Graduate Quantum Mechanics II (SPRING 2016): Prelims. PLEASE WRITE YOUR N-NUMBER AND NOT YOUR NAME. TOTAL TIME=3 HRS

- Problem 1 (15 points)
  Consider a two-dimensional isotropic harmonic oscillator $V = k(x^2 + y^2)/2$ of mass m, subjected to a weak perturbation $V_P = \alpha xy$.
  a). Using perturbation theory in V_P, determine the first non-zero correction to the ground-state energy of the oscillator.
  b). Compare your result to the exact solution and comment on the validity of perturbation theory.
  c). Using perturbation theory alone, determine the first non-vanishing correction to the energy of the first excited states. Give the corresponding leading-order wave-functions as well.

- Problem 2 (10 points)
  Consider a one-dimensional harmonic oscillator $kx^2/2$ of charge q and mass m. It is subjected to an electric field with the temporal profile $E(t) = E_0 e^{-t^2/\tau^2}$. If the particle was initially ($t = -\infty$) in the n-th eigenstate of the harmonic oscillator, using leading order perturbation theory in the electric-field, determine the probability of finding the particle in all other levels at $t = \infty$.

- Problem 3 (10 points)
  Consider a particle of mass m trapped in a one-dimensional potential $V(x) = \lambda x \ \forall x > 0$ and $V(x) = \infty \ \forall x < 0$. Thus there is a hard-wall at $x = 0$. Using the semi-classical approximation, determine the energy-eigenvalues.

- Problem 4 (10 points)
  Consider the spherically symmetric potential in the form of a truncated harmonic oscillator potential: $V(r) = \frac{1}{2}m\omega^2 r^2 \theta(R - r)$.
  a). Find a transcendental equation relating the s-wave scattering phase-shift to kR, where $E = \hbar^2 k^2/2m$ is the energy of the incident particle of mass m.
  b). When $m\omega R^2/\hbar \gg 1$, determine the energies E at which scattering resonances will occur. At approximately what energy scale will the scattering resonances cease to be sharply defined?

- Problem 5 (15 points)
  Two identical spin-1/2 fermions of mass m are trapped in a hard-wall one dimensional box of length L.
  a). Assuming the particles are not-interacting, find the lowest 3 energy eigenstates and their eigenvalues. Give their total spin quantum numbers as well.
  b). Consider the perturbation $V(x_1, x_2) = V_0 \delta(x_1 - x_2)$. Use first order perturbation theory to find the energy shifts of the three lowest states.