E & M Prelim
2016
(closed book)

1.

Current enters an infinite plane conducting sheet at a point P and leaves at infinity. A circular hole, not including P, is cut someplace in the sheet. Show that the potential difference between two points on the edge of the hole is double what it would have been in the absence of the hole.

2,

A transmission line consists of two parallel perfect conductors of arbitrary but constant cross-section. The surrounding medium has constants ϵ and μ. Show that the product of the inductance per unit length and capacitance per unit length satisfies

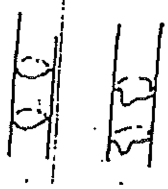
$$L\,C = (\epsilon\mu)/c^2.$$

3.

Maxwell's equations in free space, but with sources, take on a simple form in terms of the vector $Q = E + i\,H$. If the current dnsity J is generalized to $j + i\,j_m$, interpret the j_m that now appears.

4.

Show that a parallel pair of perfectly conducting solid cylinders of any given cross-sectional shape can transmit an E,M-transverse (TEM) wave at any frequency ω.



5.

Two equal and opposite charges +q and -q are connected to the ends of a rigid uncharged rod of length d. The rod rotates in the x-y plane with constant angular velocity ω about the z axis, which passes through its center. Recall that the non-relativistic acceleration field of a single charge q is given far away by

$$E = (q/c^2)\,\hat{n} \times (\hat{n} \times \dot{v})/R .$$

Compute (non-relativistically) the instantaneous radiated power $dP(\Omega,t)/d$ in the radiation zone. Integrate over solid angle Ω to show that the total radiated power P(t) is in fact independent of time.

6.

An electron e^- with kinetic energy 1.0 MeV makes a head-on collision with a positron e^+ at rest. In the collision, the two particles annihilate each other and are replaced by two photons of equal energy, each traveling at angle Θ with the electron's direction of motion.

Determine the energy E, momentum p, and angle of emission Θ of each photon.