New York University

Physics Department

PRELIMINARY EXAMINATION FOR THE PH.D. DEGREE

QUANTUM MECHANICS II

Spring, 2015

READ INSTRUCTIONS CAREFULLY

1. ANSWER ALL THE PROBLEMS, AT YOUR CHOICE.
2. Four problems, total 100 points.
3. Use a separate answer booklet for each problem. On the front cover of each booklet write the problem number and your own identification number.
4. Show ALL your work.

Problem 1 [30 points]

Consider the one-dimensional potential

$$V(x) = \lambda x^4$$

with $\lambda > 0$.

- Estimate the energy levels E_n at large n for a single particle of mass m in this potential. Your result should be accurate to within 10% at large n.
- Now consider N identical, non-interacting particles of mass m and spin $1/2$ in this potential. Estimate the ground state energy for $N \gg 1$.

Problem 2 [30 points]

Consider a particle of mass m moving in the one-dimensional potential

$$V = \frac{1}{2}\, m\, \omega(t)^2\, x^2$$

with the following time-dependent frequency: $\omega(t) = \omega_i$ for $t < 0$, $\omega(t) = 0$ for $0 < t < T$, and $\omega(t) = \omega_f$ for $t > T$.

Given that the particle was in its ground state for $t < 0$, what is the probability that a measurement of the energy made at $t = 2T$ will yield $\hbar\omega_f/2$?

Problem 3 [20 points]

Compute the differential cross-section $\frac{d\sigma}{d\Omega}(\theta, \phi)$ in the Born approximation for a particle of mass m and momentum $p = \hbar k$ scattering off a spherically symmetric potential $V = V_0$ for $r < R$, $V = 0$ for $r > R$.

Problem 4 [20 points]

Find an upper bound on the elastic scattering cross-section $\sigma(k)$, where $\hbar\vec{k}$ is the momentum of the scattered wave, in the case where the scattering amplitude $f_{\mathbf{k}}(\theta, \phi)$ is independent of the angles θ and ϕ.