E and M Prelim

1. a) A particle of rest mass M and total energy W hits a similar particle at rest. Show that if $W \gg Mc$, the maximum energy available in the c.m frame is $(2\,W\,Mc^2)^{1/2}$.

b) Show that $E^2 - B^2$ and $\underset{\sim}{E} \cdot \underset{\sim}{B}$ are invariant under uniform relative motion.

2. a) A perfectly conducting metal sphere of radius a is placed in a uniform electric field $\underset{\sim}{E}_0$. By appropriately expanding the solution of Laplace's equation in Legendre polynomials and applying boundary conditions, calculate the resulting electrostatic potential.

b) A "dielectric material" consists of a cubic array of metal spheres of radius a. The distance between centers of adjacent spheres is $d \gg a$. Calculate the dielectric constant of this material from the dipole moment per unit volume under application of a uniform electric field $\underset{\sim}{E}_0$. Assume that each sphere is influenced only by the external field.

3. a) A uniform material has dielectric constant ϵ , permeability μ , conductivity σ. Use charge conservation and any relevant E.M. equations to show that any initial charge distribution decays exponentially in time.

b) Show that the e.m.f. induced in a loop depends only on the vector potential right on the loop, and that this expression is invariant under a gauge transformation.

c) Show that the Poynting vector for a static system of charges and magnets transports no energy.

4. a) Show that a parallel pair of perfectly conducting solid cylinders of any given cross-sectional shape can transmit an EM-transverse (TEM) wave at any frequency ω.

b) Obtain an expression for the longitudinal decay rate of the transmitted power when the cylinders have large but not infinite conductivity σ.

5. a) A charged particle is in high frequency simple harmonic motion along the z-axis: $z = a \cos \omega t$. Using appropriate retarded field expressions, find the vector and scalar potentials along the z-axis at $Z \gg a$.

b) Two antennas, normal to the x-y plane, are located at $x = -\lambda/4$, $x = \lambda/4$ (and $y = 0$) where λ is the radiation wave length. They have identical current distributions. Find the radiation pattern in the xy plane.