14	1.38	0.80	1.25	1.50	0.69	1.27	1.44	
Contribution of $\alpha + \beta$		-0.14	-0.38	0.20	-0.25	-0.50	0.31	-0.27	-0.44	

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

Table 17: Model Simulation - France

	Year	$\frac{W_L}{W_M}$	$\frac{W_H}{W_M}$	L_s	M_s	H_s	$\frac{W_L L}{\sum_k W_k K}$	$\frac{W_M M}{\sum_k W_k K}$	$\frac{W_H H}{\sum_k W_k K}$	Exercise
Data	2005	0.81	1.36	0.206	0.359	0.434	0.15	0.321	0.529	
	2015	0.92	1.43	0.242	0.262	0.496	0.186	0.219	0.595	
Model	2005	0.81	1.36	0.206	0.359	0.434	0.15	0.321	0.529	
	2015	0.91	1.47	0.242	0.304	0.454	0.184	0.256	0.56	E
	2015	0.87	1.47	0.221	0.312	0.466	0.162	0.262	0.576	α+β
	2015	0.94	1.55	0.245	0.276	0.478	0.185	0.221	0.594	E+$\beta + \alpha$
Fraction of observed change[1]		1.18	2.71	1.08	0.86	0.71	0.97	0.98	0.98	
Contribution of E		0.65	0.50	0.77	0.55	0.36	0.81	0.53	0.38	
Contribution of $\alpha + \beta$		0.35	0.50	0.23	0.45	0.64	0.19	0.47	0.62	

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

Table 18: Model Simulation - UK

	Year	$\frac{W_L}{W_M}$	$\frac{W_H}{W_M}$	L_s	M_s	H_s	$\frac{W_L L}{\sum_k W_k K}$	$\frac{W_M M}{\sum_k W_k K}$	$\frac{W_H H}{\sum_k W_k K}$	Exercise
Data	2005	0.85	1.61	0.224	0.323	0.453	0.153	0.26	0.587	
	2015	0.86	1.62	0.232	0.247	0.521	0.155	0.192	0.653	
Model	2005	0.85	1.61	0.224	0.323	0.453	0.153	0.26	0.587	
	2015	0.89	1.68	0.235	0.298	0.466	0.162	0.231	0.607	E
	2015	0.88	1.75	0.222	0.286	0.492	0.146	0.213	0.642	α+β
	2015	0.92	1.82	0.233	0.263	0.504	0.154	0.188	0.658	E+$\beta + \alpha$
Fraction of observed change[1]		7.00	21.00	1.13	0.79	0.75	0.50	1.06	1.08	
Contribution of E		0.57	0.33	1.22	0.40	0.25	8.50	0.38	0.25	
Contribution of $\alpha + \beta$		0.43	0.67	-0.22	0.60	0.75	-7.50	0.63	0.75	

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

Table 19: Model Simulation - Italy

	Year	$\frac{W_L}{W_M}$	$\frac{W_H}{W_M}$	L_s	M_s	H_s	$\frac{W_L L}{\sum_k W_k K}$	$\frac{W_M M}{\sum_k W_k K}$	$\frac{W_H H}{\sum_k W_k K}$	Exercise
Data	2005	0.84	1.32	0.188	0.425	0.388	0.144	0.388	0.468	
	2015	0.87	1.34	0.262	0.359	0.379	0.209	0.328	0.463	
Model	2005	0.84	1.32	0.188	0.425	0.388	0.144	0.388	0.468	
	2015	0.89	1.39	0.203	0.389	0.408	0.159	0.343	0.498	E
	2015	0.9	1.29	0.231	0.411	0.359	0.192	0.379	0.428	α+β
	2015	0.93	1.36	0.236	0.383	0.381	0.197	0.341	0.462	E+$\beta + \alpha$
Fraction of observed change[1]		3.00	2.00	0.65	0.64	0.74	0.81	0.78	1.22	
Contribution of E		0.44	1.75	0.21	0.76	-3.00	0.19	0.88	-5.33	
Contribution of $\alpha + \beta$		0.56	-0.75	0.79	0.24	4.00	0.81	0.12	6.33	

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

Table 20: Model Simulation - Netherlands

	Year	$\frac{W_L}{W_M}$	$\frac{W_H}{W_M}$	L_s	M_s	H_s	$\frac{W_L L}{\sum_k W_k K}$	$\frac{W_M M}{\sum_k W_k K}$	$\frac{W_H H}{\sum_k W_k K}$	Exercise
Data	2005	0.85	1.26	0.166	0.309	0.525	0.127	0.277	0.595	
	2015	0.99	1.34	0.215	0.256	0.529	0.18	0.217	0.602	
Model	2005	0.85	1.26	0.166	0.309	0.525	0.127	0.277	0.595	
	2015	0.89	1.33	0.171	0.278	0.551	0.131	0.239	0.63	E
	2015	0.92	1.27	0.204	0.29	0.506	0.168	0.259	0.573	α+β
	2015	0.96	1.33	0.21	0.263	0.527	0.173	0.226	0.601	E+$\beta + \alpha$
Fraction of observed change[1]		0.79	0.88	0.90	0.87	0.50	0.87	0.85	0.86	
Contribution of E		0.36	0.93	0.13	0.63	11.75	0.10	0.70	5.25	
Contribution of $\alpha + \beta$		0.64	0.07	0.88	0.37	-10.75	0.90	0.30	-4.25	

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

To elaborate a little more on this note that:

$$\frac{(\ln(w_l z/w_h) - \mu_h)^2}{2\sigma_h^2} = \left(\left(\ln z^{\frac{1}{\sqrt{2}\sigma_h}} + \frac{1}{\sqrt{2}\sigma_h} \ln \frac{w_l}{2w_h\sigma_h} - \mu_h\right)^2 \quad (46)$$

$$= (\ln y - \tilde{\mu})^2 \quad (47)$$

So it is clear that the average changes with both μ and σ. Since we do not observe z we can always renormalize things as in the last line and only use μ or σ for the calibration.