

# MONEY RUNS\*

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## Abstract

We develop a model in which, as in practice, bank debt is both a financial security used to raise funds and a kind of money used to facilitate trade. This dual role of bank debt provides a new rationale for why banks do what they do. In the model, banks endogenously perform the essential functions of real-world banks: they transform liquidity, transform maturity, pool assets, and have dispersed depositors. Moreover, they make their debt redeemable on demand. Thus, they are endogenously fragile. We show novel effects of narrow banking, suspension of convertibility, and some other policies.

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## 1 Introduction

What is money? Some money, like the physical currency you exchange by hand, is created by central banks. But most money, like the deposit you exchange electronically by debit card or bank transfer, is created by private banks.<sup>1</sup> Such bank money is not a new thing. Bank debt has served as a means of payment for hundreds of years. For example, bank-drawn bills of exchange served as money in early modern Europe, bank-issued notes served as money in the 19th-century US, and bank-certified checks served as money more recently.

But bank debt is not only a form of money that you can use to make payments, it is also a financial security that banks use to raise funds. Thus, when banks choose what security to issue to raise funds, they should take its value as money into account. In practice, banks choose to issue securities, like banknotes and deposits, that are redeemable on demand. But such demandable debt can be an unstable form of money. Indeed, many bank panics and financial crises throughout history, from 18th-century London to contemporary Greece, seem to have followed from the failure of bank debt to be accepted as money (see below). In such crises, convertibility is often suspended. This prevents bank runs—you are unable to run on a bank if you cannot redeem your debt on demand. But it has been argued that it could also impede circulation—you could be unlikely to accept the debt as payment if you cannot redeem it on demand.<sup>2</sup> Remarkably, however, this has not always been the case. To the contrary, bank debt sometimes resumes circulation when convertibility is suspended.<sup>3</sup>

Despite historical precedents, most current theories of why banks choose a fragile financial structure are not linked to how bank debt serves as private money (see, notably, Calomiris and Kahn (1991), Diamond and Dybvig (1983), and Diamond and Rajan (2001b), discussed further in Section 7). To develop a model based on this link, we model how bank

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<sup>1</sup>E.g., the Bank of England estimates that 97% of broad money is created by banks (McLeay, Radia, and Thomas (2014)).

<sup>2</sup>See, e.g., Dewald (1972) on how the “trade journals reported that depression was accountable to suspension and a lack of loans to sustain trade” (p. 939), i.e. on how some argued that the suspension impeded payments/trade. See also Sprague (1910).

<sup>3</sup>For example, in the crisis of 1907, despite suspension of convertibility, bank debt in the form of clearing house certificates started to circulate as money. See, e.g., Andrew (1908), on how “[s]hops and stores and places of amusement...generally accepted [certificates], and it is, indeed, surprising...how little real difficulty was experienced in getting them to circulate in lieu of cash” (p. 513). Perhaps as a result, banks remained solvent in the panic. Indeed, see, e.g., Calomiris and Gorton (1991) on how “the Panic of 1907 [was] practically a non-event from the standpoint of national bank failures” (p. 156).

debt serves as a means of payment explicitly, following the new monetarist literature (see Lagos, Rocheteau, and Wright (2017) for a survey).

We use the model to address the following questions. Why is bank money almost always redeemable on demand, regardless of the form it takes, from physical banknotes to electronic deposits? Why is demandable debt a fragile form of money? Given it is, why do banks still choose to issue it, exposing themselves to sudden redemptions, and making the financial system fragile too? And what should financial regulators do about it? Should they intervene ex ante, for example by limiting bank scope to contain fragility within “narrow” banks? Or should they intervene ex post, for example by suspending convertibility?

By modeling the dual role of bank debt—to provide a means of funding to banks and a means of payment for depositors—we uncover a new rationale for why banks do what they do. Banks choose to fund themselves with demandable debt to take advantage of a “price effect of demandability”: demandable debt trades at a high price in the secondary market, and hence increases banks’ debt capacity in the primary market. But this high price is not always a good thing. Reluctant to pay it, potential counterparties may decide not to buy the debt at all, and therefore leave the holder with something he cannot trade, but only redeem on demand. Such redemption constitutes a new kind of bank run, or “money run,” resulting entirely from the failure of debt to circulate as money in the secondary market. In our model, banks are fragile because money is fragile, not the other way around (cf. Friedman and Schwartz (1963)). However, banks continue to issue demandable debt. To do so, they exploit economies of scale that arise solely from the price effect of demandability (independent of the diversification benefits in Diamond and Dybvig (1983) and Diamond (1984)). Specifically, they transform liquidity, transform maturity, pool assets, and borrow from dispersed depositors. I.e. they do something that looks like real-world banking. But, to do it effectively, they exacerbate their exposure to money runs. Narrow banking limits this fragility, but can also inefficiently constrain bank funding. Suspension of convertibility, on the other hand, can not only prevent runs, but can, in fact, facilitate circulation.

**Model preview.** Because we want to show how banking can arise endogenously, we start with a single borrower  $B$  with a single investment (e.g., a corporate loan). Ultimately, multiple borrowers will form an institution that assumes features of real-world banks. But, for now,  $B$  resembles a bank only insofar as its debt plays a dual role. To capture its role as a funding instrument, we assume that  $B$  is penniless and needs to fund an investment from a creditor  $C_0$  (i.e. a depositor). To capture its role as a means of payment, we make two assumptions. First,  $C_0$  could be hit by a liquidity shock before  $B$ ’s investment pays off, as in Diamond and Dybvig (1983). Thus,  $C_0$  could want to trade  $B$ ’s debt to get liquidity. Second,  $C_0$  must trade bilaterally in a decentralized market, similar to those in Trejos and Wright

(1995) and Duffie, Gârleanu, and Pedersen (2005). We assume that to acquire B’s debt from  $C_0$ , a counterparty  $C_1$  must pay an entry cost  $k$  to enter and bargain with  $C_0$  over the price.<sup>4</sup> Likewise, if  $C_1$  is shocked, a counterparty  $C_2$  must pay  $k$  and bargain with him to trade, and so on. The terms of trade between counterparties depend on how B designs its debt. In particular, B can make its debt redeemable on demand. In this case, B chooses a redemption value  $r$ , for which a creditor can redeem before the investment pays off. To pay  $r$ , B has to liquidate its investment (so  $r$  is bounded by the liquidation value).

**Results preview.** Our first main result is that B makes its debt redeemable on demand to borrow more from  $C_0$ .  $C_0$  is willing to pay more for demandable debt, even if he never redeems in equilibrium. The reason is that  $C_0$  still values the option to redeem off equilibrium, even if he never exercises it, because it provides him with a valuable threat (i.e. outside option) when he bargains with  $C_1$ . As a result, he can sell B’s debt at a higher price. Anticipating selling to  $C_1$  at a high price in the secondary market, he is willing to lend more to B in the primary market. This result contrasts with existing models of demandable debt as liquidity insurance, in which, roughly, you do not need the option to redeem debt on demand if you can just trade it in the secondary market (e.g., Jacklin (1987)). Here, in contrast, you do: just the option to redeem on demand props up the resale price of debt in the secondary market, even if the option is never exercised. We refer to this as the “price effect of demandability,” because it works entirely through the secondary market price, not through actual redemptions.

Our second main result is that B’s debt is susceptible to a new kind of run, which results directly from its failure to circulate in the secondary market. A sudden (but rational) change in beliefs can cause secondary-market trading to stop, leading  $C_0$  to redeem on demand and forcing B to liquidate inefficiently to pay the redemption value. Even though there are gains from trade when  $C_0$  is hit by a liquidity shock, he may not be able to get liquidity from  $C_1$ , and hence he might still end up redeeming. The reason is that  $C_1$ ’s willingness to pay the entry cost  $k$  depends on his ability to sell B’s debt in the future. Hence, if his beliefs change, and he starts to doubt whether future counterparties will enter, he will not enter himself, leaving  $C_0$  with nothing to do but redeem. The belief change may be precipitated by a shock to fundamentals, in which case the run amplifies a downturn, or by a “confidence crisis” unrelated to fundamentals, in which case the run constitutes a panic in itself. Either way, such a run can occur even though B has only a single creditor—in this case, there is not a coordination problem in which multiple creditors race to withdraw as in Diamond and Dybvig (1983), Goldstein and Pauzner (2005), or He and Xiong (2012); rather, there is a coordi-

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<sup>4</sup>The (possibly very small) cost  $k$  can capture physical costs of coming to market and trading or the opportunity cost of doing so (see Subsection 6.1).

nation problem in which a creditor cannot get liquidity in the secondary market and must withdraw as a result. We refer to this run as a “money run,” because it is the result of the failure of B’s debt to function as money in the secondary market.

We construct an equilibrium in which money runs happen on the equilibrium path due to confidence crises that occur with probability  $\lambda$ . In this case, B faces a trade-off. If he issues demandable debt, he benefits from the price effect of demandability, but exposes himself to runs with probability  $\lambda$ . Hence, we ask: what is the largest  $\lambda$  for which B still makes its debt demandable? Our model is tractable enough to admit a closed-form expression for this number. For “reasonable” parameters, we find that it is large (about 14%), suggesting that our model can plausibly explain why banks choose run-prone instruments even though doing so exposes them to costly liquidation.

Our third main result is that increasing the redemption value  $r$  has a dark side. Although it increases the price  $C_0$  can sell for (as per the price effect of demandability), it symmetrically increases the price  $C_1$  must pay. This makes  $C_1$  less willing to enter. Thus, for high  $r$ ,  $C_0$ ’s option to redeem on demand can undermine itself, putting him in such a strong bargaining position that he has no willing counterparty, and ends up redeeming on demand in a money run.

Our fourth main result is that B sets the redemption value  $r$  as high as possible. This increases the price of B’s debt, allowing it to borrow more cheaply. This has a social cost, because it increases the risk of a run, and, hence, increases the expected deadweight loss from early liquidation. But B still wants to increase  $r$ , because it has a private benefit. It helps it to extract rent from future creditors, by increasing the price they pay when they bargain to buy its debt. Hence, although financial fragility may be necessary—sometimes B must make its debt demandable to borrow enough to fund its investment—it can also be excessive—B makes the redemption value too high just to decrease its cost of funding, and hence exposes itself to more runs than necessary.

Our fifth main result is that if multiple borrowers can get together, they can exploit economies of scale that allow them to issue debt with total redemption value in excess of the total liquidation value of their investments. To show this, we consider  $N$  parallel versions of the model—we assume that there are  $N$  parallel borrowers, each of which borrows to fund an investment from one of  $N$  parallel creditors, each of whom trades bilaterally in one of  $N$  parallel markets. The only link between the parallel versions is that the borrowers can issue debt backed by the entire pool of investments. So now there are  $N$  creditors holding  $N$  securities backed by  $N$  investments, instead of one creditor holding one security backed by one investment. We assume that everything is perfectly correlated, so, unlike in Diamond and Dybvig (1983) and Diamond (1984), there is no possibility of diversification.

Despite this, we find that getting together can still benefit borrowers, because they can give each of the  $N$  creditors the option to redeem for the entire pool. Why does each creditor have a claim on the entire pool, rather than just a fraction  $1/N$  of it? Because if bank debt circulates, no one redeems on the equilibrium path; thus, if one creditor deviates, he is the only one redeeming, and he has the first claim on all of the assets. As per the price effect of demandability, the option to be first in line is valuable, even if it is never exercised. Hence, it can be enjoyed by one creditor without making it unavailable to others—in the language of public goods, the redemption option is “non-rivalrous.” To decrease their cost of funding, the borrowers continue increasing the redemption value  $r$  until the price of their debt is so high that counterparties are just indifferent between paying the entry cost  $k$  and staying out. Remarkably, by doing so, the borrowers can fund exactly the investments with positive social surplus—no more and no fewer.

With this result, we see that our model, based on only the dual role of bank debt, points to a new rationale for why banks do what they do: borrowers form a “bank” (or a banking system) only to create demandable debt, or “money”; they endogenously transform liquidity, transform maturity, pool assets, and borrow from dispersed creditors. And, like a bank, they are fragile. By doing banking, borrowers exacerbate their exposure to money runs. Unlike in the banking literature, banks are fragile because money is fragile. And, unlike in the new monetarist literature, money is fragile no matter how small counterparties’ entry cost  $k$  is (see Section 7). The reason is that, here, the redemption value  $r$  is determined endogenously. If  $k$  decreases, the bank responds by increasing  $r$ , keeping counterparties indifferent to entry. Thus, as  $k \rightarrow 0$ ,  $r$  approaches the face value—as in developed economies in normal times, bank debt is redeemable at par. But, given counterparties are indifferent to staying out, the debt remains a fragile means of payment. The bank remains vulnerable to money runs, which can now trigger liquidation of the whole pool of investments.

**Policy.** In our model, bank structure and demandable debt arise endogenously in response to the environment. Thus, the model takes the possibility of regulatory arbitrage into account. As a result, it is not easily subject to Lucas-type critiques and is well suited to policy analysis. Most notably, we explore two measures that policy makers can use to fight crises: (i) suspension of convertibility, i.e. prohibiting redemption on demand once a crisis has begun, and (ii) restricting banks to be “narrow,” i.e. separating deposit taking and lending to prevent the crisis from happening in the first place.

Our sixth main result is that suspending convertibility not only mechanically puts an end to bank runs in a crisis, but, in certain circumstances, can also restore the circulation of bank debt. The reason is that it lowers the price of debt, and thus makes counterparties more willing to enter. This result suggests that the worry that suspension could impede

circulation (see footnote 2) could be exaggerated.

In contrast, the model reveals a straightforward but important downside of narrow banking. Yes, forbidding multiple creditors to have the first claim on the same assets can prevent bank runs. However, it also prevents banks from taking advantage of the non-rivalrous redemption value (as per our fifth main result). As a result, it can prevent them from funding some investments with positive social surplus.

**Further results.** We explore three extensions. (i) We add random, heterogenous entry costs. We show that this is another way to generate runs on the equilibrium path, as well as to obtain a unique equilibrium. (ii) We show that if B can choose its investment, its choice can be distorted toward high-liquidation-value investments, which facilitate its issuing demandable debt. (iii) We study a version of the model with a continuum of creditors in which debt can be rolled over as well as traded. We show that the results of our baseline model are robust. (This setup also has the attractive feature that not every withdrawal is a run.)

**Evidence and applications.** Although our model is stylized, our findings resonate with practice. Banks borrow via demandable debt, but it seems to be a fragile means of payment. Indeed, many historical panics have features of money runs. For example, when merchants refused bank-drawn bills of exchange in 18th-century London, it led to the crisis of 1772<sup>5</sup>; when the Second Bank refused state bank notes in the early 19th-century US, it led to the crisis of 1819<sup>6</sup>; when New York clearing houses refused bank trusts' checks in the early 20th-century US, it led to the Panic of 1907<sup>7</sup>; when retailers refused checks in late 20th- and early 21st-century Argentina, it exacerbated both the banking panic of 1995 and the economic crisis of 1998–2002<sup>8</sup>; when retailers refused debit cards and wholesalers refused

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<sup>5</sup>See, e.g., Kosmetatos (2014) on how “[t]hrough drawers, acceptors, or endorsers of bills stopping [accepting them], [issuers]...quickly failed” in the 1772 crisis (p. 14). See, e.g., Bagehot’s *Lombard Street* on how something similar happened in the crisis of 1825 when “the country was...within twenty-four hours of a state of barter” and “Exchequer Bills would have been useless unless the bank cashed them...[in an] intervention...chiefly useful by the effect which it would have in increasing the circulating medium” (pp. 98–99).

<sup>6</sup>See, e.g., Blackson (1989) on how “[t]he 1818 decline began, no doubt...[when] the [Second Bank] began to press state institutions to satisfy in specie their obligations [to it]” (p. 351). Such panics were ubiquitous in the free banking era. See, e.g., Gorton (2012a) on how, as Secretary of the Treasury Howell Cobb describes it, “The merchant, the mechanic, the grocer, and the butcher began business in the morning...and their customers found that the bank note that passed freely yesterday was rejected this morning” (p. 36).

<sup>7</sup>See, e.g., Tallman and Moen (1990) on how “stop[ping] clearing checks for the Knickerbocker Trust Company” incited the 1907 panic; see also Frydman, Hilt, and Zhou (2015).

<sup>8</sup>See, e.g., the New York *Times* on how “[s]ome big businesses [were] demanding cash on delivery and refusing to accept checks” in the 1995 banking panic (“Bank crisis undermining ‘the Argentine Miracle,’” May 1, 1995) and on how “a growing number of stores...were refusing to accept any form of payment other than cash” in the 1998–2003 crisis (“In Argentina’s bank holiday, cash is most scarce commodity,” April 24, 2002).

bank transfers in contemporary Greece, it exacerbated the Greek debt crisis.<sup>9</sup>

Although we are motivated by these traditional forms of bank debt, like banknotes and deposits, we suggest that it could also apply, perhaps more loosely, to other forms of bank debt classified as money, such as repos (Section 5.2). And some results could even apply to some non-bank debt that resembles money, such as commercial paper.

**Layout.** The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes benchmarks. Section 4 includes our main results. Section 5 discusses policy, applications, and empirical content. Section 6 discusses our assumptions and explores some extensions. Section 7 discusses the related literature. Section 8 concludes. The Appendix contains all proofs and a table of notations.

## 2 Model

In this section, we present the model.

### 2.1 Players, Dates, and Technologies

There is a single good, which is the input of production, the output of production, and the consumption good. Time is discrete and the horizon is infinite,  $t \in \{0, 1, \dots\}$ .

There are two types of players, a borrower B and infinitely many deep-pocketed creditors  $C_0, C_1, \dots$ , where  $C_t$  is “born” at Date  $t$ . Everyone is risk-neutral and there is no discounting. B is penniless but has a positive-NPV investment. The investment costs  $c$  at Date 0 and pays off  $y > c$  at a random time in the future, which arrives with intensity  $\rho$ . Thus, the investment has  $\text{NPV} = y - c > 0$  and expected horizon  $1/\rho$ . B may also liquidate the investment before it pays off; the liquidation value is  $\ell < c$ .

B can fund its project by borrowing from  $C_0$ . However, there is a horizon mismatch similar to that in Diamond and Dybvig (1983): creditors may need to consume before B’s investment pays off. Specifically, creditors consume only if they suffer “liquidity shocks,” which arrive at independent random times with intensity  $\theta$  (after which they die). Hence, a creditor’s expected “liquidity horizon” is  $1/\theta$ .

For now, we focus on a single borrower funding a single investment with debt to a single creditor; this helps us to distinguish the forces in our model from those in the literature.<sup>10</sup>

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<sup>9</sup>See, e.g., the *Financial Post* on how “many retailers were not accepting card transactions” in 2015 (“Greece in limbo as it shuts banks,” June 29, 2015) and, e.g., *Reuters* on how Greek olive oil producers “want it in cash or they prefer to keep their olive oil in their tanks” (“Greek olive farmers demand cash as bank fears grow,” July 7, 2015) in the same period.

<sup>10</sup>For example, there is no coordination problem among multiple creditors (but we show there can be a different coordination problem with a single creditor) and there is no possibility to pool multiple investments



Later, we include multiple borrowers funding multiple investments from multiple creditors; this allows us to show how the forces in our model give rise to something that looks like real-world banking.

## 2.2 Borrowing Instruments

At Date 0, B borrows the investment cost  $c$  from its initial creditor  $C_0$  via an instrument with terminal repayment  $R \leq y$ , paid when the investment pays off, and redemption value  $r \leq \ell$ , paid if the instrument is redeemed earlier. Creditors can exchange the instrument among themselves and B must repay whichever creditor holds it. Hence, the instrument is tradeable demandable debt, and we refer to it as a “banknote,” although it also resembles a bank deposit or even a repo. We let  $v_t$  denote the Date- $t$  value of B’s debt to a creditor not hit by a liquidity shock.

As benchmarks, we consider instruments that may not be tradeable (so B has to repay  $C_0$ ) and/or may not be demandable, but may be “long-term” (so B makes only the terminal repayment). I.e. we allow B to borrow via the banknote or one of the following debt instruments: (i) non-tradeable long-term debt, which we refer to as a “loan,” (ii) non-tradeable demandable debt, which we refer to as a “puttable loan”; and (iii) tradeable long-term debt, which we refer to as a “bond” (although it also resembles an equity share). These instruments are summarized in Figure 1. They constitute all of the feasible Markovian instruments in the sense that they are all transfers from B to the debtholder that can depend on the state of B’s investment at Date  $t$  (but not on the date itself) and do not violate B’s limited-liability constraints.

FIGURE 1: DEBT INSTRUMENTS

	not demandable	demandable
non-tradeable	“loan”	“puttable loan”
tradeable	“bond”	“banknote”

## 2.3 Secondary Debt Market: Entry, Bargaining, and Settlement

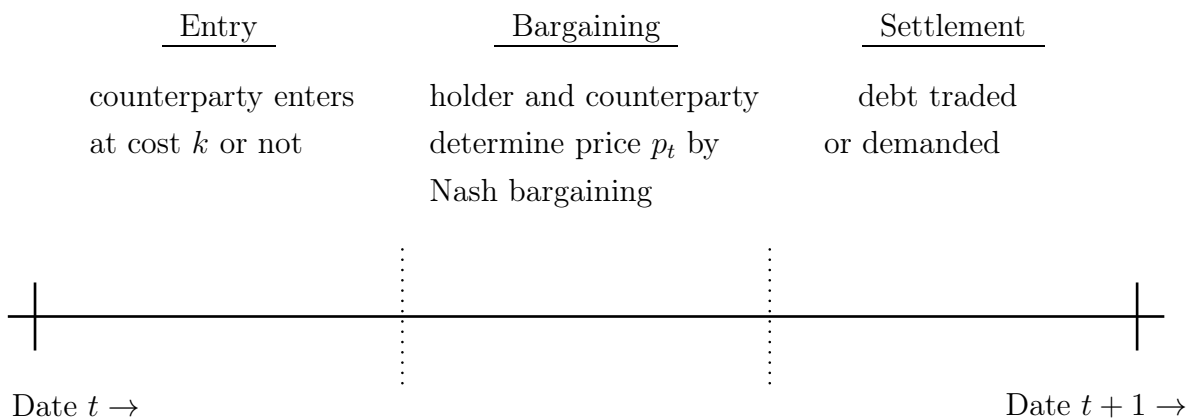
If B has borrowed via tradeable debt, then creditors can trade it bilaterally in a decentralized market. At each Date  $t$ ,  $C_t$  is the single (potential) counterparty with whom the debtholder, denoted by  $H_t$ , can trade B’s debt.  $C_t$  meets  $H_t$  whenever he pays an “entry” cost  $k$ , which

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(but we show a new reason to pool investments in an enriched environment (Subsection 4.5)).

can represent any opportunity cost of trade. In this case,  $C_t$  and  $H_t$  determine the price  $p_t$  via generalized Nash bargaining.<sup>11</sup>  $H_t$ 's bargaining power is denoted by  $\eta$ . If  $C_t$  and  $H_t$  agree on a price, then trade is settled:  $C_t$  becomes the debtholder in exchange for  $p_t$  units of the good. Otherwise,  $H_t$  retains the debt. If the debt is demandable,  $H_t$  can demand redemption from B or he can remain the debtholder at Date  $t + 1$ . This sequence of entry, bargaining, and settlement is illustrated in Figure 2.<sup>12</sup> (The entry and bargaining stages are standard in the literature; the settlement stage is our addition to model demandable debt.)

FIGURE 2: SECONDARY-MARKET TRADE



We let  $\sigma_t$  denote  $C_t$ 's mixed strategy if  $H_t$  is hit by a liquidity shock, so  $\sigma_t = 1$  means that  $C_t$  enters for sure and  $\sigma_t = 0$  means that  $C_t$  does not enter. Thus,  $\sigma_t$  also represents the probability that  $H_t$  finds a counterparty when hit by a liquidity shock. Observe that we restrict attention to  $C_t$ 's strategy given  $H_t$  is hit by a liquidity shock without loss of generality.<sup>13</sup>

<sup>11</sup>We discuss the entry cost  $k$  and Nash bargaining protocol in Subsection 6.1. Note here that what matters is just that  $C_t$  makes the decision to bear a cost to trade before he bargains with  $H_t$  and that  $C_t$  and  $H_t$  split the gains from trade after  $k$  is sunk. Further, it does not matter if  $H_t$  bears a (possibly larger) cost to trade.

<sup>12</sup>By separating bargaining and settlement, we zero in on tradability and demandability— $H_t$  agrees to trade with  $C_t$  or not at the bargaining stage and then demands redemption from B or not at the settlement stage. This structure precludes other arrangements, e.g., in which B intermediates trades between  $H_t$  and  $C_t$ . We discuss such “rollover” arrangements in Subsection 6.1 and modify our set-up to speak to them explicitly in Subsection 6.4.

<sup>13</sup>The reason that this is without loss of generality is that  $C_t$  would never enter if  $H_t$  were not shocked: if  $H_t$  is not shocked,  $H_t$  and  $C_t$  are identical and there are no gains from trade, so it is never worthwhile to pay the entry cost  $k$  for the opportunity to trade.

## 2.4 Timeline

First, B makes  $C_0$  a take-it-or-leave-it offer of a repayment and a redemption value, as described in Subsection 2.2 above. Then, if  $C_0$  accepts, he becomes the initial debtholder  $H_1$ . The debtholder may redeem on demand or may trade in the secondary market, as described in Subsection 2.3 above. Formally, the extensive form is as follows.

Date 0            B offers  $C_0$  a repayment  $R$  and a redemption value  $r$ .

If  $C_0$  accepts, then B invests  $c$ .  $C_0$  is the initial debtholder,  $H_1 = C_0$ .

Date  $t > 0$         If B's investment pays off: B repays  $R$  to  $H_t$  and B consumes  $y - R$ .

If B's investment does not pay off: there is entry, bargaining, and settlement as described in Subsection 2.3.

If there is trade,  $C_t$  becomes the new debtholder,  $H_{t+1} = C_t$ .

If there is no trade,  $H_t$  either holds the debt,  $H_{t+1} = H_t$ , or redeems on demand, in which case B liquidates its investment, repays  $r$  to  $H_t$ , and consumes  $\ell - r$ .

## 2.5 Equilibrium

The solution concept is subgame perfect equilibrium. An equilibrium constitutes (i) the repayments  $R$  and  $r$ , (ii) the price of debt in the secondary market  $p_t$  at each date, and (iii) the entry strategy  $\sigma_t$  of the potential counterparty  $C_t$  such that B's choice of instrument and  $C_t$ 's choice to enter are sequentially rational,  $p_t$  is determined by Nash bargaining, and each player's beliefs are consistent with other players' strategies and the outcomes of Nash bargaining.

For most of the paper, we focus on stationary equilibria, i.e.  $\sigma_t \equiv \sigma$  and  $p_t \equiv p$ .

## 3 Benchmarks

To begin, we consider three benchmark instruments, the loan, the puttable loan, and the bond. We verify two results in the literature in our environment: (i) demandability can increase debt capacity as in Calomiris and Kahn (1991) and (ii) tradeability can substitute for demandability as in Jacklin (1987).

### 3.1 Loan

First, we consider a loan, i.e. non-tradeable long-term debt. At Date  $t$ , the value  $v_t$  of the loan with face value  $R$  can be written recursively:

$$v_t = \rho R + (1 - \rho)(1 - \theta)v_{t+1}. \quad (1)$$

The terms are determined as follows. With probability  $\rho$ , B's investment pays off and B repays  $R$ . With probability  $(1 - \rho)\theta$ , B's investment does not pay off and the debtholder  $H_t$  is hit by a liquidity shock. Since the loan is neither tradeable nor demandable,  $H_t$  gets zero. With probability  $(1 - \rho)(1 - \theta)$ , B's investment does not pay off and  $H_t$  is not hit by a liquidity shock.  $H_t$  retains B's debt at Date  $t + 1$ , which has value  $v_{t+1}$  at Date  $t$  since there is no discounting.<sup>14</sup> By stationarity ( $v_t = v_{t+1} \equiv v$ ), equation (1) gives

$$v = \frac{\rho R}{\rho + (1 - \rho)\theta}. \quad (2)$$

Even though B will always repay eventually, the loan's value  $v$  is less than its face value  $R$ . The loan is discounted because, without the option to demand debt or trade it,  $H_t$  gets nothing in the event of a liquidity shock. Hence, the discount vanishes as shocks become unlikely,  $v \rightarrow R$  as  $\theta \rightarrow 0$ . For  $\theta > 0$ , demandability and tradeability can help to reduce the discount, as we see next.

### 3.2 Puttable Loan

Now we consider a puttable loan, i.e. non-tradeable demandable debt. At Date  $t$ , the value  $v_t$  of the puttable loan can be written recursively:

$$v_t = \rho R + (1 - \rho)\left(\theta r + (1 - \theta)v_{t+1}\right). \quad (3)$$

The terms are determined as follows. With probability  $\rho$ , B's investment pays off and B repays  $R$ . With probability  $(1 - \rho)\theta$ , B's investment does not pay off and the debtholder  $H_t$  is hit by a liquidity shock. Since the loan is demandable, but not tradeable,  $H_t$  redeems on demand and gets  $r$ . With probability  $(1 - \rho)(1 - \theta)$ , B's investment does not pay off and  $H_t$  is not hit by a liquidity shock.  $H_t$  retains B's debt at Date  $t + 1$ , which has value  $v_{t+1}$  at

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<sup>14</sup>Formally, the value of holding B's debt is the Date- $t$  *expected* value of B's debt at Date  $t + 1$ , i.e. we should write  $\mathbb{E}_t[v_{t+1}]$  instead of  $v_{t+1}$ . For now, we focus on deterministic equilibria. Thus, this difference is immaterial and we omit the expectation operator for simplicity. (In Subsection 4.4, we do keep track of the expectation operator.)

Date  $t$  since there is no discounting. By stationarity ( $v_t = v_{t+1} \equiv v$ ), equation (3) gives

$$v = \frac{\rho R + (1 - \rho)\theta r}{\rho + (1 - \rho)\theta}. \quad (4)$$

We now compare the puttable loan's debt capacity with the loan's, where "debt capacity" refers to the maximum  $B$  can borrow given limited liability. I.e. we compare equation (4) with  $R = y$  and  $r = \ell$  and equation (2) with  $R = y$ :

PROPOSITION 1. (BENCHMARK: BENEFIT OF DEMANDABILITY.) *If*

$$\frac{\rho y}{\rho + (1 - \rho)\theta} < c \leq \frac{\rho y + (1 - \rho)\theta \ell}{\rho + (1 - \rho)\theta}, \quad (5)$$

*then  $B$  can fund itself with a puttable loan but not with a loan.*

The analysis so far already points to one rationale for demandable debt. As in Calomiris and Kahn (1991), the ability to liquidate insures  $C_0$  against bad outcomes, making him more willing to lend.<sup>15</sup> Thus, by issuing demandable debt,  $B$  expands its debt capacity.

### 3.3 Bond

Now we consider a bond, i.e. tradeable long-term debt. (This instrument can also represent an equity claim; debt and equity have equivalent payoffs, since the terminal payoff  $y$  is deterministic.) At Date  $t$ , the value  $v_t$  of the bond can be written recursively:

$$v_t = \rho R + (1 - \rho)\left(\theta \sigma_t p_t + (1 - \theta)v_{t+1}\right). \quad (6)$$

The terms are determined as follows. With probability  $\rho$ ,  $B$ 's investment pays off and  $B$  repays  $R$ . With probability  $(1 - \rho)\theta$ ,  $B$ 's investment does not pay off and the debtholder  $H_t$  is hit by a liquidity shock. Since the bond is tradeable, but not demandable,  $H_t$  gets  $p_t$  if he finds a counterparty, which happens with probability  $\sigma_t$ , and nothing otherwise. With probability  $(1 - \rho)(1 - \theta)$ ,  $B$ 's investment does not pay off and  $H_t$  is not hit by a liquidity shock.  $H_t$  retains  $B$ 's debt at Date  $t + 1$ , which has value  $v_{t+1}$  at Date  $t$  since there is no discounting.

To solve for the value  $v_t$ , we must first find the secondary-market price of the bond  $p_t$ .

LEMMA 1. *The secondary-market price of the bond is  $p_t = \eta v_t$ .*

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<sup>15</sup>In Calomiris and Kahn (1991), "bad outcomes" are associated with moral hazard problems, rather than liquidity shocks.

The bond price splits the gains from trade between  $H_t$  and  $C_t$  in proportions  $\eta$  and  $1 - \eta$ . Since  $H_t$  has value zero in this case ( $H_t$  dies at the end of the period and the bond is not demandable), the gains from trade are just the value  $v_t$  of the bond to the new debtholder  $C_t$ .

By stationarity ( $v_t = v_{t+1} \equiv v$  and  $\sigma_t \equiv \sigma$ ) and the preceding lemma ( $p_t \equiv p \equiv \eta v$ ), equation (6) gives

$$v = \frac{\rho R}{\rho + (1 - \rho)\theta(1 - \eta\sigma)}. \quad (7)$$

We now compare the bond's debt capacity (equation (7) with  $R = y$  and  $\sigma = 1$ )<sup>16</sup> to the puttable loan's (equation (4) with  $R = y$  and  $r = \ell$ ):

PROPOSITION 2. (BENCHMARK: TRADEABILITY SUBSTITUTES DEMANDABILITY.) *Suppose the bond circulates in equilibrium ( $\sigma = 1$ ).<sup>17</sup> If*

$$\frac{\rho y + (1 - \rho)\theta\ell}{\rho + (1 - \rho)\theta} < c \leq \frac{\rho y}{\rho + (1 - \rho)\theta(1 - \eta)}, \quad (8)$$

*then B can fund itself with a bond but not with a puttable loan (or a loan).*

If the bond circulates, B can borrow against the full value  $y$  whenever trading frictions vanish (in the sense that  $H_t$  gets the bargaining power). I.e. if  $\sigma = 1$ , then there is no role for demandability whenever  $\eta \rightarrow 1$ . Hence, the analysis so far supports Jacklin's (1987) intuition that tradeability substitutes for demandability. If  $C_0$  is hit by a liquidity shock, he can trade B's debt in the market, rather than die with it. In other words, like the option to demand, the option to trade insures  $C_0$  against bad outcomes, making him more willing to lend. In the language of Brunnermeier and Pedersen (2009), market liquidity creates funding liquidity. Moreover, absent trading frictions ( $\eta \rightarrow 1$ ), B can expand its debt capacity more by issuing tradeable debt (a bond) than by issuing demandable debt (a puttable loan). However, we will see next that with trading frictions ( $\eta < 1$ ), there is a role for demandability, even if debt is never redeemed in equilibrium (Proposition 3).

## 4 Banknote and Banking

In this section, we analyze the banknote and present our main results.

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<sup>16</sup>The debt capacity of a tradeable instrument refers to the maximum B can borrow *if* it circulates, or  $\sigma = 1$ . Thus, since  $\sigma$  is chosen by  $C_t$ , the debt capacity is an upper bound on what B can borrow. I.e. the condition that the debt capacity exceeds  $c$  is necessary but not sufficient for B to invest.

<sup>17</sup>As we will see below (setting  $r = 0$  in equation (14)), there is an equilibrium in which the bond circulates as long as  $C_t$ 's entry cost  $k$  is sufficiently small.

## 4.1 The Price Effect of Demandability

Now we consider a banknote, i.e. tradeable, demandable debt. At Date  $t$ , the value  $v_t$  of the banknote can be written recursively:

$$v_t = \rho R + (1 - \rho) \left( \theta (\sigma_t p_t + (1 - \sigma_t) r) + (1 - \theta) v_{t+1} \right). \quad (9)$$

The terms are determined as follows. With probability  $\rho$ , B's investment pays off and B repays  $R$ . With probability  $(1 - \rho)\theta$ , B's investment does not pay off and the debtholder  $H_t$  is hit by a liquidity shock. Since the banknote is both tradeable and demandable,  $H_t$  gets  $p_t$  if he finds a counterparty, which happens with probability  $\sigma_t$ , and otherwise redeems on demand and gets  $r$ . With probability  $(1 - \rho)(1 - \theta)$ , B's investment does not pay off and  $H_t$  is not hit by a liquidity shock.  $H_t$  retains the banknote at Date  $t + 1$ , which has value  $v_{t+1}$  at Date  $t$  since there is no discounting.

To solve for the value  $v_t$ , we must first give the secondary-market price of the banknote  $p_t$ .

LEMMA 2. *The secondary-market price of the banknote is  $p_t = \eta v_t + (1 - \eta)r$ .*

The price of the banknote splits the gains between  $H_t$  and  $C_t$  in proportions  $\eta$  and  $1 - \eta$ . Since  $H_t$  has value  $r$  ( $H_t$  redeems on demand and gets  $r$  if he does not trade with  $C_t$ ), the gains from trade are  $v_t - r$ , the value to the new debtholder  $C_t$  minus the value to the current debtholder  $H_t$ . The price that splits these gains is  $p_t = r + \eta(v_t - r) = \eta v_t + (1 - \eta)r$ .<sup>18</sup>

By stationarity ( $v_t = v_{t+1} \equiv v$  and  $\sigma_t \equiv \sigma$ ) and the preceding lemma ( $p_t \equiv p = \eta v + (1 - \eta)r$ ), equation (9) gives

$$v = \frac{\rho R + (1 - \rho)\theta(1 - \eta\sigma)r}{\rho + (1 - \rho)\theta(1 - \eta\sigma)}. \quad (10)$$

We now compare the banknote's debt capacity ( $v$  with  $R = y$ ,  $r = \ell$ , and  $\sigma = 1$ ) to the benchmark instruments' (Section 3). We find that B can borrow more via a banknote than via any other instrument.

PROPOSITION 3. (PRICE EFFECT OF DEMANDABILITY.) *Suppose the banknote circulates ( $\sigma = 1$ ).<sup>19</sup> If*

$$\max \left\{ \frac{\rho y + (1 - \rho)\theta\ell}{\rho + (1 - \rho)\theta}, \frac{\rho y}{\rho + (1 - \rho)\theta(1 - \eta)} \right\} < c \leq \frac{\rho y + (1 - \rho)\theta(1 - \eta)\ell}{\rho + (1 - \rho)\theta(1 - \eta)}, \quad (11)$$

<sup>18</sup>This result depends on how outside options determine the division of surplus in bargaining. See Subsection 6.1 for a discussion.

<sup>19</sup>We will see below (equation (14)) that there is an equilibrium in which the banknote circulates as long as  $C_t$ 's entry cost  $k$  is sufficiently small.

then  $B$  can fund itself only with the banknote.

Unlike the puttable loan, the banknote need not be redeemed in equilibrium. Like the bond, it can circulate in the secondary market until maturity. But it is still more valuable than the bond. The reason is that just the option to redeem the banknote on demand (off equilibrium) puts the debtholder in a strong bargaining position in the secondary market, increasing its price. Thus, given secondary market trading frictions ( $\eta < 1$ ), demandability *complements* tradability: your option to demand debt increases the price you trade at. This high price leads to a high debt capacity: in anticipation of being able to sell at a high price in the secondary market,  $C_0$  is willing to pay a high price in the primary market.

What kind of borrower needs to issue the banknote? To answer, we rewrite the condition of Proposition 3. From the left-hand inequality in equation (11):

$$\frac{1}{\rho} > \frac{1}{\theta} \cdot \frac{y - c}{(1 - \rho) \min\{c - \ell, (1 - \eta)c\}}. \quad (12)$$

This says that creditors' expected liquidity horizon  $1/\theta$  is small relative to  $B$ 's expected investment horizon  $1/\rho$ . Hence,  $B$ 's debt is a kind of inside money, since a creditor generally does not hold it for its entire maturity; rather he holds it for a short time and then uses it to get liquidity from another creditor—as Kiyotaki and Moore (2001) put it, “[w]henver paper circulates as a means of short-term saving (liquidity), it can properly be considered as money, or a medium of exchange, because agents hold it not for its maturity value but for its exchange value” (p. 1). Moreover, it implies that  $B$  intermediates between short-horizon creditors and a long-horizon investment. Hence,  $B$  is starting to resemble a bank, as maturity transformation is one of banks' defining features. But this is just the first step in our argument that  $B$  is a bank. Below, we will see that  $B$  will endogenously look a lot like a real-world bank: it will not only transform maturity, but pool assets and engage in other canonical banking activities as well, all to create valuable money (Subsection 4.5).

## 4.2 Money Runs

Having established how a banknote helps  $B$  raise funds in the primary market, we now turn to how it trades in the secondary market, and whether it could be in fact redeemed early. In other words, does the banknote always circulate ( $\sigma = 1$ ), as we assumed above? To answer, we assume that  $B$  has issued a banknote at Date 0 with terminal repayment  $R$  and redemption value  $r$ , and we look at the equilibria of the subgames for  $t > 0$ . (We determine  $R$  and  $r$  in equilibrium in Proposition 7.)

First, observe that  $C_t$  enters as long as the value he captures minus the price he pays



exceeds his entry cost, or

$$v - p = \frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta(1 - \eta\sigma)} \geq k. \quad (13)$$

Observe that his payoff depends on other strategies'  $\sigma$ , which reflect his beliefs about whether B's banknote circulates. Indeed, the note circulates as long as  $\sigma_t = 1$  is a best response to the belief that  $C_{t'}$  plays  $\sigma_{t'} = 1$  for all  $t' > t$ . This is the case as long as  $C_t$  is willing to pay the entry cost  $k$  to gain the surplus  $v - p$  given  $\sigma = 1$ , or

$$k \leq v - p \Big|_{\sigma=1} = \frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta(1 - \eta)}, \quad (14)$$

having substituted in from Lemma 2 and equation (10).

But there may also be another equilibrium in which B's banknote does not circulate. B's banknote does not circulate as long as  $\sigma_t = 0$  is a best response to the belief that  $C_{t'}$  plays  $\sigma_{t'} = 0$  for all  $t' > t$ . This is the case as long as  $C_t$  is *not* willing to pay the entry cost  $k$  to gain the surplus  $v - p$  given  $\sigma = 0$ , or

$$k \geq v - p \Big|_{\sigma=0} = \frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta}, \quad (15)$$

again having substituted in from Lemma 2 and equation (10). If  $r$  were fixed, this "bad" equilibrium would arise only for sufficiently high  $k$ . But  $r$  is endogenous, not fixed. We show below that it can increase if  $k$  decreases, so that this equilibrium can arise even for arbitrarily small  $k > 0$  (see Subsection 4.5).

**PROPOSITION 4. (MONEY RUNS.)** *Suppose that B borrows via a banknote with terminal repayment  $R$  and redemption value  $r$ . If the entry cost  $k$  is such that*

$$\frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta} \leq k \leq \frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta(1 - \eta)}, \quad (16)$$

*then the  $t > 0$  subgame has both an equilibrium in which B's debt circulates ( $\sigma = 1$ ) and there is no early liquidation and an equilibrium in which B's debt does not circulate ( $\sigma = 0$ ) and there is early liquidation. There is also a mixed equilibrium, with  $\sigma \in (0, 1)$  given in the proof.*

If a counterparty  $C_t$  doubts future liquidity, i.e. he doubts that he will find a counterparty in the future, then  $C_t$  will not enter. As a result, the debtholder  $H_t$  indeed will not find a counterparty. There is a self-fulfilling dry-up of secondary-market liquidity. With demandable

debt, this has severe real effects: unable to trade,  $H_t$  redeems his debt on demand, leading to the costly liquidation of  $B$ 's investment. In other words, a change in just the beliefs about future liquidity leads to the failure of  $B$ 's debt as a medium of exchange in the secondary market—the failure of  $B$ 's debt as money. As a result, there is sudden withdrawal of liquidity from  $B$ , i.e. a bank run, or a *money run*.

COROLLARY 1. *Suppose  $k$  satisfies condition (16). If  $C_t$ 's beliefs change from  $\sigma_{t'} = 1$  to  $\sigma_{t'} = 0$  for  $t' > t$ , the debtholder  $H_t$  “runs” on  $B$ , i.e.  $H_t$  unexpectedly demands redemption of his debt, forcing  $B$  to liquidate its investment.*

The literature has stressed bank failures resulting from shocks to fundamentals (e.g., Allen and Gale (1998) and Gorton (1988)) or beliefs about primary market withdrawals (Diamond and Dybvig (1983)). Friedman and Schwartz (1963) emphasize that such bank failures, whatever their root cause, disrupt economic activity because banks create money—e.g., they issue banknotes—which facilitates trade. Our model also connects bank failure with money creation. But the chain of causation goes in the opposite direction: the banknote is redeemed only because it fails to circulate. Thus, a run can occur even with a single creditor, who redeems his debt when he cannot trade it. It need not be the result of many creditors racing to be the first to redeem from a common pool of assets. Thus, our model explains runs on repos and 19th-century banknotes, which are individually collateralized, not backed by common assets (see Subsection 5.2 and Section 7).

In our model, financial fragility is a necessary evil. It is necessary because  $B$  must issue a fragile instrument—the banknote—to fund itself (Proposition 3). And it is evil because money runs lead to inefficient liquidation. This contrasts with the literature on the necessity of financial fragility, which stresses its virtue, not evil (Allen and Gale (1998), Diamond and Rajan (2001a, 2001b)).

### 4.3 Dark Side of Demandable Debt

Although financial fragility is necessary in our model, it can still be excessive. To see why, first observe that increasing the redemption value  $r$  makes runs “more likely”: high  $r$  puts  $H_t$  in a strong bargaining position, increasing the price  $C_t$  pays. This makes it less attractive for him to enter. And if  $C_t$  does not enter,  $H_t$  is unable to trade and must redeem early—must run.

PROPOSITION 5. (DARK SIDE.) *Increasing the redemption value  $r$  makes the banknote less likely to circulate in the following senses:*

(i) each counterparty  $C_t$  enters only for lower entry cost  $k$  (given the strategy of other counterparties);

(ii)  $\sigma = 1$  is an equilibrium of the  $t > 0$  subgame only for lower  $k$ ;

(iii)  $\sigma = 0$  is not an equilibrium of the  $t > 0$  subgame for lower  $k$ .

Hence, demandability cuts both ways. It is both the thing that allows B to fund itself and the thing that exposes B to runs. It increases B's debt capacity, since it props up the price of B's debt (Proposition 3). But it also increases B's liquidation risk, since it makes  $C_t$  reluctant to enter.

Does B internalize the full cost of liquidation risk? Given there are multiple equilibria, it is not straightforward to know how to address this question. Although Proposition 5 says that increasing  $r$  makes a run equilibrium more likely to exist, it does not say how an incremental increase in  $r$  affects the probability that a run occurs within the multiple-equilibria region. To make progress, we focus on possible equilibria in which  $\sigma_t$  is an increasing function  $f$  of  $(R - r)$ , directly reflecting the form of  $C_t$ 's payoff if he enters (equation (13)). The restriction holds in the equilibria we solve for explicitly below, both in (i) the sunspot equilibrium in Proposition 4.4 and in (ii) the unique stationary cut-off equilibrium in Proposition 10, which reflects an exercise akin to equilibrium selection using global games.

With this additional structure, we find that the answer is no, B does not internalize liquidation risk:

**PROPOSITION 6. (REDEMPTION VALUE.)** *Suppose that the probability that each counterparty enters is an increasing function  $f$  of  $(R - r)$ . As long as the derivative  $f'$  is not too large, B sets the maximum redemption value,  $r = \ell$ .*

Intuitively, there is a benefit to B of increasing the redemption value  $r$ :  $C_0$  requires less compensation for the risk of having to sell at a discount in the secondary market. This benefit is a cost to  $C_0$ 's future counterparty, who pays a high price for the banknote. But he is not there at Date 0, when B and  $C_0$  are bargaining. Hence, although B and  $C_0$  maximize their joint surplus, they do not fully internalize this cost, and B continues to increase  $r$  even when it has no social benefit.

#### 4.4 Equilibrium Runs

We now turn to characterizing an equilibrium in which B borrows via a banknote and money runs arise on the equilibrium path. To do this, we introduce a "sunspot" coordination variable at each date,  $s_t \in \{0, 1\}$ . We will interpret  $s_t = 1$  as "normal times" and  $s_t = 0$

as a “confidence crisis,” since the sunspot does not affect economic fundamentals, but serves only as a way to coordinate beliefs. We assume that  $s_0 = 1$ , that  $\mathbb{P}[s_{t+1} = 0 | s_t = 1] =: \lambda$ , and that  $\mathbb{P}[s_{t+1} = 0 | s_t = 0] = 1$ , where we think about  $\lambda$  as a small number. In words: the economy starts in normal times and a permanent<sup>20</sup> confidence crisis occurs randomly with small probability  $\lambda$ .

We now look for a Markov equilibrium, i.e. an equilibrium in which the sunspot (rather than the whole history) is a sufficient statistic for  $C_t$ 's action:

$$\sigma_t = \begin{cases} \sigma^1 & \text{if } s_t = 1, \\ \sigma^0 & \text{if } s_t = 0. \end{cases} \quad (17)$$

We can now write the banknote's value  $v$  as a function of  $s_t$  (cf. the analogous equation for the stationary case in equation (9)):

$$v^0 = \rho R + (1 - \rho) \left( \theta \left( \sigma^0 p^0 + (1 - \sigma^0) r \right) + (1 - \theta) v^0 \right), \quad (18)$$

$$v^1 = \rho R + (1 - \rho) \left( \theta \left( \sigma^1 p^1 + (1 - \sigma^1) r \right) + (1 - \theta) \left( \lambda v^0 + (1 - \lambda) v^1 \right) \right). \quad (19)$$

The next proposition characterizes an equilibrium in which the “confidence crisis” induces a money run.

**PROPOSITION 7. (EQUILIBRIUM WITH SUNSPOT RUNS.)** *Suppose that the condition in equation (11) is satisfied strictly. As long as  $\lambda$  is sufficiently small, there exists  $k$  such that  $B$  can fund its investment only with tradeable, demandable debt (a banknote), even though it admits a money run when  $s_t = 0$ . Specifically,  $C_t$  plays  $\sigma_t = s_t$ , and the value of the banknote when  $s_t = 0$  is*

$$v^0 = \frac{\rho R + (1 - \rho)\theta\ell}{\rho + (1 - \rho)\theta}, \quad (20)$$

*the value of the banknote when  $s_t = 1$  is*

$$v^1 = \frac{\left( \rho + (1 - \rho)(\lambda(1 + \theta\eta) + (1 - \lambda)\theta) \right) c - (1 - \rho)\lambda\theta\eta\ell}{\rho + (1 - \rho)(\lambda + (1 - \lambda)\theta)}, \quad (21)$$

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<sup>20</sup>We assume that the crisis is an absorbing state only to be able to solve easily in closed form.

the repayment  $R$  is

$$R = c + \frac{(1 - \rho)\theta\left(\rho(\lambda + (1 - \lambda)(1 - \eta)) + (1 - \rho)(\lambda + (1 - \lambda)\theta(1 - \eta))\right)}{\rho\left(\rho + (1 - \rho)(\lambda + (1 - \lambda)\theta)\right)} (c - \ell), \quad (22)$$

and the redemption value is  $r = \ell$ .

With these closed-form expressions, it is easy to see how the price of debt depends on parameters.

COROLLARY 2. (COMPARATIVE STATICS.) *The (net) interest rate  $(R - c)/c$  is*

- (i) *decreasing in the liquidation value  $\ell$ ;*
- (ii) *decreasing in debtholders' bargaining power  $\eta$ ;*
- (iii) *decreasing in creditors' liquidity horizon  $1/\theta$ ;*
- (iv) *increasing in the probability of a confidence crisis  $\lambda$ ;*
- (v) *increasing in the investment size  $c$ ;*
- (vi) *increasing in the investment horizon/expected maturity  $1/\rho$ . Moreover, the term structure is upward sloping, in the sense that the yield<sup>21</sup>  $\rho(R - c)/c$  is also increasing in  $1/\rho$ .*

In our model, the interest rate is compensation for liquidity risk. The results (i)–(iv) capture that increasing  $\ell$  and  $\eta$  decrease liquidity risk and increasing  $\theta$  and  $\lambda$  increase it. (v) says that bigger investments are effectively riskier (all else equal). The reason is that, for fixed liquidation value  $\ell$ , they are liquidated at a larger discount in a confidence crisis. (vi) says that longer maturity investments demand not only higher repayments, but also higher per-period interest rates, even though there is no discounting in preferences. The reason is that as maturity increases both the probability that  $C_0$  has to trade at a discount before maturity and the size of the discount he trades at increase. So illiquidity in the secondary market generates the term structure.<sup>22</sup>

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<sup>21</sup>This un-compounded yield is approximately equal to the continuously compounded yield, which you might be more used to. For a zero-coupon instrument:

$$\text{continuously compounded yield} \equiv \rho \log \frac{R}{c} = \rho \log \left(1 + \frac{R - c}{c}\right) \approx \rho \frac{R - c}{c},$$

for small  $(R - c)/c$  (given the Taylor expansion of  $\log(1 + x)$ ).

<sup>22</sup>See Kozlowski (2017) for a macroeconomic model in which trading frictions generate the yield curve.

**A word on welfare and a numerical example.** Given its multiple equilibria, our model does not admit a general welfare analysis. To speak to welfare, we make the following assumption, motivated by the idea that confidence crises are likely only if there is the risk of early redemption: confidence crises can occur if B borrows via the banknote, but not if B borrows via the bond. We ask: if both the banknote and the bond are feasible, how small does the probability  $\lambda$  of a confidence crisis have to be for B to prefer the banknote?

LEMMA 3. *Suppose confidence crises can occur only if B borrows via the banknote. If the bond is feasible, B still borrows via the banknote whenever the probability  $\lambda$  of a confidence crisis is below the threshold  $\lambda^*$ ,*

$$\lambda^* = \frac{\rho(\rho + (1 - \rho)\theta)(1 - \eta)\ell}{\rho(y - \ell) + (\rho + (1 - \rho)\theta)(1 - \eta)(\rho\ell - c)}, \quad (23)$$

*and borrows via the bond otherwise.*

For example, if  $\theta = 1/4$ ,  $\rho = 1/10$ ,  $y = 20$ ,  $c = 10$ ,  $\ell = 8$ , and  $\eta = 3/4$ , B chooses the banknote whenever  $\lambda \leq \lambda^* \approx 14.4\%$ .<sup>23</sup> This points to a potentially attractive feature of our model: unlike in many quantitative bank run models, a borrower chooses the run-prone instrument for “reasonable” parameters even when the probability of a run is relatively high.

## 4.5 Banking

We now suppose that the horizon mismatch (equation (11)) is so severe that the borrower cannot raise  $c$  to fund its investment, not even via a banknote. In this case, direct finance is not possible. But perhaps a form of intermediated finance is?

To address this question, we now consider  $N$  parallel versions of our baseline model:  $N$  identical borrowers  $B^1, \dots, B^N$  can do parallel investments at Date 0 and  $N$  identical creditors  $C_t^1, \dots, C_t^N$  can enter parallel markets at each Date  $t > 0$ . At Date 0, the borrowers can issue mutualized instruments, backed by the whole pool of their investments. Redeeming creditors are paid first come, first served à la Diamond and Dybvig (1983). At each Date  $t > 0$ , each version of the model proceeds exactly as in the baseline, as described in Section 2. Note that we assume that the parallel versions of the model are

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<sup>23</sup>We think about these as annual numbers.  $\theta = 1/4$ , the number used in Ennis and Keister (2003), implies creditors suffer liquidity shocks on average once every four years.  $\rho = 1/10$  implies the investment is long-term, taking ten years to complete on average. Given this maturity,  $y = 20$  and  $c = 10$  imply the investment has annual return of 7.2%.  $\ell = 8$  implies the investment has a 20% liquidation discount relative to its book value.  $\eta = 3/4$  implies that debtholders get most of the surplus, but far from all of it; this is intended to capture some degree of competition among counterparties.

identical in every state, i.e. investments/liquidity shocks are perfectly correlated across borrowers/creditors. Thus, there are no diversification benefits from pooling loans/deposits as in Diamond (1984)/Diamond and Dybvig (1983).

Even absent diversification, the borrowers can benefit from pooling their investments to increase their debt capacity and raise  $c$ . The reason is that pooling allows borrowers to increase the redemption value  $r$  of each banknote up to  $N\ell$ , rather than just up to  $\ell$ .

Why does each creditor have a claim on the whole liquidation value  $N\ell$  rather than just on a fraction  $1/N$  of it? The answer is that in an equilibrium in which banknotes circulate, no one redeems on the equilibrium path; thus, if one creditor deviates, he is the only one redeeming, and can get paid up to  $N\ell$ . As per the price effect of demandability, a high redemption value  $r$  is valuable, even if the redemption option is never exercised. Thus, increasing  $r$  can benefit all  $N$  creditors simultaneously—it is “non-rivalrous.” And now  $r$  can become arbitrarily large as the number of borrowers  $N$  pooling assets increases.

But that does not mean that borrowers should make  $r$  arbitrarily large. If it is too large, banknotes do not circulate, viz. creditors do not enter if they anticipate being in a weak bargaining position. Their entry condition (equation (14)) puts an upper bound  $r^{\max}$  on  $r$ :

$$r \leq r^{\max} := R - \frac{\rho + (1 - \rho)\theta(1 - \eta\sigma)}{\rho(1 - \eta)}k. \quad (24)$$

Now, to find the debt capacity of a banknote, we substitute  $r = r^{\max}$ ,  $R = y$ , and  $\sigma = 1$  into the value of the banknote (equation (10)), to get

$$\max v = \frac{\rho R + (1 - \rho)\theta(1 - \eta\sigma)r}{\rho + (1 - \rho)\theta(1 - \eta\sigma)} \Bigg|_{r=r^{\max}, R=y, \sigma=1} \quad (25)$$

$$= y - \frac{(1 - \rho)\theta}{\rho}k. \quad (26)$$

Given borrowers can undertake an investment only if its debt capacity exceeds its cost ( $\max v > c$ ), equation (26) implies that the borrowers can undertake investments if and only if the NPV,  $y - c$ , exceeds creditors’ total expected entry costs,  $\frac{(1 - \rho)\theta}{\rho}k$ .<sup>24</sup> Thus, by forming a “bank,” the borrowers can issue banknotes to fund all (and only) investments with positive total surplus. There is a money-creation rationale for banking:

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<sup>24</sup>This expression for the expected entry costs can be understood as follows: from Date 1 onward, creditors pay  $k$  at Date  $t$  if debtholders are shocked while investments are still underway, which occurs with probability  $(1 - \rho)^t\theta$ . Hence,

$$\text{total expected entry costs} = \sum_{t=1}^{\infty} (1 - \rho)^t\theta k = \frac{(1 - \rho)\theta}{\rho}k.$$

PROPOSITION 8. (BANKING.) *Suppose*

$$N \geq \frac{1}{\ell} \left( y - \frac{\rho + (1 - \rho)(1 - \eta)}{\rho(1 - \eta)} k \right). \quad (27)$$

*There is an equilibrium in which borrowers successfully fund all investments, raising  $c$  by issuing a banknote to each of the Date-0 creditors, if and only if the investments have positive total surplus, i.e. the NPV is higher than the total expected entry costs, or*

$$y - c \geq \frac{(1 - \rho)\theta}{\rho} k. \quad (28)$$

To fund all positive-surplus investments, borrowers have to set  $r$  so high that counterparties are indifferent between entering and staying out. This makes them especially susceptible to runs, since an arbitrarily small change in a counterparty's belief about others' strategies makes him stay out, leading to a money run.

And now a money run has severe consequences. As in a real-world bank run, there is mass liquidation: with  $r > \ell$ , multiple investments need to be liquidated to redeem each banknote. In addition to this fragility, the coalition of borrowers has other defining features of a real-world bank.

1. **Liquidity transformation.** The bank funds illiquid assets (non-tradeable investments that are costly to liquidate early) with liquid liabilities (circulating demandable debt).
  - Issuing liquid (tradeable) liabilities gives creditors insurance against liquidity shocks.
2. **Maturity transformation.** The bank funds long-term investments with short-term (demandable) liabilities.
  - Issuing demandable liabilities allows creditors to trade at a high price given liquidity shocks.
3. **Asset pooling.** The bank pools borrowers' investments, reusing their liquidation value to back demandable debt.
  - Issuing debt backed by a pool of assets gives creditors a high redemption value.
4. **Dispersed depositors (creditors).** The bank borrows from a large number of dispersed creditors.



- Issuing debt to many creditors gives them the option to redeem against the same assets (hence dispersed creditors are necessary for asset pooling to help).

5. **Fragility.** The bank borrows via debt that is susceptible to runs, and runs force early liquidation of multiple investments.

- Issuing run-prone debt, i.e. demandable debt with high redemption value, is necessary to make the secondary market price high enough that the bank can fund efficient investments.

In the banking equilibrium, financial fragility is not necessarily the result of monetary fragility. With dispersed creditors, there is a common pool problem, which makes creditors want to redeem if they believe others are going to. Thus, not all runs need be money runs; there can be Diamond–Dybvig runs too, and these different types of runs could exacerbate each other.

The banking equilibrium also makes it easier to apply our model to contemporary deposit markets, in which entry costs often seem to be small and deposits are redeemable at par:

*COROLLARY 3. Consider the banking equilibrium in Proposition 8 in which  $r = r^{\max}$ . As entry costs become small, i.e.  $k \rightarrow 0$ , the redemption value and secondary market price approach the face value, i.e.  $r \rightarrow R$  and  $p \rightarrow R$ .*

Here, unlike in the baseline model, a run can occur no matter how small the entry cost  $k$  is. If  $k$  is small, the borrowers make  $r$  high, so counterparties are still indifferent between entering and staying out (cf. equation (15)).

The flip side of this result is that for  $k > 0$ , we should expect to see a penalty for early withdrawal (i.e.  $r < R$ ), in line with those we see for less liquid forms of bank debt, such as savings accounts and certificates of deposit.

## 5 Policy, Applications, and Empirical Content

We now turn to our model’s policy implications, applications, and empirical content.

### 5.1 Policy

Our analysis stresses how the secondary market in which bank debt circulates interacts with the primary market in which banks issue debt and debtholders demand redemption. Our focus on this interaction gives new perspectives on a number of policies. Most notably, it

admits an analysis of suspension of convertibility in crises. It affirms how this can protect against runs, as in the literature so far. But it also suggests that it can restore the circulation of banknotes, in contrast to received thought, but in line with some historical evidence (see footnotes 2 and 3).

**Suspension of convertibility.** To analyze suspension of convertibility, we consider a set-up akin to that in Subsection 4.4, in which there are two sunspot states,  $s_t \in \{0, 1\}$ , where  $s_t = 0$  represents a confidence crisis and  $s_t = 1$  normal times. Here, we assume that there is a circulating banknote with face value  $R$ . We assume it has redemption value  $r^1$  in normal times, and ask what happens if the bank (or a regulator) suspends convertibility in a confidence crisis, setting  $r^0 = 0$ . Does this mitigate the confidence crisis by preventing runs? Or does it exacerbate it by inhibiting circulation? To address these questions, we assume the economy is in a confidence crisis today that persists with probability  $1 - \mu$  at each date or comes to a permanent end,<sup>25</sup> i.e.  $s_0 = 0$ ,  $\mathbb{P}[s_{t+1} = 0 | s_t = 0] =: 1 - \mu$ , and  $\mathbb{P}[s_{t+1} = 1 | s_t = 1] = 1$ .

We find that suspending convertibility can not only prevent runs, but also restore circulation:

PROPOSITION 9. (SUSPENSION OF CONVERTIBILITY.) *Suppose  $r^1 = r^{max}$  as in the banking equilibrium in Proposition 8. If the banknote circulates in normal times  $\sigma^1 = 1$ , then, as long as*

$$R > \frac{(1 - \eta)\mu(1 - \rho)\theta + \rho(\rho + (1 - \rho)(\theta + (1 - \theta)\mu))}{(1 - \eta)\rho(\rho + (1 - \rho)\mu)} k, \quad (29)$$

*suspending convertibility in a crisis ensures the banknote always circulates.*

Intuitively, if convertibility is suspended, the debt holder  $H_t$  can no longer redeem his note from the bank if trade fails. This decreases his outside option when bargaining with his counterparty  $C_t$ , putting  $C_t$  in a relatively strong bargaining position. Under the condition in the proposition, this makes  $C_t$ 's bargaining position so strong that he is willing to enter even if he believes no one else will enter until the crisis is over. However, suspension should only be temporary. As we point out in the proof of Proposition 9, if  $C_t$  anticipates redeeming at a high price in the future, it can help circulation in a crisis. In line with these prescriptions, in practice, suspension is temporary, after which bank debt is typically repaid in full.<sup>26</sup>

**Other policies.** Our model gives new perspectives on a number of other policies.

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<sup>25</sup>As in Section 4.4, we assume that  $s_t = 1$  is an absorbing state to be able to solve easily in closed form.

<sup>26</sup>See, e.g., Calomiris and Gorton (1991) on how, despite frequent suspensions, “[t]he worst loss per deposit dollar...in the National Banking Era was 2.1 cents per dollar of deposits” (p. 114).

1. **Narrow banking.** In our model, a bank can fund all worthwhile investments if it can pool them and issue demandable debt backed by the whole pool (Proposition 8). This suggests a downside to the idea of narrow banking, which suggests that real investments should be separate from deposit-taking (its financial stability benefits notwithstanding).
2. **Asset purchase guarantees.** In 2008 the US Treasury opened its Temporary Guarantee Program, in which it promised to buy the shares of money market mutual funds at a guaranteed price. This off-equilibrium promise to buy money-like securities could eliminate the “bad” equilibrium, in which counterparties do not enter the secondary market fearing it will dry up in the future.
3. **Capital requirements.** In our model, capital requirements are a double-edged sword. They can help, by curbing banks’ incentive to use too much demandable debt (Proposition 5). But they can also hurt, by inefficiently constraining investment (Proposition 3).

## 5.2 Applications

By zeroing in on the dual role of bank debt—as a funding instrument and payment instrument—we have abstracted from some things stressed in contemporary discussions of money, banking, and financial stability. Notably, (i) we assume that all money is created by banks, abstracting from outside money and assuming that debt is redeemable for real goods. (ii) We assume that there is a single bank, abstracting from interbank competition, trade, and clearing. We also make stark assumptions about the secondary market, modeling decentralized trade in the simplest way we can, with (iii) costly entry/trade and (iv) prices determined by bilateral bargaining.

In the Free Banking Era, our model assumptions seem to be mostly satisfied. Before Greenbacks were introduced in 1861, (i) all paper money was created by banks, and it was redeemable for gold and silver. Sometimes, (ii) there was only one bank in a geographical region (see, e.g., Helderman (1931)). As a result, counterparties would trade largely that bank’s notes. (iii) Bilateral trade was costly for a number of reasons, including spatial separation, and the fact that “[e]ach time a transaction took place the seller [of goods] had to make some judgment about the quality of the particular set of bank notes being offered...[and the] process of making this judgment used real resources” (Rockoff (1974), p. 144). Moreover, banknotes traded at different discounts, which varied depending on who traded and where, reflecting that (iv) prices were determined bilaterally. Indeed, notes tended to trade at higher

prices when redemption values were higher and when physical redemption was cheaper (i.e. the issuer was physically closer), consistent with the effect of the redemption value  $r$  on the secondary market price through bargaining (see, e.g., Gorton (1996) and Weber (2005)). However, the mapping from  $r$  to the price was not one-to-one, consistent with the effect of bargaining on the division of surplus (for  $\eta \in (0, 1)$ ). And it was not homogenous across notes and time, consistent with the effect of a self-fulfilling aspect to prices, as in our model.

More often, however, our model assumptions are not satisfied so literally. But we think our model still speaks to these circumstances, albeit with a broader interpretation. (i) Central banks create paper and electronic money, which is what bank debt can be redeemed for. In this case, the good in our model should be interpreted as outside money, which can be used freely for consumption/investment. Typically, (ii) there are multiple banks in a geographical region. In this case, the bank in our model should be interpreted as the banking system, and rejecting a bank's notes as not accepting transfers from it. For example, in the recent crises in Argentina and Greece, merchants sometimes refused transfers from the domestic banking system, but accepted foreign ones (see footnotes 8 and 9). These crises also highlight that although (iii) trading costs and (iv) bilateral negotiations are less salient in developed economies in normal times, they can quickly reemerge in crisis. This is consistent with our model, which suggests that notes trade at par for small  $k$  (Corollary 3).

Our model can also capture some aspects of other important money-like securities, such as repos. Repos are financial contracts that banks use as a primary source of liquidity. Some repo transactions are centralized through tri-party agents or CCPs, but trillions of dollars of them are negotiated bilaterally in decentralized (OTC) markets (CGFS (2017)). Moreover, repos are effectively redeemable on demand (repo positions are typically left open until creditors demand they be closed). In these aspects, our model applies well to repos. In another aspect, however, it seems like it might not apply as well. That is that repo contracts do not circulate per se—they are formally bilateral agreements, not tradeable instruments. But the collateral underlying them does, making them closer to the bank money in our model than they might seem.<sup>27</sup> As Gorton and Metrick (2010) put it,

[An] important feature of repos is that the...collateral can be “spent”...used as collateral in another, unrelated, transaction.... This...means that there is a money velocity associated with the collateral. In other words, the same collateral can support multiple transactions, just as one dollar of cash can lead to a multiple of demand deposits at a bank. The collateral is functioning like cash (p. 510).

Thus, “repo runs,” salient events of the 2008–2009 financial crisis, could be money runs. As

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<sup>27</sup>See also Donaldson, Lee, and Piacentino (2018), Donaldson and Micheler (2018), Singh (2010), and Singh and Aitken (2010).

such, our framework casts light on the puzzle of how runs arise even though each repo is individually collateralized, and the common pool problem necessary to generate Diamond–Dybvig runs is absent (see Gorton and Metrick (2010, 2012) and Krishnamurthy, Nagel, and Orlov (2014)).

## 6 Discussion of Assumptions and Extensions

In this section, we discuss some of our key assumptions and then analyze extensions.

### 6.1 Discussion of Assumptions

**Entry costs.** As we stress in Subsection 4.5, money runs can arise no matter how small  $k$  is. Whether  $k$  reflects the literal cost of entering a trade or the opportunity cost of the time taken in trading, such a small  $k$  could be realistic for some contemporary markets, like retail markets in which consumers trade deposits for goods via debit cards. But a larger entry cost also has natural interpretations. Historically, it could represent the physical/temporal costs of coming to market or, alternatively, of acquiring the expertise/technology to check for counterfeit instruments. Today, it could represent the cost of setting up a trading desk to participate in a specific market (e.g., the repo market) or, alternatively, of establishing the legal infrastructure to handle certain instruments (e.g., the GMRA master agreement for repos). More generally, it could represent any relative cost of searching for a counterparty as in the search money literature, of trading/transacting as in the finance literature, or of posting a vacancy as in the labor literature. Any cost sunk before counterparties meet suffices for our results.

**Rollover.** To focus on trade in the secondary market, we want to abstract from rollover in the primary market (its practical importance notwithstanding). Indeed, the entry-bargaining-settlement setup in Subsection 2.3 deliberately precludes strategies in which B borrows via one-period contracts, and issues new debt to  $C_t$  to settle its existing debt with  $H_t$  at each date: since B would have to settle first with  $C_t$  and then with  $H_t$ , this would require an additional settlement stage. Moreover, such a one-period rollover strategy would typically be less desirable than demandable debt in our baseline environment anyway: someone would have to pay the cost  $k$  to enter and buy the new issue in every period, rather than to enter and trade existing debt only in periods in which  $H_t$  is hit by a liquidity shock. More practically, secondary market trade allows the borrower to avoid floatation costs, which could be prohibitive if borne in every period in the rollover strategy.

That said, below we include rollover in our environment (under some additional assump-

tions), and show that money runs can still occur (Subsection 6.4).

**Bargaining protocol.** In our model, demandability matters because the redemption value  $r$  serves as the outside option in bargaining. Thus, security design can substitute for market design: the borrower can adjust the terms of trade in the secondary market, choosing  $r$  to calibrate the division of surplus between counterparties, even though the bargaining power  $\eta$  is immutable.

Our results hold for bargaining protocols, like Nash bargaining, in which the outside option determines the division of the surplus. Not every non-cooperative bargaining game has this feature in equilibrium (Sutton (1986)). But many do. Indeed, the Nash outcome coincides with the equilibrium of a game in which bargainers either (i) risk having the bargaining process suddenly break down or (ii) have the ability to make take-it-or-leave-it offers (see, e.g., Binmore, Rubinstein, and Wolinsky (1986)). Within our model, the risk of a breakdown could reflect the probability that a counterparty abandons the negotiation because he is hit by a liquidity shock himself or because he finds another, more profitable trade to execute. And the ability to make take-it-or-leave-it offers could reflect the situation in modern “‘hi-tech’ markets [like the repo market] in which binding deals are made quickly over the telephone [or Bloomberg chat]” (Binmore, Osborne, and Rubinstein (1992)), p. 190; see Shaked (1994)).

**Infinite horizon.** Money runs arise due to dynamic coordination—a counterparty enters if he believes his future counterparty will, who enters if he believes his future counterparty will... Thus, if it is common knowledge that any counterparty is the last one, he will never enter, and the “good” equilibrium would unravel by backward induction. We avoid this by assuming that the horizon is infinite, so every counterparty has a future counterparty. Indeed, there is no date at which the banknote expires for sure; as such, tradable, demandable debt may have more in common with perpetual debt than with short-term debt.

The infinite horizon is one way to capture the idea that each counterparty believes that an instrument could continue for one more period with positive probability. It is the way used in the new monetarist literature, following Kiyotaki and Wright (1989, 1993), but it is not the only way; for example, counterparties could be uncertain about their position in a finite trading sequence (see, e.g., Moinas and Pouget (2013)).

## 6.2 Random Entry Costs

We now extend the model so  $C_t$ 's entry cost is a random variable  $\tilde{k}_t$ , so that  $C_t$  enters only if his entry cost is below a threshold  $k^*$ . This adds generality and realism, by allowing for some random, fundamental-based withdrawals. Just as importantly, it shows that the

probability that  $C_t$  enters, i.e.  $\mathbb{P}[\tilde{k}_t \leq k^*]$  is an increasing function of  $R - r$ , providing a micro-foundation for the class of functions considered in Proposition 6.

We assume that the distribution of entry costs is Pareto, with support  $[k_0, \infty)$  and exponent one,  $\tilde{k}_t \sim \text{Pareto}(k_0, 1)$ . This allows us to solve for cut-off equilibria in closed-form. To do so, we suppose that  $C_t$  believes that future creditors  $C_{t'}$  enter whenever  $k_{t'} \leq k^*$ , so his entry condition is

$$v - p = \frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta \left(1 - \eta \mathbb{P}[\tilde{k}_{t'} \leq k^*]\right)} \geq k_t. \quad (30)$$

This is the entry condition in equation (13) with future creditors' entry probability given by the probability their entry cost exceeds the cut-off  $k^*$ , rather than by their mixing probability  $\sigma$ . Using the Pareto distribution to substitute in for this probability,  $\mathbb{P}[\tilde{k}_{t'} \leq k^*] = 1 - k_0/k^*$ , gives the next proposition:

PROPOSITION 10. (CUT-OFF EQUILIBRIUM WITH RANDOM ENTRY COSTS.) *Define*

$$k^* = \frac{\rho(1 - \eta)(R - r) - (1 - \rho)\theta\eta k_0}{\rho + (1 - \rho)\theta(1 - \eta)}. \quad (31)$$

*If entry costs follow the Pareto distribution described above and  $k^* > k_0$ , then there is a unique stationary cut-off equilibrium of the  $t > 0$  subgame in which  $C_t$  enters if and only if  $k_t \leq k^*$ .*

Observe that, in this equilibrium,  $H_t$  withdraws with positive probability on the equilibrium path, namely whenever he suffers a liquidity shock and his counterparty  $C_t$  does not enter.  $C_t$  enters only if  $k_t \leq k^*$ , or

$$\mathbb{P}[\tilde{k}_t \leq k^*] = 1 - \frac{k_0}{k^*} = 1 - \frac{(\rho + (1 - \rho)\theta(1 - \eta))k_0}{\rho(1 - \eta)(R - r) - (1 - \rho)\theta\eta k_0}. \quad (32)$$

Recall that in Proposition 6, we establish a dark side of demandable debt, under the assumption that the probability that creditors enter is an increasing function  $f$  of  $(R - r)$ . This is the case here. Hence, this extension provides an equilibrium foundation for that assumption (we verify that the expression above satisfies the other conditions of the proposition in Appendix A.19); cf. the discussion in Subsection 4.3.

### 6.3 Asset Choice

What if a single borrower B chooses the type of its investment before borrowing from  $C_0$ ? Do frictions in the secondary market distort its choice? Yes, toward high-liquidation-value investments:

PROPOSITION 11. (EXCESSIVE LIQUIDITY.) *Suppose that B can choose between an investment with payoff  $y$  and liquidation value  $\ell$  and another investment that is otherwise identical but has lower payoff  $y' < y$  and higher liquidation value  $\ell'$ , where*

$$\ell' > \ell + \frac{\rho}{(1-\rho)\theta(1-\eta)}(y - y'). \quad (33)$$

*There exists an investment cost  $c$  such that in any equilibrium in which investment occurs, B chooses the low-NPV, high-liquidation-value investment  $(y', \ell')$ .*

Intuitively, with a high-liquidation-value investment, B can issue a high-redemption-value banknote and borrow more. Thus, to make its debt money-like, B chooses to increase its liquidation value even at the expense of NPV.

### 6.4 Partial Rollover

We now turn to a version of the model in which counterparties are matched with debtholders in a single market via a homogenous matching technology. (This differs from Subsection 4.5, in which they trade in parallel markets.) This set-up allows us to show that money runs can occur even if (i) there are no aggregate shocks to liquidity and (ii) B can raise money from new creditors at the beginning of each date, thereby rolling over its debt to meet redemptions.<sup>28</sup> Moreover, unlike in the baseline model, not every withdrawal is a run. Rather, some debtholders redeem at each date.

Here we do not model funding/investment, but focus on the secondary market, assuming that banknotes are held by a unit continuum of debtholders, a fraction  $\theta$  of which needs liquidity at each date. Counterparties can enter at cost  $k$ , in which case they are matched with debtholders via a homogenous matching function. Thus, the probability  $\sigma_t$  with which a debtholder meets a counterparty depends on the number of counterparties who enter. The fraction  $\theta\sigma_t$  of debtholders who meet counterparties trade in the secondary market. The

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<sup>28</sup>This distinguishes our run risk from rollover risk, where we use “run risk” to mean the risk of an unexpectedly large number of withdrawals and “rollover risk” to mean the risk that B attempts to raise new debt and fails. Below, we assume B can roll over costlessly—there is no rollover risk—but B cannot go back to the market to meet a large number of withdrawals without some delay—there is run risk.



remaining  $\theta(1 - \sigma_t)$  redeem for  $r$ . We assume that B issues new identical banknotes to raise exactly enough to meet these redemptions at the beginning of each date.<sup>29</sup>

The next result says that this set-up has multiple steady state equilibria. Indeed, there is a “good equilibrium,” in which many counterparties enter and few debtholders are left unmatched. In this equilibrium, there are relatively few withdrawals at each date, so B chooses its rollover strategy to raise a relatively small amount of liquidity. But there is also a “bad equilibrium,” in which few counterparties enter and many debtholders are left unmatched. In this equilibrium, there are more withdrawals at each date, so B has to choose a rollover strategy to raise more liquidity. Thus, a change in beliefs can lead to a money run analogous to that in Corollary 1: if counterparties today believe that few of their future counterparties will enter, then few of them enter today; this leads to an unexpectedly high number of withdrawals—a money run.

PROPOSITION 12. (MONEY RUNS WITH PARTIAL ROLLOVER.) *Let the matching technology be given by  $\sigma = m\sqrt{q}$ , where  $q$  is the number of counterparties that enter and  $m > 0$  is a parameter. Suppose that B borrows via banknotes from a continuum of creditors. The  $t > 0$  subgame has two stationary equilibria, one in which many counterparties enter,*

$$\sigma = \frac{k(\rho + (1 - \rho)\theta) + \sqrt{k^2(\rho + (1 - \rho)\theta)^2 - 4m^2k\rho(1 - \rho)(R - r)\theta\eta(1 - \eta)}}{2k(1 - \rho)\theta\eta} =: \sigma_+ \quad (34)$$

—banknotes are liquid—and another in which few counterparties enter,

$$\sigma = \frac{k(\rho + (1 - \rho)\theta) - \sqrt{k^2(\rho + (1 - \rho)\theta)^2 - 4m^2k\rho(1 - \rho)(R - r)\theta\eta(1 - \eta)}}{2k(1 - \rho)\theta\eta} =: \sigma_- \quad (35)$$

—banknotes are illiquid—as long as  $\sigma_+$  and  $\sigma_-$  above are well-defined probabilities.

This result implies that money runs can occur even with no aggregate risk, no rollover risk, and no sequential-service constraint. This affirms that money runs result only from intertemporal coordination in the secondary market and helps distinguish our model of bank fragility from models of rollover risk (e.g., Acharya, Gale, and Yorulmazer (2011) and He and Xiong (2012)).

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<sup>29</sup>Assuming that B decides how much to raise at the beginning of the date makes runs possible and assuming that all banknotes are identical (with face value  $R$  and redemption value  $r$ ) keeps the model stationary.

## 7 Related Literature

We make four main contributions to the literature.

First, we offer a new rationale for demandable debt. This adds to the literature in two ways. (i) It complements the literature that shows how demandability can help to mitigate moral hazard problems (Calomiris and Kahn (1991) and Diamond and Rajan (2001a, 2001b)).<sup>30</sup> In particular, we show how demandability can help to increase the value of bank debt as “private money.” Thus, our model connects two of the main features of bank liabilities: they circulate as money and are redeemable on demand. (ii) It provides a counterpoint to the literature that suggests that tradeability can substitute for demandability. Notably, Jacklin (1987) shows that, in Diamond and Dybvig’s (1983) environment, you do not need to redeem debt on demand if you can just trade it in the secondary market.<sup>31,32</sup> We show that if bank debt is traded in a decentralized market, like, e.g., banknotes, deposits, and repos are, then demandability complements tradeability by increasing the price at which it trades.

Second, we uncover a new kind of bank run. By connecting the fragility of money to the fragility of banks, this adds both to the literature on coordination-based bank-run models following Diamond and Dybvig (1983) and to the literature on search-based money models following Kiyotaki and Wright (1989, 1993). In these money models, monetary exchange is fragile since trade is self-fulfilling. Similarly, in the bank run models, bank deposits are fragile since withdrawals are self-fulfilling. To the best of our knowledge, we are the first to show that such bank fragility follows immediately from such monetary fragility<sup>33</sup> and, hence,

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<sup>30</sup>In their conclusion, Diamond and Rajan (2001a) make the link between demandability and circulating banknotes informally, saying that

deposits are readily transferable, and liquid, because buyers of deposits have no less ability to extract payment than do sellers of deposits. Thus, the deposits can serve as bank notes or checks that circulate between depositors. This could explain the special role of banks in creating inside money (p. 425).

We make this link formally in this paper.

<sup>31</sup>However, Jacklin (1987) does point out that tradeable debt can have one disadvantage relative to demandable debt: investments at the initial date can be distorted in anticipation of trading later on (see also Allen and Gale (2004), Farhi, Golosov, and Tsyvinski (2009), and Kučinskas (2017)).

<sup>32</sup>Other papers show that there may still be a role for demandability if tradeability is limited (Allen and Gale (2004), Antinolfi and Prasad (2008), Diamond (1997), and von Thadden (1999)). In these models, banks issue demandable debt *in spite of* trade in secondary markets, e.g., to overcome trading frictions, such as limited market participation. In our model, banks issue demandable debt *because of* trade in secondary markets—the option to redeem on demand improves the terms of trade in the secondary market.

<sup>33</sup>A number of papers study bank money creation independently of financial fragility (e.g., Donaldson, Piacentino, and Thakor (2018), Gu, Mattesini, Monnet, and Wright (2013), Kiyotaki and Moore (2001, 2002, 2005)) and some others embed Diamond–Dybvig runs in economies with private money (e.g., Champ, Smith, and Williamson (1996) and Sanches (2015); see also Sultanum (2018)). Relatedly, Sanches

coordination-based bank runs can occur even with a single depositor—i.e. without multiple depositors racing to withdraw from a common pool of assets.<sup>34,35</sup> Thus, we explain historical runs on assets backed by individual collateral, such as banknotes and repos.<sup>36</sup> In so doing, we respond to what Gorton (2012b) argues remains a theoretical challenge, saying that

Generating [a run] event in a model seems harder when...the form of money [is such that] each “depositor” receives a bond as collateral. There is no common pool of assets on which bank debtholders have a claim. So, strategic considerations about coordinating with other agents do not arise. This is a challenge for theory and raises issues concerning notions of liquidity and collateral, and generally of the design of trading securities—private money (p. 2).

Third, we show that the need to create circulating demandable debt gives rise to numerous other banking activities. This adds to the literature on the foundations of banking, connecting pooling assets and dispersed liabilities (e.g., Boyd and Prescott (1986), Diamond (1984), Diamond and Dybvig (1983), and Ramakrishnan and Thakor (1984)) with money creation (e.g., Donaldson, Piacentino, and Thakor (2018) and Gu, Mattesini, Monnet, and Wright (2013)). Notably, in contrast to papers that emphasize that pooling helps banks meet redemptions in equilibrium via diversification, we show that it improves creditors’ option to redeem off equilibrium even absent diversification. The benefit of being first in line off equilibrium also appears in Diamond and Rajan (2001b), where it helps curb moral hazard.

Fourth, by studying security design when securities are traded in a decentralized secondary market, our paper adds to the literature in three ways. (i) It complements the search-based money literature which analyzes which type of asset is the socially optimal medium of exchange for trade in the secondary market (e.g., Kiyotaki and Wright (1989) and Burdett, Trejos, and Wright (2001)). We analyze which type of contract is the privately optimal circulating instrument for funding in the primary market. And we show that this link with security design matters for the fragility of money as a means of payment. In our model, securities with positive fundamental value remain fragile—in the sense that there are multiple steady-state equilibria—as trading/entry costs become vanishingly small. In the new monetarist literature, in which securities are not designed to raise funds, they do

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(2016) argues that banks’ inability to commit to redeem deposits can make private money unstable.

<sup>34</sup>Our focus on runs that result from dynamic coordination failures among counterparties in the secondary market complements models that focus on runs that result from dynamic coordination failures among depositors in the primary market (the dynamic analog of Diamond–Dybvig-type runs), such as He and Xiong (2012); see also Qi (1994).

<sup>35</sup>Bond and Rai (2009) uncover another kind of run that can occur with a single depositor, or even with no depositors whatsoever: a “borrower run.”

<sup>36</sup>This complements the models in Donaldson, Lee, and Piacentino (2018), Kuong (2015) and Martin, Skeie, and von Thadden (2014a, 2014b).

not (see Lagos, Rocheteau, and Wright (2017)).<sup>37</sup> (ii) It extends results in the literature on corporate bonds that suggest short-maturity bonds can have the benefit of high resale prices in the secondary market, but the cost of frequent debt issuances (Bruche and Segura (2016) and He and Milbradt (2014)). These papers restrict attention to debt contracts as in Leland and Toft (1996). We point out that with more general contracts, the benefit can come without the cost: demandable debt props up the secondary market price by giving sellers the option to redeem on demand, an option that need never be exercised.<sup>38</sup> (iii) It provides a counterpoint to the literature that suggests that security design may prevent bank runs (e.g., Andolfatto, Nosal, and Sultanum (2018), Green and Lin (2003), and Peck and Shell (2003)). This literature suggests that if the space of securities is rich enough, then bank runs do not arise in Diamond and Dybvig’s (1983) environment. Our analysis suggests that the security designs proposed in this literature may not prevent all kinds of bank runs. This is because, in our environment, it is exactly the possibility of a run, i.e. the option to redeem on demand, that makes the banknote the optimal funding instrument.

More broadly, this paper complements the related line of research that focuses on information, rather than trading frictions, in secondary-market trade, which started with Gorton and Pennacchi (1990).<sup>39</sup> This literature generally focuses on fundamental risk, and suggests that information frictions in the secondary market lead banks to do risk transformation, and this improves social efficiency. We focus on coordination risk, and suggest that trading frictions in the secondary market lead banks to do liquidity transformation but that this can decrease social efficiency.

## 8 Conclusion

What is a bank? A bank is something that creates money, i.e. debt that facilitates trade in decentralized markets. By thinking about a bank this way, we found a new rationale for demandable debt, a new type of bank run—a “money run”—and a new explanation for the other quintessential things banks do, such as pooling assets and maturity/liquidity transformation. The perspective matters for policy. Among other things, it suggests a cost of narrow banking and a benefit of suspension of convertibility, both new to the literature.

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<sup>37</sup>See Rocheteau and Wright (2013) for a model in which multiple (non-steady state) equilibria arise in a decentralized market without a fixed cost.

<sup>38</sup>In an extension, Bruche and Segura (2016) do consider a version of puttable debt. However, they effectively assume it is not tradeable, which shuts down the interaction of demandability and tradeability that is critical to our results.

<sup>39</sup>See, Chemla and Hennessy (2014), Dang, Gorton, and Holmström (2015a, 2015b), Dang, Gorton, Holmström, and Ordoñez (2017), Gorton and Ordoñez (2014), Jacklin (1989), and Vanasco (2017).

## A Proofs

### A.1 Proof of Proposition 1

For an instrument  $i$ , let  $\max v_i$  be an instrument's debt capacity, i.e. its maximum value over any  $R$ ,  $r$ , and  $\sigma$ :

$$\max v_i := \sup \{ v_i \mid r \leq \ell, R \leq y, \sigma \in [0, 1] \}. \quad (36)$$

So,  $C_0$  lends against instrument  $i$  only if  $\max v_i \geq c$ . Hence, B can fund itself with the puttable loan but not with the loan if and only if

$$\max v_{\text{loan}} < c \leq \max v_{\text{putt. loan}}. \quad (37)$$

Substituting  $r = \ell$  and  $R = y$  into the expressions for their values in equations (2) and (4) gives the condition in the proposition.  $\square$

### A.2 Proof of Lemma 1

When  $C_t$  and  $H_t$  are matched,  $H_t$  has been hit by a liquidity shock. Thus,  $C_t$ 's value of the bond is  $v_t$  and  $H_t$ 's value of the bond is zero (since  $H_t$  consumes only at Date  $t$  and the bond is not demandable). The total surplus is thus  $v_t$ , which  $C_t$  and  $H_t$  split in proportions  $1 - \eta$  and  $\eta$ , in accordance with the Nash bargaining solution. Thus the price is  $p_t = \eta v_t$ .  $\square$

### A.3 Proof of Proposition 2

The proof is analogous to that of Proposition 1. B can borrow via a bond but not with a puttable loan if and only if

$$\max v_{\text{putt. loan}} < c \leq \max v_{\text{bond}}, \quad (38)$$

where  $\max v$  is as defined in equation (36). Substituting  $r = \ell$ ,  $R = y$ ,  $\sigma = 1$  into the expressions for their values in equations (4) and (7) gives the condition in the proposition.  $\square$

### A.4 Proof of Lemma 2

When  $C_t$  and  $H_t$  are matched  $H_t$  has been hit by a liquidity shock. Thus,  $C_t$ 's value of the banknote is  $v_t$  and  $H_t$ 's value of the banknote is  $r$  (since  $H_t$  consumes only at Date  $t$ , it redeems on demand if it does not trade). The gains from trade are thus  $v_t - r$ , which  $C_t$  and

$H_t$  split in proportions  $1 - \eta$  and  $\eta$ , in accordance with the Nash bargaining solution, i.e.  $p_t$  is such that

$$H_t \text{ gets } \eta(v_t - r) + r = p_t, \quad (39)$$

$$C_t \text{ gets } (1 - \eta)(v_t - r) = v_t - p_t, \quad (40)$$

or  $p_t = \eta v_t + (1 - \eta)r$ . □

### A.5 Proof of Proposition 3

The proof is analogous to those of Proposition 1 and Proposition 2. B can borrow via a banknote but not with a puttable loan or a bond if and only if

$$\max \left\{ \max v_{\text{putt. loan}}, \max v_{\text{bond}} \right\} < c \leq \max v_{\text{b.note}}, \quad (41)$$

where  $\max v$  is as defined in equation (36). Substituting  $r = \ell$ ,  $R = y$ ,  $\sigma = 1$  into the expressions for their values in equations (4), (7), and (10) gives the condition in the proposition. □

### A.6 Proof of Proposition 4

For the pure equilibria, the argument is in the text (see equations (14) and (15)).

For the mixed equilibrium,  $C_t$  must be indifferent between entering and staying out,  $k = v - p$ , or

$$k = \frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta(1 - \eta\sigma)}. \quad (42)$$

Solving for  $\sigma$  gives

$$\sigma = \frac{1}{\eta} \left( 1 - \frac{\rho}{(1 - \rho)\theta k} \left( (1 - \eta)(R - r) - k \right) \right). \quad (43)$$

□

### A.7 Proof of Corollary 1

The result follows immediately from Proposition 4. □

### A.8 Proof of Proposition 5

We prove points (i)–(iii) in turn.

(i) Consider  $C_t$ 's best response given other counterparties play  $\sigma$  in equation (13):

$$k \leq v - p = \frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta(1 - \eta\sigma)}. \quad (44)$$

The RHS above is decreasing in  $r$  (for fixed  $\sigma$ ).

(ii) The result follows immediately from equation (14).

(iii) The result follows immediately from equation (15).  $\square$

## A.9 Proof of Proposition 6

We prove the result by setting up B's maximization problem over  $R$  and  $r$  given  $\sigma = f(R - r)$  and showing that B optimally sets  $r = \ell$ . We proceed in the following steps.

(i) We write down B's utility as a function of  $R$  and  $r$ .

(ii) We set up the constrained maximization problem to find  $R$  and  $r$ .

(iii) We show that the constraint in the maximization problem binds.

(iv) We show that the objective in the maximization problem is increasing in  $r$  given the constraint binds.

(v) We conclude that  $r = \ell$ , its maximum possible value.

**B's utility.** Let  $u$  denote B's expected utility, which can be written recursively as

$$u_t = \rho(y - R) + (1 - \rho)\left(\theta(\sigma_t u_{t+1} + (1 - \sigma_t)(\ell - r)) + (1 - \theta)u_{t+1}\right) \quad (45)$$

The terms are determined as follows. With probability  $\rho$ , B's investment pays off and B repays  $R$ , keeping  $y - R$ . With probability  $(1 - \rho)\theta$ , B's investment does not payoff and the debtholder  $H_t$  is hit by a liquidity shock. With conditional probability  $\sigma_t$ ,  $H_t$  finds a counterparty and B continues its investment, getting  $u_{t+1}$ , since there is no discounting. Otherwise, with conditional probability  $1 - \sigma_t$ ,  $H_t$  does not find a counterparty and redeems on demand. B must liquidate its investment and repay  $r$ , so it gets  $\ell - r$ . With probability  $(1 - \rho)(1 - \theta)$ , B's investment does not pay off and  $H_t$  is not hit by a liquidity shock. Again, B continues and gets  $u_{t+1}$ . Given ( $u_t = u_{t+1} \equiv u$ ), substituting  $\sigma_t \equiv f \equiv f(R - r)$  in accordance with the hypothesis of the proposition and solving for  $u$  gives

$$u = \frac{\rho(y - R) + (1 - \rho)\theta(1 - f)(\ell - r)}{\rho + (1 - \rho)\theta(1 - f)}. \quad (46)$$

**B's maximization problem.** B will choose  $R$  and  $r$  to maximize  $u$  subject to the constraint that  $v \geq c$  (so  $C_0$  lends). Substituting for  $u$  from equation (46) and for  $v$  from equation (10) with  $\sigma = f(R - r)$ , this reads:

$$\left\{ \begin{array}{l} \text{maximize} \\ \text{s.t.} \end{array} \right. \quad \begin{array}{l} \frac{\rho(y - R) + (1 - \rho)\theta(1 - f)(\ell - r)}{\rho + (1 - \rho)\theta(1 - f)} \\ \frac{\rho R + (1 - \rho)\theta(1 - \eta f)r}{\rho + (1 - \rho)\theta(1 - \eta f)} \geq c. \end{array} \quad (47)$$

**Constraint binds.** To show that the constraint binds, we show that decreasing  $R$  (i) increases the objective and (ii) tightens the constraint:

(i) By differentiation,  $\frac{\partial u}{\partial R} < 0$  as long as

$$f' \left[ (1 - \rho)\theta(y - R - (\ell - r)) \right] < \rho + (1 - \rho)\theta(1 - f) \quad (48)$$

If the term in square brackets is negative, this is always satisfied, since  $f' > 0$ . If it is positive, then it is satisfied as long as  $f'$  is sufficiently small, which is required by hypothesis.

(ii) By differentiation,  $\frac{\partial v}{\partial R} > 0$  as long as

$$f' > -\frac{\rho + (1 - \rho)\theta(1 - \eta f)}{(1 - \rho)\theta\eta(R - r)}. \quad (49)$$

This is always satisfied given  $f' > 0$ .

**Optimal  $r$ .** To show that  $r = \ell$ , we show that  $u$  is increasing in  $r$  given the constraint binds. To see this, compute the total derivative of  $u = u(r, f(R - r), R(r))$  “along the constraint”:

$$\frac{du}{dr} = \frac{\partial u}{\partial r} + \frac{\partial u}{\partial f} \frac{df}{dr} + \frac{\partial u}{\partial R} \frac{dR}{dr} \quad (50)$$

$$= \frac{\partial u}{\partial r} + \frac{\partial u}{\partial f} f' \left( \frac{dR}{dr} - 1 \right) + \frac{\partial u}{\partial R} \frac{dR}{dr}, \quad (51)$$

where  $dR/dr$  comes from differentiating the constraint (given it binds) and the partial deriva-



tives follow from direct computation:

$$\frac{dR}{dr} = -\frac{\theta(1-\rho)\left((1-\eta f)(\rho + \theta(1-\rho)(1-\eta f)) - \eta\rho(R-r)f'\right)}{\rho(\rho + \theta(1-\rho)(1-\eta f)) + \eta\theta(1-\rho)(R-r)f'}, \quad (52)$$

$$\frac{\partial u}{\partial R} = -\frac{\rho}{\rho + (1-\rho)\theta(1-f)}, \quad (53)$$

$$\frac{\partial u}{\partial f} = \frac{(y-R-(\ell-r))\theta(1-\rho)\rho}{(\rho + (1-\rho)\theta(1-f))^2}, \quad (54)$$

$$\frac{\partial u}{\partial r} = -\frac{(1-\rho)\theta(1-f)}{\rho + (1-\rho)\theta(1-f)}. \quad (55)$$

Substituting equations (52), (53), (54), and (55) into equation (51) and manipulating, we see that the derivative  $du/dr > 0$  as long as so long as

$$\begin{aligned} & \left[ (\rho + \theta(1-\rho)(1-\eta f))^2 (y - \ell - (R-r)) + \eta(\rho + (1-\rho)\theta(1-f))^2 (R-r) \right] f' < \\ & < (1-\eta)f(\rho + (1-\rho)\theta(1-\eta f))(\rho + (1-\rho)\theta(1-f)). \end{aligned} \quad (56)$$

If the term in square brackets is negative, this is always satisfied, since  $f' > 0$ . If it is positive, then it is satisfied as long as  $f'$  is small, which is required by hypothesis.  $\square$

## A.10 Proof of Proposition 7

We first solve for the values  $v^0$  and  $v^1$  in terms of  $r$  and  $R$  given the strategies  $\sigma^0 = 0$  and  $\sigma^1 = 1$ . We then show that these strategies are indeed best responses (for some  $k$ ). Finally, we argue that  $r = \ell$  and compute the repayment  $R$ . Finally, we substitute  $r$  and  $R$  back into the values to get the expressions in the proposition. Then, the fact that B can borrow via the banknote and only via the banknote for  $\lambda$  sufficiently small follows immediately from Proposition 3 and the continuity of  $v^1$  in  $\lambda$ .

**Values.** From equation (18) with  $\sigma^0 = 0$ , we have immediately that

$$v^0 = \frac{\rho R + (1-\rho)\theta r}{\rho + (1-\rho)\theta} \quad (57)$$

(this is just the value of the puttable loan in equation (4)). From Lemma 2 (the logic of which is not affected by the presence of sunspots), we have the prices

$$p^0 = \eta v_0 + (1-\eta)r, \quad (58)$$

$$p^1 = \eta(\lambda v^0 + (1-\lambda)v^1) + (1-\eta)r. \quad (59)$$

Thus, equation (19) with  $\sigma^1 = 1$  reads

$$v^1 = \rho R + (1 - \rho) \left( \theta \left( \eta (\lambda v^0 + (1 - \lambda)v^1) + (1 - \eta)r \right) + (1 - \theta) \left( \lambda v^0 + (1 - \lambda)v^1 \right) \right), \quad (60)$$

so

$$v^1 = \frac{\rho R + (1 - \rho) \left( \lambda (1 - \theta(1 - \eta))v_0 + \theta(1 - \eta)r \right)}{\rho + (1 - \rho) \left( \lambda (1 - \theta(1 - \eta)) + \theta(1 - \eta) \right)}. \quad (61)$$

**Best responses.**  $\sigma^1 = 1$  and  $\sigma^0 = 0$  are best responses if

$$v^0 - p^0 \leq k \leq \lambda v^0 + (1 - \lambda)v^1 - p^1 \quad (62)$$

or

$$(1 - \eta)(v^0 - r) \leq k \leq (1 - \eta)(\lambda v^0 + (1 - \lambda)v^1 - r). \quad (63)$$

This is satisfied for some  $k$  as long as  $v^1 \geq v^0$ . This is the case as long as  $R \geq r$ , which must be the case since  $R > c > \ell \geq r$ .

**Repayments.**  $r = \ell$  since  $v^1$  is (uniformly) increasing in  $r$  but, for  $\lambda$  small, B's payoff does not depend on  $r$  (directly).<sup>40,41</sup>

Now, the repayment  $R$  is determined by solving

$$c = \lambda v^0 + (1 - \lambda)v^1. \quad (64)$$

Substituting in for  $v^0$  and  $v^1$  from equations (57) and (61) and solving for  $R$ , we find

$$R = c + \frac{(1 - \rho)\theta \left( \rho(\lambda + (1 - \lambda)(1 - \eta)) + (1 - \rho)(\lambda + (1 - \lambda)\theta(1 - \eta)) \right)}{\rho \left( \rho + (1 - \rho)(\lambda + (1 - \lambda)\theta) \right)} (c - \ell), \quad (65)$$

as expressed in the proposition.

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<sup>40</sup>Intuitively, if you are “close” to the good equilibrium (so the banknote almost always circulates), you get all of the benefit increasing  $r$  (via the increased price), but almost none of the cost (via the increased payout given early redemption). Formally,  $\frac{\partial v}{\partial r} > 0$  uniformly in  $\lambda$ , but  $\frac{\partial u}{\partial r} \rightarrow 0$  as  $\lambda \rightarrow 0$  (see the expressions for B's payoffs in equations (71) and (72)).

<sup>41</sup>Note that we are calculating the optimal values of  $R$  and  $r$  as if they do not affect the equilibria of the  $t > 0$  subgames. I.e. counterparties enter in state  $s_t = 1$  and not in state  $s_t = 0$ , as described in the proposition, off the equilibrium path as well as on it. However, other equilibria are possible too, supported by different off-equilibrium behavior.

We can then use the expressions for  $R$  and  $v^0$  above and substitute them into  $v_1$  to find

$$v^1 = \frac{\left(\rho + (1 - \rho)(\lambda(1 + \theta\eta) + (1 - \lambda)\theta)\right)c - (1 - \rho)\lambda\theta\eta\ell}{\rho + (1 - \rho)(\lambda + (1 - \lambda)\theta)}, \quad (66)$$

as expressed in the proposition.  $\square$

### A.11 Proof of Corollary 2

The results follow directly from differentiation given the expression for  $R$  in equation (22).  $\square$

### A.12 Proof of Lemma 3

We first solve for B's Date-0 utility if it issues a bond, which we label  $u|_{\text{bond}}$ . Then we solve for B's utility if it issues a banknote, which we label  $u|_{\text{b.note}}$ . Then we show  $u|_{\text{b.note}} \geq u|_{\text{bond}}$  whenever  $\lambda \leq \lambda^*$ .

**Bond.** Suppose B issues a bond. By assumption, the bond always circulates. Hence, B never liquidates early and eventually gets  $y$  and repays  $R$ . Since there is no discounting, B's utility is  $u = y - R$ . Since the bond is like a banknote that always circulates with redemption value zero,  $R$  is given by equation (22) with  $\lambda = 0$  and  $\ell$  replaced by zero (since  $r = 0$  instead of  $r = \ell$ ). We have

$$u = y - \frac{\rho + (1 - \rho)\theta(1 - \eta)}{\rho}c =: u|_{\text{bond}}. \quad (67)$$

**Banknote.** Suppose B issues a banknote. Denote B's utility in state  $s_t$  by  $u^{s_t}$ . First, consider  $s_t = 0$ .  $u^0$  solves

$$u^0 = \rho(y - R) + (1 - \rho)(\theta(\ell - r) + (1 - \theta)u^0), \quad (68)$$

where the terms are determined as follows. With probability  $\rho$ , B's investment pays off and B repays  $R$ , keeping  $y - R$ . With probability  $(1 - \rho)\theta$ , B's investment does not payoff and the debtholder  $H_t$  is hit by a liquidity shock. Since  $s_t = 0$ ,  $\sigma_t = 0$  and  $H_t$  redeems on demand and B must liquidate its investment and repay  $r$ , getting  $r - \ell$ . With probability  $(1 - \rho)(1 - \theta)$ , B's investment does not pay off and  $H_t$  is not hit by a liquidity shock. B gets  $u^0$ , since  $s_{t+1} = 0$  given  $\mathbb{P}[s_{t+1} = 0 | s_t = 0] = 1$ . Solving for  $u^0$  with  $r = \ell$  gives

$$u^0 = \frac{\rho(y - R)}{\rho + (1 - \rho)\theta}. \quad (69)$$

Now, consider  $s_t = 1$ .  $u^1$  solves

$$u^1 = \rho(y - R) + (1 - \rho)(\lambda u^0 + (1 - \lambda)u^1), \quad (70)$$

where the terms are determined as follows. With probability  $\rho$ , B's investment pays off and B repays  $R$ , keeping  $y - R$ . With probability  $1 - \rho$ , B's investment does not payoff. In this case, B continues its investment to the next date (it does not matter if  $H_t$  is shocked, since B's debt always circulates given  $s_t = 1$ ). Hence, with conditional probability  $\lambda$ ,  $s^{t+1} = 0$  and B gets  $u^0$  and, with conditional probability  $1 - \lambda$ ,  $s^{t+1} = 1$  and B gets  $u^1$ . Solving for  $u^1$  gives

$$u^1 = \frac{\rho(y - R) + (1 - \rho)\lambda u^0}{\rho + (1 - \rho)\lambda}. \quad (71)$$

B's Date-0 utility is thus

$$u_{\text{b.note}} = \lambda u^0 + (1 - \lambda)u^1. \quad (72)$$

Substituting for  $u^0$ ,  $u^1$ , and  $R$  from equations (69), (71), and (22) and differentiating immediately gives the following lemma, which is useful below.

LEMMA 4.  $u|_{\text{b.note}}$  is continuously decreasing in  $\lambda$ .

*Proof.* Direct computation gives

$$\frac{\partial}{\partial \lambda} \left( u|_{\text{b.note}} \right) = - \frac{(1 - \rho)\theta \left( \rho y - (\rho + (1 - \rho)\theta(1 - \eta))c + \rho\eta(c - \ell) + (1 - \rho)\theta(1 - \eta)\ell \right)}{(\rho + (1 - \rho)\theta)(\lambda + (1 - \lambda)\rho)^2}. \quad (73)$$

This is negative since each term in the numerator is positive, given  $\rho y - (\rho + (1 - \rho)\theta(1 - \eta))c \geq 0$  by the assumption that the bond is feasible (equation (8)).

□

**Comparison.** B prefers to issue a banknote than a bond whenever  $u|_{\text{b.note}} \geq u|_{\text{bond}}$ . From the expressions above, equality holds if

$$\lambda^* = \frac{\rho(\rho + (1 - \rho)\theta)(1 - \eta)\ell}{\rho(y - \ell) + (\rho + (1 - \rho)\theta)(1 - \eta)(\rho\ell - c)}. \quad (74)$$

And given  $u|_{\text{bond}}$  does not depend on  $\lambda$  and  $u|_{\text{b.note}}$  is increasing in  $\lambda$  (Lemma 4),  $u|_{\text{b.note}} \geq u|_{\text{bond}}$  exactly when  $\lambda \leq \lambda^*$ . □

### A.13 Proof of Proposition 8

Most of the argument is in the text preceding the proposition. It remains only to show that creditors' entry condition puts a tighter bound on the redemption value than the liquidation value does, i.e.  $r \leq r^{\max}$  is a tighter constraint than  $r \leq N\ell$ . And, indeed, the assumption in equation (27) says exactly that  $r^{\max} \leq N\ell$ .  $\square$

### A.14 Proof of Corollary 3

The result follows immediately from the expressions for  $r^{\max}$  in equation (24) and  $p$  in Lemma 2 (given the expression for  $v$  in equation (10)).  $\square$

### A.15 Proof of Proposition 9

Here, we start with the value functions, then the prices, and then compute  $C_t$ 's entry condition in the crisis under the assumption that the note does not circulate in the crisis. We then show that, under the condition in the proposition,  $C_t$  does enter if convertibility is suspended, i.e. if  $r^0 = 0$ , contradicting the assumption that the note does not circulate.

(Note that we assumed in the proposition that counterparties always enter in normal times; this is a best response given the definition of  $r^1 = r^{\max}$ .)

**Values.** We have that

$$v^0 = \rho R + (1 - \rho) \left( \theta \left( \sigma^0 p^0 + (1 - \sigma^0) r^0 \right) + (1 - \theta) \left( (1 - \mu) v^0 + \mu v^1 \right) \right), \quad (75)$$

$$v^1 = \rho R + (1 - \rho) \left( \theta \left( \sigma^1 p^1 + (1 - \sigma^1) r^1 \right) + (1 - \theta) v^1 \right), \quad (76)$$

which are the analogs of equations (18) and (19) in Subsection 4.4. These values depend on the prices, which are determined by Nash bargaining, to give

$$p^0 = \eta \left( (1 - \mu) v^0 + \mu v^1 \right) + (1 - \eta) r^0, \quad (77)$$

$$p^1 = \eta v^1 + (1 - \eta) r^1. \quad (78)$$

Substituting back into the value functions, and supposing that the note circulates in normal times and (in anticipation of a contradiction) that it does not circulate in a crisis,  $\sigma^1 = 1$  and  $\sigma^0 = 0$ , we have that

$$v^0 = \frac{\rho R + \theta(1 - \rho)r^0 + (1 - \theta)\mu(1 - \rho)v^1}{\rho + (1 - \rho)(\theta + (1 - \theta)\mu)} \quad (79)$$

and

$$v^1 = \frac{\rho R + (1 - \rho)\theta(1 - \eta)r^1}{\rho + (1 - \rho)\theta(1 - \eta)}, \quad (80)$$

which, with  $r = r^{\max}$  from equation (24), simplifies to

$$v^1 = R - \frac{\theta(1 - \rho)}{\rho}k. \quad (81)$$

**C<sub>t</sub>'s entry condition.** C<sub>t</sub>'s entry condition in a crisis ( $s_t = 0$ ) is

$$(1 - \mu)v^0 + \mu v^1 - p^0 \Big|_{\sigma=0} \geq k, \quad (82)$$

which is the analog of equation (13) in the model without sunspots. Using  $v^0$  and  $p^0$  above, this reads

$$\frac{(1 - \eta)(\mu v^1 + (1 - \mu)\rho R - (\rho + (1 - \rho)\mu)r^0)}{\rho + (1 - \rho)(\theta + (1 - \theta)\mu)} \geq k. \quad (83)$$

Note that the RHS is decreasing in  $r^0$ : a high redemption value in a crisis makes C<sub>t</sub> more reluctant to enter, as stressed in Proposition 5 and Proposition 6. Moreover, the RHS is increasing in  $v^1$ , which in turn is increasing in  $r^1$ : a high redemption value in normal times makes C<sub>t</sub> more willing to enter, because he anticipates a higher value in the future.

From here, we substitute for  $v^1$  and  $r^1 = r^{\max}$  to find that C<sub>t</sub> enters if and only if

$$r^0 \leq R - \frac{(1 - \eta)\mu(1 - \rho)\theta + \rho(\rho + (1 - \rho)(\theta + (1 - \theta)\mu))}{(1 - \eta)\rho(\rho + (1 - \rho)\mu)}k. \quad (84)$$

Suspension of convertibility restores circulation if this equation is satisfied for  $r^0 = 0$ , or the RHS is positive, which is the condition in the proposition.  $\square$

## A.16 Proof of Proposition 10

Given  $\tilde{k}_t \sim \text{Pareto}(k_0, 1)$ , we can replace  $\sigma$  in C<sub>t</sub>'s entry condition (equation (13)) by  $\mathbb{P}[\sigma_t = 1] = 1 - k_0/k^*$ . C<sub>t</sub> must be indifferent at the cut-off  $k^*$ :

$$k^* = \frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta[1 - \eta(1 - \frac{k_0}{k^*})]}. \quad (85)$$

Solving for  $k^*$  gives

$$k^* = \frac{\rho(1 - \eta)(R - r) - (1 - \rho)\theta\eta k_0}{\rho + (1 - \rho)\theta(1 - \eta)}. \quad (86)$$

$\square$

## A.17 Proof of Proposition 11

By Proposition 3, B can invest in  $(y', \ell')$  but not in  $(y, \ell)$  if and only if

$$\max v_{\text{b.note}}|_{(y, \ell)} < c \leq \max v_{\text{b.note}}|_{(y', \ell')}, \quad (87)$$

where  $\max v$  is as defined in equation (36). Substituting for  $R$ ,  $r$  and  $\sigma$  in the value of the banknote (equation (10)), this says that

$$\frac{\rho y + (1 - \rho)\theta(1 - \eta)\ell}{\rho + (1 - \rho)\theta(1 - \eta)} < c \leq \frac{\rho y' + (1 - \rho)\theta(1 - \eta)\ell'}{\rho + (1 - \rho)\theta(1 - \eta)}. \quad (88)$$

There exists  $c$  satisfying the above inequalities whenever the left-most term is less than the right-most term. This reduces to the condition in the proposition (equation (33)).  $\square$

## A.18 Proof of Proposition 12

Observe first that the value of the banknote is given by the same expression as in the baseline model (equation (10)). But now an interior value of  $\sigma$  is determined by counterparties' entry condition. Recall that the matching function is homogenous, so each counterparty is matched with a debtholder with probability  $\sigma/q$ . Counterparties' entry condition is thus

$$\frac{\sigma}{q}(v - p) \geq k, \quad (89)$$

where  $q$  represents the steady-state mass of counterparties entering at each date. Since each counterparty is small, the inequality above will bind. Substituting in for  $v$  and  $p = \eta v + (1 - \eta)r$ , we have

$$\frac{\sigma}{q} \left( \frac{\rho(1 - \eta)(R - r)}{\rho + (1 - \rho)\theta(1 - \eta\sigma)} \right) = k. \quad (90)$$

With  $\sigma = m\sqrt{q}$ , this can be re-written as

$$mk(1 - \rho)\theta\eta q - k(\rho + (1 - \rho)\theta)\sqrt{q} + m\rho(1 - \eta)(R - r) = 0. \quad (91)$$

This is a quadratic equation in  $\sqrt{q}$ . It has the two solutions, i.e. there are two steady states,

$$\sqrt{q}_{\pm} = \frac{k(\rho + (1 - \rho)\theta) \pm \sqrt{k^2(\rho + (1 - \rho)\theta)^2 - 4m^2k\rho(1 - \rho)(R - r)\theta\eta(1 - \eta)}}{2mk(1 - \rho)\theta\eta}. \quad (92)$$

Substituting  $\sigma_{\pm} = m\sqrt{q}_{\pm}$  gives the expressions in the proposition.  $\square$

## A.19 Verifying the Conditions of Proposition 6 in the Set-up of Subsection 6.2

Following Proposition 6, call the probability that creditors enter  $f$ . In the set-up of Subsection 6.2, this probability is given by equation (32):

$$f(R - r) := 1 - \frac{(\rho + (1 - \rho)\theta(1 - \eta))k_0}{\rho(1 - \eta)(R - r) - (1 - \rho)\theta\eta k_0}. \quad (93)$$

This is an increasing function of  $R - r$ , as long as the denominator of the second term is positive, which is the case by the hypothesis that  $k^* > k_0$  (Proposition 10). Indeed, we can write  $f'$  as

$$f' = \frac{\rho(1 - \eta)(1 - f)}{\rho(1 - \eta)(R - r) - (1 - \rho)\theta\eta k_0} > 0. \quad (94)$$

Now we verify that the condition of the lemma that  $f'$  not be too large is satisfied. The specific condition we need is given in equation (48) in the proof. One sufficient condition is immediate:

$$(1 - \rho)\theta(y - R - (\ell - r)) < 0. \quad (95)$$

If the above condition is not satisfied, it is enough that

$$f' < \frac{\rho + (1 - \rho)\theta(1 - f)}{(1 - \rho)\theta(y - R - \ell + r)}. \quad (96)$$

To re-write this, we substitute for  $f'$  from equation (94). We find that the following condition must be satisfied:

$$\frac{\rho(1 - \eta)(1 - f)}{\rho(1 - \eta)(R - r) - (1 - \rho)\theta\eta k_0} < \frac{\rho + (1 - \rho)\theta(1 - f)}{(1 - \rho)\theta(y - R - \ell + r)}. \quad (97)$$

A sufficient condition for the above is that the LHS is less than  $\frac{1-f}{y-R-\ell+r}$ . This form allows us to cancel  $1 - f$  and derive that the conditions of Proposition 6 hold in the random-entry costs set-up if

$$y - \ell + \frac{(1 - \rho)\theta\eta k_0}{\rho(1 - \eta)} > 2(R - r), \quad (98)$$

(and the condition in equation (95) is violated).  $\square$



## B Table of Notations

Players and Indices	
$t$	time index
B	borrower or “bank”
$C_t$	(potential) creditor/counterparty at Date $t$
$H_t$	debtholder at Date $t$
Technologies and Preferences	
$y$	payoff of B’s investment
$c$	cost of B’s investment
$\ell$	liquidation value of B’s investment
$\rho$	probability B’s investment pays off each date
$\theta$	probability $C_t$ is hit by liquidity shock at each date
$u$	B’s utility (used only in the Appendix)
$k$	$C_t$ ’s entry cost
Prices, Values, and Strategies	
$R$	terminal repayment (face value of debt)
$r$	redemption value
$v_t$	value of B’s debt to a creditor at Date $t$
$p_t$	secondary-market price of B’s debt at Date $t$
$\sigma_t$	mixed strategy of counterparty $C_t$
Other Variables	
$s_t$	sunspot at Date $t$ (Subsection 4.4 and Subsection 5.1)
$\lambda$	$\mathbb{P}[s_{t+1} = 0   s_t = 1]$ , “confidence crisis” probability (Subsection 4.4)
$\mu$	$\mathbb{P}[s_{t+1} = 1   s_t = 0]$ , “confidence crisis” probability
$\max v$	debt capacity/maximum value of an instrument (equation (36))
$r^{\max}$	maximum redemption value s.t. $C_t$ enters (equation (24))
$m$	matching parameter in Subsection 6.4

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