New York University
Physics Department
2014 Preliminary Examination in Statistical Physics

This is closed book, closed notes, no cell phones, and no internet exam.
No use of any computational equipment is allowed.

Solve any 3 of these 4 problems.

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1. Air conditioner usually works by consuming electric energy. Is it possible to design an air conditioner receiving energy from lukewarm water, which is colder than outside air? To address this question, suppose the air temperature outside is T_0, while you want to maintain a temperature $T_r < T_0$ in the room, and you have a source of water at some temperature T_w intermediate between T_0 and T_r: $T_0 > T_w > T_r$. How would you characterize the efficiency of such a device? What does fundamental physics have to say about this efficiency? Given the three temperatures T_0, T_r, and T_w and nothing else, is it possible to find how much water per minute will be needed?

2. Consider a system of non-interacting spins in a magnetic field B pointing in the z-direction. The work done by the field is given by $B\Delta M_z$, with a magnetization $M_z = \mu \sum_{i=1}^{N} m_i$. For each spin, m_i takes only two values, $-1/2$ and $+1/2$.

   (a) Calculate the Gibbs partition function $Z(T, B, N)$ (note that the ensemble corresponding to the macrostate (T, B, N) includes the work by magnetic field).

   (b) Calculate the Gibbs free energy $G(T, B, N)$ and find its asymptotics at small values of magnetic field B. (Note: what you have to find is not the limit when $B \to 0$, but asymptotics when B is small – and you have to identify the characteristic scale for which B is small).

   (c) Calculate the zero field susceptibility $\chi = \partial M_z/\partial B|_{B=0}$ and show that it satisfies the Curie law (i.e., it is inversely proportional to temperature).

3. Consider some extensive variable X, and look at how its equilibrium average value changes depending on the applied force (or field) f conjugate to X. To achieve this, consider the quantity

$$\chi = \frac{\partial \langle X \rangle}{\partial f} \ . \tag{1}$$

   (a) Show how this quantity χ is related to the amount of fluctuations of X away from its average value when the system is in equilibrium at a given f.

   (b) The relation you just obtained is called fluctuation-response theorem. Can you explain this name? Identify fluctuation and response.

   Hint: Consider Gibbs canonical ensemble, in which free energy is a function of applied force f.

4. Figure 1 shows experimental data for heat capacity of potassium in specially chosen coordinates, as C_V/T versus T^2. Explain the result of this experiment. Explain which physical properties of the system can be found from the fit coefficients 2.08 and 2.57 (you do not have to give explicit formulae).

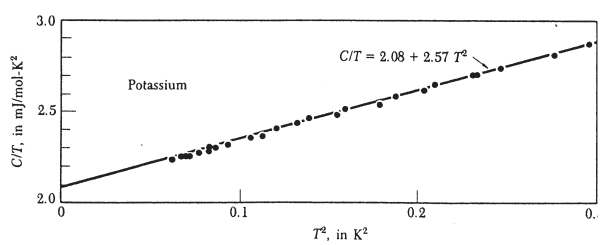

FIG. 1: Experimental values of heat capacity for solid potassium. The figure copied from the book [1].

Hint: Remember that potassium is a metal, there are free conductance electrons inside. Accordingly, think of both electron and phonon contributions to the free energy and to all other thermodynamic quantities. You do not have to provide explicit formulae, only their origin and temperature dependence.

[1] C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, Inc, 1996).