This is a closed-book exam. No reference materials of any sort are permitted.
Full credit will be given for complete solutions to the following five questions.

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1. An impenetrable sphere of radius a carries a single charge $+e$. How many oppositely charged point particles will be attracted to the sphere at zero temperature? What is the total (net) charge of the final cluster of charges? How does this answer change if the point particles carry charge $-2e$? Assume that the sphere has the same dielectric constant as the surrounding medium.

2. Suppose that a sphere of radius R is composed of two conducting halves separated by an infinitesimally thin insulating ring at the equator. The top hemisphere is connected to a battery and thereby held at potential $V = V_0$. The bottom hemisphere is grounded, and thus is at potential $V = 0$. Under these conditions, the sphere will have a non-zero surface charge density σ.

   Find the electric dipole moment

   $$\vec{p} = \int \sigma \, \vec{x} \, dA$$

   for this charge distribution.

3. A plane electromagnetic wave of angular frequency ω propagates along the $\hat{z}$ axis through an optically active medium. Such materials have the ability to rotate the plane of polarization of linearly polarized light about the direction of propagation. The polarization vector of the medium is given by

$$\vec{P} = \gamma \nabla \times \vec{E},$$

where $\vec{E}$ is the light's electric field and γ is a real constant.

(a) The wave will experience two different refraction coefficients. Find the two indexes of refraction.

(b) Find the electric field configurations that correspond to the two refractive indexes obtained in (a).

4. If the proton is treated as a point charge, the ground state wavefunction of the electron in a hydrogen atom is

$$\psi(\vec{r}) = \frac{1}{\sqrt{\pi a^3}} \exp\left(-\frac{r}{a}\right)$$

where $a = \hbar^2/(me^2)$ is the Bohr radius, e is the magnitude of the electron charge, and m is the mass of the electron. The ground state energy is $E = -me^4/(2\hbar^2)$.

It would be more accurate to treat the proton as a uniform ball of charge centered at the origin. Assuming the proton's radius R to be much smaller than a, calculate the corresponding correction to the ground state energy of hydrogen.

5. A particle of charge q and mass m is affixed to the end of a spring of spring constant k. At time $t = 0$, the particle is displaced by a distance x_0 from its equilibrium position, after which it is released and allowed to oscillate. Assume that the subsequent motion is non-relativistic ($v \ll c$) and that $|q|$ is small enough that the particle's oscillation is only weakly damped by emission of electromagnetic radiation (recall Larmor emission). Treating the motion as that of a damped harmonic oscillator,

$$m\ddot{x} + \gamma\,\dot{x} + kx = 0,$$

determine the value of the damping coefficient γ in terms of q, m, and k.