New York University

Physics Department

PRELIMINARY EXAMINATION FOR THE PH.D. DEGREE

ELECTROMAGNETISM

Spring, 2014

READ INSTRUCTIONS CAREFULLY

1. ANSWER 4 OUT OF 5 PROBLEMS, AT YOUR CHOICE.
2. All problems have the same point value; when a problem is broken into parts, every part has the same value.
3. Use a separate answer booklet for each problem. On the front cover of each booklet write the problem number and your own identification number.
4. Show ALL your work.

1. A metal sphere of radius a carries a charge Q. It is surrounded, out to radius b, by a dielectric material of dielectric constant ε.

a) If the potential at infinity is zero, find the potential at the center of the metal sphere.

b) Find the bound charge at the outer surface (r=b) and the inner surface (r=a). (Hint: First compute the polarization in the dielectric.)

2. a) Consider a long wire carrying a steady current I. The wire is composed of a simple metal and so obeys Ohm's Law V=IR. Using the Poynting vector, calculate the energy flux into the wire. How do you interpret this result?

b) Using the Maxwell stress tensor $T_{ij} = \varepsilon_0 [E_iE_j-(1/2)\delta_{ij}E^2] + (1/\mu_0)[B_iB_j-(1/2)\delta_{ij}B^2]$, calculate the total force on the upper hemisphere of a uniformly charged sphere of radius R and total charge Q. (Assume that no other charges exist anywhere.)

3. a) A nonconducting medium with dielectric constant ε and magnetic permeability approximately equal to μ_0 occupies the half-space $z>0$; the half-space $z<0$ is occupied by vacuum. The boundary between the two consists of the xy-plane (z=0). There are no free charges or currents anywhere. A plane wave of frequency ω, traveling in the z-direction and linearly polarized in the x-direction, approaches the z=0 boundary from the vacuum side. What fraction of the incident energy is reflected, and what fraction transmitted? Express your answer in terms of the index of refraction of the medium $n \equiv (\varepsilon/\varepsilon_0)^{1/2}$.

b) Now suppose that the medium is a conductor with conductivity σ, dielectric constant ε, and magnetic permability μ. Use Maxwell's equations and Ohm's Law ($\mathbf{J} = \sigma \mathbf{E}$) to find the skin depth (the distance into the conductor at which the wave amplitude is reduced by a factor of 1/e).

4.a) A center-fed, linear antenna with length *d* oriented along the *z*-axis emits electromagnetic radiation due to an oscillating current $I = I_0(1-2|z|/d)e^{-i\omega t}$ for $|z| \leq d/2$. Find the angular distribution of radiated power $dP/d\Omega$ and the total radiated power P. Assume $d \ll \lambda$, where λ is the wavelength of the emitted radiation.

b) Do the same for a circular current loop of radius *a* and oscillating current $I = I_0 e^{-i\omega t}$.

5.a) Suppose an electron undergoes a constant negative acceleration *a* from some initial velocity v_0 ($\ll c$) down to 0. What fraction of its initial kinetic energy is lost to radiation?

b) To get a sense of the numbers involved, assume the initial velocity is thermal, i.e., $v_0 \approx 10^5$ m/sec, and the distance the electron goes is 30 Å. The charge of the electron is 1.6×10^{-19} C, and the permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ V·sec/A·m. What (order of magnitude) fraction of the electron energy is lost to radiation through collisions? What can you conclude about radiation losses for the electrons in an ordinary conductor?