Graduate Quantum Mechanics II (SPRING 2017): Prelims. PLEASE WRITE YOUR N-NUMBER AND NOT YOUR NAME. TOTAL TIME=3 HRS, TOTAL POINTS=50

- Problem 1 (15 points)
  Consider a quantum system with three states and a Hamiltonian given by

$$H = V_0 \begin{pmatrix} 1 & 2\epsilon & 0 \\ 2\epsilon & 1 & 3\epsilon \\ 0 & 3\epsilon & 4 \end{pmatrix} \tag{1}$$

  with V_0 a constant and $\epsilon \ll 1$. Consider ϵ to be a small perturbation.

  a). Write down the eigenvalues and eigenvectors of the unperturbed Hamiltonian $\epsilon = 0$.
  b). Find the leading correction to the energy of the state that is non-degenerate in the zero-th order Hamiltonian.
  c). Use degenerate perturbation theory to find the first order corrections to the two initially degenerate energies.

- Problem 2 (10 points)
  A particle in a square well potential (with walls at $x = 0$ and $x = L$, i.e, $V(x) = 0$ for $0 < x < L$; $V(x) = \infty$ otherwise) starts out in the ground state $|\psi(t = 0)\rangle = |1\rangle$, where $|n\rangle$ is the normalized eigenstates of the unperturbed Hamiltonian. Starting at $t = 0$, a time-dependent perturbation $V(x,t) = e^{-\gamma t} V_0 \sin \frac{\pi x}{L}$ is applied.

  a). Calculate the probability for the particle to make a transition to an excited state $|n\rangle$ after a long time. Define "long time".
  b). What are the selection rules for this transition?

- Problem 3 (10 points)
  Consider a triangular barrier of the form $U = -F_0|x|$, where $F_0 > 0$.
  a). Write the WKB wave-functions for a particle of mass m and energy $E > 0$ and energy $E < 0$.
  b). For particle with energy $E < 0$, determine the transmission amplitude within the WKB approximation.

- Problem 4 (15 points)
  a). Using the Born approximation, find the scattering amplitude and the total scattering crossection of a particle of energy $E = \hbar^2 k^2/(2m)$ on a delta-function potential $U(r) = \alpha\delta(r - R)$.
  b). Simplify your expression for two opposite limits, one for slow particles, and the other for fast particles. HINT: Scattering amplitude $f(\vec{q}) = -\frac{m}{2\pi\hbar^2}\tilde{U}(\vec{q})$ where $\tilde{U}(\vec{q})$ is the Fourier transform of the potential. The scattering crossection is $\sigma(E) = \int d\Omega |f|^2$