Quantum Mechanics II Preliminary Exam

3 hours; no books, notes, calculators; need 5 correct answers to questions 1-10 (to be written below) and at least one correct solution of either Problem 11 or 12 to pass

NAME:

A1: $< x >=$

A2: $E \sim$

A3: $E =$

A4: $\mu \sim$

A5: $\lambda \sim \qquad\qquad \approx$

A6: $S =$

A7: $S =$

A8: $S =$

A9: $S =$

A10: $T \sim \qquad\qquad \approx$

1. Consider a particle in one dimension in a state with the wave function $\psi(x)$,

$$\psi(x) = 0, \qquad x < 10,$$

$$\psi(x) = i, \qquad 10 < x < 11,$$

$$\psi(x) = 0, \qquad 11 < x.$$

Calculate $< x >$ (the expectation value of x).

2. Quartic oscillator is a 1D particle of mass m in a potential $V(x) = \kappa x^4$. Estimate the ground state energy.

3. Calculate the ground state energy of Li^{++} (doubly ionized; in eV, please).

4. Estimate the magnetic dipole moment of electron (in terms of mass m, charge e and fundamental constants).

5. A hydrogen atom is in the ground state, except that the electron spin is flipped. Estimate the wavelength of the photon emitted when the electron spin flips back (in terms of mass m, charge e and fundamental constants, and also numerically).

6. There are 2 distinguishable non-interacting spin-1 particles. What are the possible values of the total spin?

7. There are 2 identical non-interacting spin-1 bosons in the same orbital state. What are the possible values of the total spin?

8. There are 3 distinguishable non-interacting spin-1 particles. What are the possible values of the total spin?

9. There are 3 identical non-interacting spin-1 bosons in the same orbital state. What are the possible values of the total spin?

10. Estimate the Bose condensation temperature for an ideal gas of density $\rho = 0.1 \mathrm{g/cm}^3$ and a molecule mass $M = 10^{-23}$g (in terms of ρ, M and fundamental constants, and also numerically).

Problem 11

I

Solve the Schrödinger equation for the S-wave in the potential $V(r) = (1/2ma)\delta(r - R)$, where m is the scattered particle mass, while R and a are positive constants with $a \gg R$.

II

At low momenta k, the S-wave scattering phase in potential scattering can be expanded as

$$k \cot \delta_0 = -\frac{1}{\alpha} + \frac{r_0}{2} k^2 + O(k^3).$$

α is the *scattering length* and r_0 is the *effective range.*

Compute α and r_0 for the potential given in point I.

[δ_l is defined in terms of the asymptotic radial wave function χ_l as: $\chi_l = \sin(kr - \pi l/2 + \delta_l)$].

Problem 12

Show that the free Dirac equation $[\gamma^\mu \partial_\mu + m\cos(\theta) + im\sin(\theta)\gamma_5]\psi$ is invariant under a parity transformation defined by $P\psi(\mathbf{x},t) = C\psi(-\mathbf{x},t)$. Find C in terms of the matrices γ^μ and $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$.

Conventions

$(\gamma^0)^\dagger = -\gamma^0$, $(\gamma^i)^\dagger = \gamma^i$, $i = 1,2,3$. $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$, $\eta^{\mu\nu} = \mathrm{diag}\,(-1,1,1,1)$.