Electromagnetism Prelim Exam 2012

Use separate booklets for each problem.

1. A rocket ship of proper length 10 m is moving away from the Earth at speed 4c/5. A light signal is sent after it that arrives at the rocket's tail at time zero according to rocket clocks and Earth clocks. Calculate in both frames the time at which the light signal reaches the head of the rocket (5 points). It is there reflected back by a mirror. Calculate in both frames the time at which the light signal again reaches the tail of the rocket (5 points).

2. A rocket ship flies at a velocity v through a large circular hoop of radius a along its axis. How far beyond the hoop is the rocket ship when the hoop appears exactly lateral to the pilot? Assume relativistic speed v (10 points).

3. Two momentarily coincident observers travel towards a small and distant object. To one observer the object looks twice as large (linearly) as to the other. Prove that their relative velocity is 3c/5 (20 points).

4. Calculate the energy necessary (in MeV) for a cosmic γ-ray to create an electron-positron pair (1 MeV) by a head-on collision with a photon of the cosmic microwave background of energy 3 kT (T = 2.7K) (20 points).

5. Null electromagnetic fields are characterized by the conditions $\mathbf{E}^2 = \mathbf{B}^2$ and $\mathbf{E.B} = 0$ (c =1). Show that null fields have the following properties: a)null fields are invariant under Lorentz transformations (5 points); b) energy density flows in a null field with the speed c (5 points); c) give a non-trivial null field that is a time-independent solution of Maxwell's vacuum equations (5 points); Give a time-dependent solution of Maxwell's vacuum equations (5 points).

6. If $\mathbf{E.B} \neq 0$, prove that there are infinitely many frames (with common relative direction of motion, and only those) in which **E** is parallel to **B**; precisely one of these moves in the direction **ExB,** its velocity being given by the smaller root of the equation $\beta^2 - R\,\beta + 1 = 0$, where β v/c and $R = (E^2 + c^2 B^2)/|\mathbf{ExB}|$. For the reality of β it is necessary to show that $R > 2$. (20 points).