QM Prelim

need 4 correct to pass
"correct" means correct answer written below (both formula and number must be correct) plus correct solution in the notebook
3 hours, 1 formula sheet, no calculators, textbooks, etc.

NAME:

A1:

A2:

A3:

A4:

A5:

A6:

A7:

1. Calculate the ground state energy of positronium (non-relativistic approximation).
2. Estimate the hyper-fine energy shift in hydrogen.
3. Estimate the energy shift between para and ortho-positronium.
4. Calculate the wavelength shift of the Lyman alpha line (n=2 to 1) between hydrogen and deuterium.
5. An electron spin is along the z-axis. The spin projection onto the axis \( \hat{n} \) making an angle \( \theta \) with \( \hat{z} \) is measured. Calculate the probability to get 1/2.
6. There is a 1D particle in the ground state in a box \( |x| < a/2 \). The box size \( a \) doubles instantaneously: \( a \rightarrow 2a \). Calculate the probability to stay in the ground state.
7. Two distinguishable spin-1/2 particles are in a pure state with total spin \( S \) and the total spin projection \( S_z \). For which values of \( S, S_z \) will the first particle be in a pure state?
Answers and Solutions:

1. \[ E = -\frac{m_e^4}{4\hbar^2} = -6.8 \text{ eV}. \]
   Solution: Hydrogen, with the reduced mass \( m/2 \).

2. \[ \delta E \sim \frac{m_e^4}{m_p} \alpha^4 m_e c^2 \sim 1 \mu \text{eV}. \]
   Solution: \( \delta E \sim \frac{\mu_p \mu_e}{a^3}, \mu \sim \frac{e\hbar}{m_e}, \alpha \sim \frac{\hbar^2}{m_e c^2} \).

3. \[ \delta E \sim \alpha^4 mc^2 \sim 1 \text{ meV}. \]
   Solution: Same as above, with \( m_p \to m_e \).

4. \[ \delta \lambda = \frac{8\pi}{3} \frac{c\hbar^3}{m_p e^4} \approx 0.3 \text{Å}. \]
   Solution: \( \frac{\delta \lambda}{\lambda} = \frac{\delta \mu}{\mu}, \mu \approx m(1 - m/M), \frac{\delta \mu}{\mu} \approx \frac{m_e^4}{2m_p}, \lambda = \frac{2\pi c\hbar}{\Delta E}, \Delta E \approx \frac{m_e^4 c^2}{2\hbar^2}(1 - \frac{1}{4}). \)

5. \[ p = \cos^2 \frac{\theta}{2}. \]
   Solution: Without loss of generality,

\[ \sigma_n \equiv \hat{n} \cdot \vec{\sigma} = \cos \theta \sigma_z + \sin \theta \sigma_x = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix}. \]

\[ \langle \uparrow | \sigma_n | \uparrow \rangle = \cos \theta = p - (1 - p). \]

6. \[ p = \frac{64}{9\pi^2} \approx 0.7 \]
   Solution: \( p = A^2, A = \int_{-a/2}^{a/2} dx \Psi_a \Psi_{2a}, \Psi_a = (\frac{2}{a})^{1/2} \cos \frac{\pi}{a} x. \)

7. \[ S = 1, |S_z| = 1. \]
   Solution: \( |1, 1 \rangle = |\uparrow\uparrow\rangle, |1, 0 \rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle), ... \)