**New York University**
Physics Department
**Preliminary Examination "Statistical Physics", Winter 2015**

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*This is closed book, closed notes, and no internet exam.*
*No use of any computational equipment or cell phones is allowed.*

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1. Find zero field susceptibility and mean squared fluctuations of order parameter $\eta$ for the system in 3D described by Landau theory with single scalar order parameter. Consider both above and below the transition cases. Landau theory is based on the Hamiltonian

$$F = \int \left[ a(T - T_c)\eta^2 + b\eta^4 - h\eta + c\left(\nabla\eta\right)^2 \right] d^3r \ , \tag{1}$$

   where $\eta$ is an order parameter, $T$ is temperature, $T_c$ is critical temperature, $h$ is an externally imposed field, and $a$, $b$, and $c$ are constant coefficients (properties of the system in question).

2. Consider a system of $N$ spins $\{s_i\}$ in a thermostat at temperature $T$. Suppose spins interact in some complex way, but such that their energy $E_0(s)$ satisfies $E_0(-s) = E_0(s)$, where $-s$ means that all spins are flipped. Suppose now the system is placed in a magnetic field $h$, which means the system energy is now $E(s) = E_0(s) - hm(s)$, where magnetization is $m(s) = \sum_i s_i$. Consider probability distribution of magnetization $P(m)$ and find $P(m)/P(-m)$.

3. Consider an equilibrium "black body radiation" field at sufficiently high temperature $T$, such that electron-positron pairs can be born or annihilated:

$$\gamma \leftrightharpoons e^- + e^+ \ . \tag{2}$$

   Consider this "chemical reaction" and find equilibrium concentration of electrons as a function of temperature $T$. Assume that all gases in question are ideal. Give an estimate of a typical distance between electrons as it depends on temperature and world constants.

   **Hint:** You may follow these steps: Write down the general condition of chemical equilibrium. You should know what is chemical potential of photons; this should allow you to find chemical potential of electrons. Knowing the chemical potential of electrons, and their statistics, find their number. Do not forget that thermal speed of electrons at high temperature $T$ might be comparable to the speed of light, such that energy of an electron moving with momentum $p$ is equal to $\varepsilon_p = \sqrt{(mc^2)^2 + (pc)^2}$.

   **Useful mathematical formula:** $\int_0^\infty \frac{x^2 dx}{e^x + 1} = \frac{3}{2}\zeta(3)$.

4. Thermodynamic efficiency as a concept usually seems rather obvious. It is in fact quite subtle.

   (a) Consider a car moving at a speed of 62.5 mph (which is 100 km/h) along a highway – horizontally, or up a hill, or down. What is the efficiency of this car?

   (b) Consider a chemical machine, such as our muscles, which utilizes energy extracted from certain chemical reactions and performs mechanical work. Not knowing anything about internal machinery of our muscles, what can you say about efficiency?

   **Hint:** Remember the definition of thermodynamic efficiency. Remember the thermodynamic theorems you know about this efficiency. Carefully think of the conditions when this theorem and this concept applies, and how it can or cannot be applied to an automobile or to a muscle..