QMII, FALL 2011, PRELIM EXAM

Natural units $\hbar = c = 1$ are used throughout.
Total Point Value: 20 Points

Problem 1

Consider a bound state made of an α particle (i.e. a $Z = 2$ nucleus), a muon (negatively charged) and an electron.

I

Check that the potential of the alpha particle plus the muon in its ground state is well approximated by the Coulomb potential with $Z = 1$ at distances much larger than the Bohr radius *of the muon.*

[2 points]

Useful Formulae

The wave function of the 1S state normalized to $\int d^3x |\psi(x)|^2 = 1$ is $\psi(r) = \frac{1}{\sqrt{\pi a^3}} \exp(-r/a)$. The Bohr radius of a particle of mass m is $a = 1/me^2$. The relation between electron, muon and Nucleon mass are approximately $m_N = 2000 m_e$, $m_\mu = 200 m_e$.

II

Compute the ionization energy of the electron neglecting effects due to the finite size of the muon cloud.

[2 points]

III

Estimate the finite-size effects by approximating the charge due to the muon cloud with a constant density distributed inside a sphere of radius a_μ.

[3 points]

Problem 2

An electron scatters off a Hydrogen *atom* in the ground state. Using Born's approximation compute the inelastic differential cross section when the Hydrogen final state is $n = 2, l = 0$. You do NOT need to compute all integrals in the formula, but you must find the behavior in $q = p' - p$ of the cross section for $|q| \gg 1/a$.

[6 points]

Useful formulae

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{m^2 p'}{4\pi^2 p} |\langle f|\langle p'|V|p\rangle|i\rangle|^2,$$

θ is the angle between the initial momentum of the electron, p and its final momentum p'. The scattering electron's plane wave is normalized as $|p\rangle = \exp(ipx)$; the wave function of the $n = 2$ S-wave (the atom's final state $|f\rangle$) is $\psi_{20} = \frac{1}{2\sqrt{2\pi a^3}}(1 - r/2a)\exp(-r/2a)$, $a = 1/e^2 m$ =Bohr's radius.

[Hint: integration over the position of the scattering electron gives a Fourier transform on the bound electron's potential: $\int d^3x \exp(iqx)V(x-r) = \exp(irq)\int d^3x V(x)\exp(iqx) = \exp(irq)\tilde{V}(q)$.]

Problem 3

A particle scatters off a cubic lattice of lattice spacing d. The interaction potential is

$$V(\mathbf{x}) = \sum_{\mathbf{n}\in Z^3} v(\mathbf{x} + d\mathbf{n}),$$

with $v(\mathbf{x}) = C\delta^3(\mathbf{x})$.

Treat the scattering in Born approximation. Show that the condition for non-vanishing scattering is that the Bragg law is satisfied.

[Hint: use the formula $\sum_{n\in Z}\exp(idnx) = \sum_{m\in Z}(2\pi/d)\delta(x - 2\pi m/d)$.]

[3 points]

Problem 4

Show that the free Dirac equation $[\gamma^\mu\partial_\mu + m\cos(\theta) + im\sin(\theta)\gamma_5]\psi$ is invariant under a parity transformation defined by $P\psi(\mathbf{x},t) = C\psi(-\mathbf{x},t)$. Find C in terms of the matrices γ^μ and $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$.

[4 points]

Conventions

$(\gamma^0)^\dagger = -\gamma^0$, $(\gamma^i)^\dagger = \gamma^i$, $i = 1,2,3$. $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$, $\eta^{\mu\nu} = \mathrm{diag}\,(-1,1,1,1)$.

[Hint: Find first a matrix that anticommutes with the γ^i and commutes with γ^0. Next show that $\cos(\theta) + i\gamma^5\sin(\theta) = \exp(i\gamma^5\theta)$.]