Abstract

We document that income elastic-sectors are more intensive in high- and low-skill occupations than income inelastic sectors, which are relatively more middle-skill intensive. As a result, increases in aggregate expenditure have an asymmetric effect on labor demand across occupations and cause labor-market polarization. We quantify the importance of this demand-driven labor market polarization for the US using a general equilibrium model with nonhomothetic demand and endogenous job assignment. Our model is calibrated to match aggregate variables in 1980 and household-level estimates of sectoral income elasticities. We find that the increase in aggregate expenditure from 1980 to 2016 accounts for 50% of the increase in the wage bill share of high-skill occupations, 60% of the decline for medium-skill occupations and virtually all of the increase in the wage bill share of low-skill occupations. This mechanism is also quantitatively important to understand the evolution of labor market outcomes across occupations in the period 1950-1980 and in other developed economies.

Keywords: Inequality, Nonhomothetic Demand, Occupations, Workers’ Skills.

1 Introduction

Labor market outcomes in the US have polarized since the 1980s. The wage bill accrued to US high-skill workers relative to middle-skill has increased six-fold since the 1980, and the wage bill accrued to low-skill workers relative to middle-skill has increased three-fold. The polarization of wage bills has been driven by a polarization in both hours worked and the average wage of high and low-skilled workers relative to medium-skilled workers. What drives polarization? This question has motivated an enormous literature that has focused on skilled-biased technical change, computerization trade and offshoring, de-unionization, etc.\(^1\)

This paper documents and quantifies a novel mechanism to account for labor-market polarization that builds on the nonhomotheticity of demand. We document the novel fact that there is a strong positive correlation across sectors between the intensity of high- and low-skill occupations and the income elasticity of sectoral value added. Since income-elastic sectors represent a high share of US value added, this implies that the sectoral distribution of wage bills is concentrated in high-income elastic sectors for high and low-skill occupations, while the wage bill of middle-skill occupations is concentrated mostly in income inelastic sectors.\(^2\)

Our demand-driven polarization mechanism follows from this fact. As aggregate expenditure grows, demand shifts to high-income-elastic sectors. Since these sectors are intensive in high- and low-skill occupations, the reallocation of sectoral demand causes an increase in the relative demand for high- and low-skilled workers. This leads to a hollowing out of the wage bill distribution and polarization of workers earnings.\(^3\)

The empirical relevance of our mechanism relies on two empirical observations. First, that income elasticity is an important driver of sectoral growth. We document that sectoral value added growth over the period 1980-2016 has been fastest in more income-elastic sectors. We estimate the income elasticity parameters of a nonhomothetic CES demand system using US household expenditure data, as in Comin et al. (2015).\(^4\) A key property of this demand system is that there is a one-to-one mapping between sectoral income elasticity parameters and the observed average expenditure elasticity in the data. Figure 1 depicts the estimated income elasticity parameter in the x-axis and sectoral value-added growth across 8 broad sectors of the US economy on the y-axis.\(^5\) There is a clear, strong (0.86) and statistically significant relationship

\(^1\)See, among many others, Acemoglu and Autor (2011) and the references therein.
\(^2\)Expenditure elasticities of demand differ significantly across sectors. Among others, Aguiar and Bils (2015) document this heterogeneity across sectors using US household expenditure data. They also show that expenditure elasticities are stable both over time and across the income distribution.
\(^3\)Furthermore, as we discuss below, the share in total value added of high-income elastic sectors tends to increase over this period. As a result, the positive correlation between income elasticity and sectoral distribution of the wage bill for high and low-skill occupations tends to persist over time (see Table 1).
\(^4\)Demand is specified over sectoral value added. We thus combine data from the consumer expenditure survey (CEX) and BEA’s input-output tables to estimate demand over value added, as in Buera et al. (2015).
\(^5\)The relationship is robust to both finer and broader sectoral aggregations. We use 8 sectors in our analysis because it is a compromise between the tradition in the structural change literature, which typically focuses on 2 or 3 sectors, and the 15 sectors available in the BEA Input-Output tables.
Figure 1: Sectoral Nominal Value-Added Growth (1980-2016) and Income Elasticity Parameters

The second empirical requirement for our mechanism is the correlation between the sectoral distribution of the wage bill and sectoral income elasticity parameters across different occupations. Following Acemoglu and Autor (2011), we classify occupations into three skill categories (high, middle and low skill) according to their average wage in 1980. We then compute the share of the total wage bill accrued by any given occupation that comes from each of the 8 broad sectors. Figure 2 plots, for each occupation, the income elasticity parameter in the x-axis and the sectoral share of the wage bill in 1980 on the y-axis. Figures 2a and 2c show that there is a strong positive correlation between income elasticity and the low- and high-skill shares. The corresponding correlations are 0.57 and 0.95, respectively. In contrast, Figure 2b shows a negative relationship for middle-skilled occupations (with a correlation of -0.36).

These correlation patterns between income elasticity and sectoral distribution of wage bill shares emerge for two reasons. First, high income elastic sectors are intensive in high- and low-skill occupations, while the low-income elastic sectors are intensive in medium-skill occupations. This fact is documented in Figure 3 that plots the share of the sectoral wage bill accrued by a given occupation (in 1980) against the income elasticity of sectoral demand. The correla-

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6See Appendix B for details.
Notes: Each dot represents the share of the wage bill in occupation $j$ coming from sector $s$ in 1980. Employment shares are computed from the decennial census. Wages come from the Current Population Survey. See Appendix B for details.
Figure 3: Sectoral Factor Intensity and Income Elasticity

Notes: Each dot represents the share of the total wage bill in sector $s$ that is paid to occupation $j$ in 1980. Employment shares are computed from the decennial census. Wages come from the Current Population Survey. See Appendix B for details.

Table 1: Correlation of Sectoral Distribution of Occupation Wage Bill and Income Elasticity

<table>
<thead>
<tr>
<th></th>
<th>High</th>
<th>Middle</th>
<th>Low</th>
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</thead>
<tbody>
<tr>
<td>1980</td>
<td>0.58</td>
<td>-0.34</td>
<td>0.95</td>
</tr>
<tr>
<td>1990</td>
<td>0.50</td>
<td>-0.32</td>
<td>0.88</td>
</tr>
<tr>
<td>2000</td>
<td>0.46</td>
<td>-0.33</td>
<td>0.87</td>
</tr>
<tr>
<td>2016</td>
<td>0.53</td>
<td>-0.33</td>
<td>0.87</td>
</tr>
</tbody>
</table>

The correlation between these two variables is 0.9 for high-skill, 0.56 for low-skill and -0.9 for medium-skill occupations. Second, since high-income elastic sectors account for a significant part of total expenditures the vast majority of labor demand for low- and high-skill workers comes from high-income elastic sectors. Once these two observations are established, The positive correlation between wage bill shares and income elasticities for low- and high-skill workers documented in Figure 2 follows naturally from these two observations. Furthermore, since the correlations between the sectoral intensity of each occupation and the income elasticity of sectoral value added are very persistent and the value added shares of high income elastic sectors increase over time, we would expect the correlation patterns between income elasticity and sectoral wage bill shares to persist over time. Table 1 documents that indeed the correlation patterns between income elasticity and sectoral wage bill shares across occupations have been remarkably stable over the period 1980-2016.

To quantitatively evaluate the significance of nonhomotheticities for labor-market polarization, we develop a general equilibrium multi-sector economy with three key features: (i) nonhomothetic preferences with sectoral differences in the income elasticity of demand, (ii)
production factor intensities that vary over time and across sectors and (iii) endogenous assignment of workers to occupations.

We calibrate the key demand parameters in the model from household-level estimates from the CEX and by matching the distribution of value added across sectors in 1980. The traditional explanations of polarization are captured by the time-varying and sector-specific intensities of each occupation. These are calibrated to the share in value added of the wage bill accrued by each occupation. Our key finding is that, because of demand nonhomotheticities, an increase in aggregate expenditures consistent with the US historical increase in Personal Consumption Expenditures (PCE) per capita and with the increase in the PCE deflator is a key driver of the polarization of labor markets in all its dimensions. This increase in expenditures is responsible for roughly 50% (100%) of the observed increase in the wage bill share of high-skill (low-skill) occupations and 60% of the decline in the wage bill share of medium-skill occupations.

These results highlight two key conclusions. Our findings emphasize the importance of using a multi-sector framework for studying labor market dynamics. A one-sector model would incorrectly attribute to changes in (aggregate) skill-intensity the observed reallocation of the wage bill across occupations. Instead, in our setting, we can directly measure the skill-intensity at the sector level, and in this way, separately identify the role of changes in the composition of demand and in skill intensity for the evolution of relative wage bills. The main conclusion that emerges from our analysis is that non-homotheticities in demand play a key role for the evolution of the wage bill. Our findings prove that what drives the bulk of the change in the composition of value added in the economy and the associated reallocation of the wage bill across occupations is a pervasive force that has largely gone unnoticed: how we change our consumption patterns as we become richer. This finding must have profound implications on the design of policy interventions that intend to revert the polarization of labor markets.

Unlike wage bills, the model predictions for the evolution of relative wages and hours worked across occupations depend on the details of specifications for job assignment and the production functions. We explore whether the simple specifications we use divide the effect of changes in labor demand between changes in relative wages and hours worked in a way that resembles the data. We find that the model largely accounts for the observed evolution of both relative wages and hours worked across occupations. Furthermore, non-homotheticities in demand are responsible for similar shares of the evolution of relative wages and hours worked across occupations.

Our findings are robust to a number of extensions. In particular, to accounting for the wedges between sectoral production and expenditures introduced by net exports, to the rules used to assign capital income across households, and to allowing for sector-specific technical change.

Labor market polarization has been documented in other developed economies (Goos, Manning and Salomons, 2009 and 2014) and time periods (Barany and Siegel, 2018). We use
our model to study the role of nonhomotheticities in the evolution of labor market polarization in a large number of European economies since 1990, and the US during the 1950-1980 period. We find that nonhomotheticities in demand are also key to explain the evolution of distributional outcomes in labor markets in Europe and in the period 1950-1980. In particular, we find that they played a similar role to what we have documented for the US during 1980-2016. We conclude our paper by looking at the future transformation of labor markets from 2016 to 2035. Our model predicts that the wage bill will continue to polarize and that by 2035 the age bill share for medium skill workers will have declined to 35% (from 42% in 2016), while the wage bill shares of low- and high-skilled workers will increase to 10.4% and 54.9% from (8.8% and 49.1% in 2016).

Related literature This paper relates to different strands of literature. First, a vast and rich literature has explored the drivers of wage polarization. Wage polarization was probably first documented by Acemoglu (1999) for the US. Goos et al. (2009) document similar polarization patterns for European countries. Technical change has been the leading explanation to account for labor market polarization. Autor et al. (2003) proposed the “routinization” hypothesis whereby computer capital substituted for workers in routine-intensive occupations. Autor and Dorn (2009, 2013); Goos et al. (2014) also document a substantial increase in low-skill service sector jobs and link it to the fact that they are not routine-intensive. A larger number of papers have subsequently explored different aspects of routinization and polarization.7

The paper also relates to the relatively small literature that has linked structural change and relative wages. Lee and Shin (2017) and Bárány and Siegel (2018) also emphasize the importance of sectoral composition of the economy for labor market polarization. In contrast to ours, their models are set so that the structural transformation is entirely driven by skill-biased technical change completely omitting non-homotheticities. The rest of this literature has focused on the skill premium and compared labor market outcomes of college versus non-college workers. Our focus is on labor-market polarization, allowing for three skill-levels of occupations in which agents select into.8 To the best of our knowledge, Schimmelpfennig (1998) was the first to propose structural change as a complementary explanation to skill-biased technical change to account for the rise in the skill premium. He shows using a shift-share design in German data, that once the input-output structure of the economy is taken into account, structural change accounts for 40% of the rise in the skill premium. Buera et al. (2015) propose a two-sector, two-skill level (linked to education) model of structural change to account for the evolution of the

7Offshoring and international fragmentation of production has also been proposed as a potential channel. Basco and Mestieri (2013) show how trade costs have declined more in middle-skill intensive industries and how this can lead to wage polarization in rich countries. See the references therein for trade-based accounts of polarization.

8Our proposed channel is present in any model with nonhomotheticities, technological progress and heterogeneity in skill intensity across sectors. As such, the models proposed in Buera et al. (2015) and Cravino and Sotelo (2017) feature our proposed mechanism in a two-skill level setting. However, they do not analyze our proposed channel in isolation from other shocks to the economy.
skill premium. The model borrows from the insights in Buera and Kaboski (2012) who also uses a two-sector model, in which services is more skill intensive. Cravino and Sotelo (2017) extend the previous setting to allow for international trade and show that international trade also affects structural change and the evolution of the skill premium. Our formulation and estimation of the nonhomothetic CES demand system follows Comin et al. (2015). This formulation has the advantage of allowing for an arbitrary number of sectors (and nesting) with non-vanishing income effects relative to Stone-Geary demand systems.9

Finally, our work also relates to the micro literature that has used household expenditure data to document heterogeneity of income elasticities across sectors and differences in consumption patterns across the income distribution. Heterogeneity of income elasticities across sectors is a well-known fact, see Aguiar and Bils (2015) and references therein. Leonardi (2015) uses the household expenditure survey to show that educated households consume more high-skill and, to a lesser extent, very low-skill services. Leonardi then links an “education-specific elasticity of demand” to the rise of the skill premium in a static general equilibrium model. He finds a small effect for demand shifters in his calibrated model (consistent with Autor and Dorn, 2013 and Goos et al., 2014).10 Mazzolari and Ragusa (2013) also analyze changes in product demand using household consumption data. They focus their analysis on the consumption of low-skill-intensive services. They argue that when relative wages of skilled workers increases, demand for low-skill services should increase. They estimate that this channel accounts for one-third of the growth of employment of noncollege workers in low-skill services in the 1990s.

We find instructive to illustrate our mechanism by introducing its key elements one at a time. The structure of the paper reflects this strategy. In Section 2, we focus on the drivers of the relative wage bills across occupations in a simplified framework. We first introduce the multi-sector supply-side of model and document the importance of changes in the composition of value added for the evolution of wage bills. Then, we make the evolution of sectoral value added endogenous by introducing nonhomothetic preferences. We quantify the importance of aggregate expenditures for the evolution of relative wage bills across occupations in this simple set-up. After understanding the key driving forces in the simplest possible setting, we present the fully-fledged model in Section 3. This model features endogenous job assignment and heterogeneous households, in addition to the multi-sector production structure and non-homothetic preferences. In contrast to the simple model, this richer setting has implications for the evolution of both wages and hours worked across occupations. In Section 4, we extend our

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9We abstract from the changes in the quality of goods consumed within a sector as recently emphasized in Jaimovich et al. (2019).
10Autor and Dorn, 2013 conclude that demand-pull effects are likely not to drive polarization by studying whether wage increases at the very top of the distribution affected demand for services. In contrast, our results account for the overall increase income at across the income distribution and their effect on the overall sectoral composition of demand.
analysis to the 1950-1980 period for the U.S., to various European countries, and we look into the future polarization of US labor markets circa 2035. Section 5 concludes.

2 Relative Wage Bills: A first pass

This section presents a simple multi-sector model that contains the key quantitative drivers of polarization. We proceed in two steps. First, we introduce the production side of our economy. We use it to perform a decomposition exercise of the evolution of the relative wage bill and quantify the importance of sectoral reallocation of economic activity. Second, we present the demand-side of the economy. It features a representative household with nonhomothetic preferences. This setting suffices to study the sources of changes in the sectoral composition of the economy and assess the importance of nonhomotheticities in driving labor-market polarization.

2.1 Production

We consider an economy with $S = \{1, \ldots, S\}$ distinct sectors. Each sector $s \in S$ produces output $Y_{st}$ at time $t$ according to the constant-returns-to-scale Cobb-Douglas production function

$$Y_{st} = A_{st} \prod_{j \in \{H,M,L\}} X_{jst}^{\alpha_{jst}}, \quad \text{with} \quad \sum_{j \in \{H,M,L\}} \alpha_{jst} = 1,$$

where $A_{st}$ is a Hicks-neutral technological term in sector $s$, $\alpha_{jst}$ is the intensity of occupation $j$ in sector $s$ at time $t$, and $X_{jst}$ is the number of hours from occupation $j$ used in sector $s$. Since we are allowing $\{\alpha_{jst}\}$ to vary arbitrarily over time, we can flexibly capture any patterns of substitution across factors of production implied by skill-biased technological change, offshoring of production, changes in production wedges, etc.\textsuperscript{11} The normalization $\sum_{j \in \{H,M,L\}} \alpha_{jst} = 1$ implies that the contribution of capital (and other omitted factors) to production is captured by the TFP term $A_{st}$. In Section 3 we relax this assumption by allowing for time-varying sectoral labor shares.

Under competitive equilibrium, given a price of sectoral output $p_{st}$ and wages $w_{jt}$, the de-
mand for occupation $j$ in sector $s$ at time $t$ is

$$w_{jt}X_{jst} = \alpha_{jst}p_{st}Y_{st}. \tag{2}$$

The ratio of the wage bill accrued by two occupations, $j$ and $j'$, in sector $s$, is

$$\frac{w_{jt}X_{jst}}{w_{jt'}X_{j'st}} = \frac{\alpha_{jst}}{\alpha_{j'st}}. \tag{3}$$

Thus, in any sector $s$, the wage bill of occupation $j$ relative to occupation $j'$ is entirely determined by the ratio of their occupation intensities in production, $\alpha_{jst}$ and $\alpha_{j'st}$, which we can compute using readily available data.

Next, we compute the aggregate wage bill of occupation $j$. Let $X_{jt}$ denote total employment in occupation $j$ (i.e., $X_{jt} = \sum_{s \in S} X_{jst}$), and $VA_{st}$, nominal value added in sector $s$. In this simple model, sectoral output coincides with sectoral value-added, $VA_{st} = p_{st}Y_{st}$ (this assumption is relaxed in Section 3). Adding Equation (2) across sectors, we obtain that the aggregate wage bill of occupation $j$ is

$$w_{jt}X_{jt} = \sum_{s \in S} \alpha_{jst}VA_{st}. \tag{4}$$

This expression simply states that the total wage bill accrued to occupation $j$ is the sum of sectoral value added weighted by the share of the wage bill going to occupation $j$ in sector $s$.

**Accounting for Polarization**  The production structure laid down so far allows us to unpack observed changes in the wage bill. In particular, we use Equation (4) to decompose the evolution of the wage bill in occupation $j$. We rewrite $\alpha_{jst}VA_{st}$ as

$$\alpha_{jst}VA_{st} = (\alpha_{jst0} + \Delta \alpha_{jst})(VA_{st0} + \Delta VA_{st}), \tag{5}$$

where $\Delta$ denotes the time difference operator between time 0 and $t$, i.e., $\Delta VA_{st} \equiv VA_{st} - VA_{s0}$. Replacing (2) in (5) and dividing by the initial wage bill, $w_{j0}X_{j0}$, we obtain

$$\frac{w_{jt}X_{jt}}{w_{j0}X_{j0}} - 1 = \frac{\sum_{s \in S} \alpha_{jst0} \Delta VA_{st}}{w_{j0}X_{j0}} + \frac{\sum_{s \in S} \Delta \alpha_{jst}VA_{st0}}{w_{j0}X_{j0}} + \frac{\sum_{s \in S} \Delta \alpha_{jst} \Delta VA_{st}}{w_{j0}X_{j0}}.$$

Using Equation (4) to replace $w_{j0}X_{j0}$ by $\sum_{s} \alpha_{jst0}VA_{s0}$ implies that the growth in wage bill is

$$\frac{\Delta (w_{jt}X_{jt})}{w_{j0}X_{j0}} = \sum_{s \in S} \gamma_{jso} \Delta VA_{st} + \sum_{s \in S} \gamma_{jst0} \Delta \alpha_{jst} + \sum_{s \in S} \gamma_{jst0} \frac{\Delta VA_{st} \Delta \alpha_{jst}}{VA_{s0} \alpha_{jst0}} \tag{6}$$

<table>
<thead>
<tr>
<th>Term</th>
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<tbody>
<tr>
<td>$\gamma_{jso} \frac{\Delta VA_{st}}{VA_{s0}}$</td>
<td>$\gamma_{jst0} \frac{\Delta \alpha_{jst}}{\alpha_{jst0}}$</td>
<td>$\gamma_{jst0} \frac{\Delta VA_{st} \Delta \alpha_{jst}}{VA_{s0} \alpha_{jst0}}$</td>
</tr>
</tbody>
</table>
To interpret expression (6), first note that the weights $\gamma_{js0} \equiv \frac{\alpha_{js0}VA_{s0}}{\sum_s \alpha_{js0}VA_{s0}}$ represents the share of the total wage bill of occupation $j$ that comes from sector $s$. Expression (6) illustrates the key insight of the multi-sector setting. Changes in the wage bill are not only driven by changes in the factor intensity (Term 2), but also by changes in the sectoral composition of the economy (Term 1), and the interaction between changes in factor intensity and changes in the sectoral composition of the economy (Term 3). As noted by Buera et al. (2015, 2018) or Cravino and Sotelo (2017), the structural transformation of the economy may impact the wage bill of occupation $j$ if the sectors where this occupation was initially more intensive have grown faster. Similarly, the impact of changes in the intensity of an occupation on the wage bill depend on how they co-vary with the initial sectoral share distribution of the occupation.

**Contrast with a One-Sector Model** In the particular case that there is only one sector in the economy, $S = 1$, Equation (6) simplifies to

$$
\frac{\Delta (w_j X_{jt})}{w_{j0} X_{j0}} = \left( \frac{\Delta \alpha_{jt}}{\alpha_{j0}} + \frac{\Delta VA_t}{VA_0} + \frac{\Delta \alpha_{j0}}{\alpha_{j0}} \frac{\Delta VA_t}{VA_0} \right).
$$

This implies that the growth in the relative wage bill across occupations $j$ and $j'$ is

$$
\frac{\Delta (w_{jt} X_{jt})}{w_{j0} X_{j0}} - \frac{\Delta (w_{j't} X_{j't})}{w_{j'0} X_{j'0}} = \left( \left( \frac{\Delta \alpha_{jt}}{\alpha_{j0}} - \frac{\Delta \alpha_{j'0}}{\alpha_{j'0}} \right) \left( + \frac{\Delta VA_t}{VA_0} \right) \right).
$$

Expression (7) shows that, in the one-sector setting, differences in the growth of the relative wage bill of workers in occupation $j$ relative to $j'$ can only arise from differences in the growth rate of their factor intensities, $\alpha_{jt}$ and $\alpha_{j't}$. Consequently with this observation, most of the existing theories proposed to account for polarization – such as skill-biased technical change, de-unionization or offshoring – operate through sectoral changes in occupational intensities $\{\alpha_{jst}\}$.\(^{12}\)

This analysis makes clear that assuming a one-sector model imposes that any change in the wage bill must come from differential trends in $\alpha_{jt}$ and $\alpha_{j't}$. This effect corresponds to Term 2 in our decomposition (6). Thus, assuming a one-sector model shuts down the effect of differential output growth across sectors with heterogeneous skill intensity (Term 1) and its interaction with cross-sectional changes in sectoral skill intensity (Term 3). If these latter two terms are important in driving changes in the wage bill, using a one-sector model may offer a very incomplete picture of the evolution of the wage bill. We turn to this question next.

\(^{12}\)See, among others, Acemoglu and Autor, 2011, Acemoglu et al. (2001), Basco and Mestieri (2013) and the references therein.
Assessing the Contribution of the Terms in Decomposition (6) To quantify the contribution of each term in Equation (6), we group the US economy into 8 broad sectors. We take the nominal sectoral value added growth from the BEA. The occupation intensities in each sector are computed from Equation (3) as the share in the sectoral wage bill of each occupation. The weights $\gamma_{js0}$ are computed as the share of sector $s$ in occupation $j$’s total wage bill. The subscript 0 denotes 1980 and the subscript $t$ denotes the final year, 2016.

Table 2 and Figure 4 report the contribution to wage-bill growth of each of the three terms in the decomposition in Equation (6) for high-, middle- and low-skill occupations. For all three skill levels, the change in the sectoral composition of value added, Term 1 in Equation (6), has been the key driver of the growth in the wage bill at all skill levels. The contribution of changes in factor intensities (Term 2) appears to be modest and it is mostly operative through the interaction term (Term 3). The contribution of the interaction term (Term 3) implies that changes in factor shares have been skill-biased in sectors that have fast value-added growth. To assess the overall contribution of value-added growth and changes in factor intensity, we  

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13 These come from the aggregation of the 15 sectors appearing in the BEA’s input-output tables. These are i) primary sector and utilities, ii) manufacturing, iii) construction, iv) retail and wholesale trade, v) finance, insurance, real estate, information and professional services, vi) health and education vii) food and entertainment and viii) government. This grouping is based on two factors: the traditional aggregation of sectors and the estimates of the income elasticity of demand. The results from the decomposition based on Equation 6 are robust to alternative disaggregations of the economy both with larger and smaller number of sectors.

14 These numbers are not exactly the same as the observed growth in the wage bill because this decomposition is assuming a constant labor share across sectors while the sectoral labor shares have changed over the period 1980-2016. However, the magnitudes from both exercises are similar.
Table 2: Decomposition of Wage Bill Growth 1980-2016 based on Equation (6)

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<thead>
<tr>
<th></th>
<th>High</th>
<th>Middle</th>
<th>Low</th>
<th>Difference</th>
<th>H − M</th>
<th>L − M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Measured Change</td>
<td>10.19</td>
<td>3.18</td>
<td>6.61</td>
<td>7.01</td>
<td>3.43</td>
<td></td>
</tr>
<tr>
<td>Δ Sectoral Change (Term 1)</td>
<td>7.05</td>
<td>4.66</td>
<td>7.09</td>
<td>2.39</td>
<td>2.43</td>
<td></td>
</tr>
<tr>
<td>Δ Factor Intensity (Term 2)</td>
<td>0.45</td>
<td>-0.22</td>
<td>0.00</td>
<td>0.67</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>Interaction (Term 3)</td>
<td>2.69</td>
<td>-1.26</td>
<td>-0.47</td>
<td>3.95</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>Contribution of Term 1</td>
<td>62%</td>
<td>82%</td>
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split evenly the contribution of the interaction term (Term 3) between the first two terms. This decomposition implies that sectoral value added growth accounts for 62% (= 4.37/7.01) of the differential increase in the wage bill of the high-skill relative to the medium-skill occupations (as measured by the difference between the growth rate in the two occupations), and for 82% (= 2.82/3.43) of the increase in the wage bill of low- relative to medium-skill occupations.

Why does this happen? The key driver of our findings is the positive correlation between the sectoral value-added growth and the sectoral share of high- and low-skill occupations in 1980 (0.58 and 0.87, respectively) and the negative correlation for middle-skill occupations (−0.36). These correlation patterns explain how Term 1 can account for the differential growth in the relative wage bill of high- and low-skill. In contrast, the correlation between the growth rate of the factor intensities and the initial sectoral shares for each type of occupation are more negative for high- and low-skill occupations (−0.23 and −0.22) than for middle-skill occupations (0.38). Hence, the little contribution of the second term to the relative wage bills.15,16

In sum, these findings suggest that, to explain the aggregate variation in the relative intensity across occupations, it is critical to incorporate mechanisms that shift the composition of economic activity to sectors that were initially more intensive in high- and low-skill occupations.

2.2 Households

Next, we propose and quantify a demand-driven theory that attributes the changes in the distribution of value added to sectoral heterogeneity in expenditure elasticities. Our theory also

15We still find a positive contribution of Term 2 to the relative wage bill because, on average, the growth in factor intensity is higher for high- and low-skill occupations than for middle-skill occupations. A similar logic explains the positive contribution of Term 3 to the relative wage bill.

16In Appendix A, we report the results from applying the decomposition in Equation (6) to three subperiods 1980-1990, 1990-2000 and 2000-2016. The dominance of sectoral reallocation of value added over changes in intensity for the evolution of relative wage bills holds in all three subperiods. However, we find that the reallocation of value added becomes more important for the evolution of both relative wage bills over time. In particular its contribution increases from 54% in 1980-90 to 67% in 2000-16 (for the high vs. middle occupations wage bill) and from 83% in 1980-90 to 98% in 2000-16 (for the low vs. medium occupations wage bill).
encompasses the theories that traditionally have been regarded as culprits for inequality such as skill-biased technical change, robotization and international trade. To explore this mechanism, we need to specify household preferences for the goods and services produced in the economy. We use the nonhomothetic CES preferences developed by Hanoch (1975) and Sato (1975), and introduced by Comin et al. (2015) in a general equilibrium setting.

We make two simplifying assumptions. First, we assume that all income accrued by the workers is spent in consumption. Second, we study an economy with a representative household. That is, all the resources earned by the household are pulled together and, given the prevailing prices, the household decides its consumption bundle. We extend the model to heterogeneous households in Section 3.

Preferences  The utility of the representative household at time $t$, $U_t$, is a nonhomothetic CES aggregator defined over consumption goods $\{c_st\}_{s \in S}$ available in the economy,

$$\sum_{s \in S} \left( \frac{U_t^{e_s} \zeta_s}{\varepsilon_s} \right)^\frac{1}{\sigma - 1} = 1.$$  \hspace{1cm} (8)

The parameter $\sigma$ controls the price elasticity of substitution across goods, while the income elasticity parameter $\varepsilon_s$ controls the expenditure elasticity of sector $s$. The constant $\zeta_s$ captures the constant taste component of preferences. This formulation of preferences allows for the level of utility $U_t^{e_s}$ to enter symmetrically in (8) as an additional taste component. In contrast to $\zeta_s$, this term is variable and endogenously determined. As a result, the weight attached to the consumption of each good depends on the level of utility itself, $U_t$, with an elasticity $\varepsilon_s$. If $\varepsilon_s$ were constant across all $s$, e.g., $\varepsilon_s = 1 - \sigma$, we would obtain homothetic CES preferences.$^{17}$

Given a set of prices $\{p_{st}\}_{s \in S}$ and total expenditure $E_t$, a household maximizing utility (8) subject to the budget constraint $\sum_{s \in S} p_{st}c_{st} \leq E_t$ chooses $\{c_{st}\}_{s \in S}$

$$c_{st} = \zeta_s \left( \frac{E_t}{p_{st}} \right)^{\sigma} U_t^{e_s}. \hspace{1cm} (9)$$

The corresponding expenditure function is given by $E_t^{1-\sigma} = \sum_{s \in S} \zeta_s U_t^{e_s} p_{st}^{1-\sigma}$.

We can normalize one taste parameter $\zeta_s \equiv 1$ (as with homothetic CES) and one income elasticity parameter $\varepsilon_s \equiv 1$ for some $s \in S$. These normalizations cardinalize (8) and uniquely define a cost-of-living index $P_t$ and a real consumption index $C_t$ of the representative house-

$^{17}$This parametrization of preferences requires that $\varepsilon_s > 0$ if $0 < \sigma < 1$ and $\varepsilon_s < 0$ otherwise. It is possible to parametrize nonhomothetic CES preferences such that the income elasticity parameters do not change signs depending on whether goods are gross complements or substitutes. For example, the alternative specification of preferences $\sum_s \zeta_s \left( \frac{E_t}{p_{st}} \right)^{\sigma} = 1$, is well-defined for $\tilde{\varepsilon}_s > 0$ and any $\sigma > 0$. Note that we can go from this specification to our baseline specification (8) with the change of variables $\tilde{\varepsilon}_s = \varepsilon_s / (1 - \sigma)$. See Appendix A of Comin et al. (2015) for a more general discussion.
hold, \( U_t = \frac{E_t}{P_t} \equiv C_t \). The cost-of-living index can be expressed in terms of observables and demand parameters,

\[
P_t = \left[ \sum_{s \in S} \left( \xi_s P_{st}^{1-\sigma} \right)^{\chi_s} \left( x_{st} E_t^{1-\sigma} \right)^{1-\chi_s} \right]^{\frac{1}{1-\sigma}}, \tag{10}
\]

where \( x_{st} = p_{st} c_{st} / E_t \) denotes the expenditure share in sector \( s \), and \( \chi_s \equiv (1 - \sigma) / \varepsilon_s \). With this notation, the expenditure share in sector \( s \) is

\[
x_{st} = \xi_s \left( \frac{p_{st}}{P_t} \right)^{1-\sigma} E_t^{\varepsilon_s - (1-\sigma)}. \tag{11}
\]

Finally, the model expenditure elasticity of sector \( s \), \( \eta_s \) is

\[
\eta_s \equiv \frac{\partial \ln p_{st} c_{st}}{\partial \ln E_t} = \sigma + (1 - \sigma) \frac{\varepsilon_s}{\sum_{s \in S} x_{st} \varepsilon_s}. \tag{12}
\]

Thus, whether a good has an expenditure elasticity higher (or lower) than 1 depends on whether \( \varepsilon_s > (\leq) \sum_{s \in S} x_{st} \varepsilon_s \), which depends on the total level of expenditure of the household \( E_t \). This implies that the same good can be a luxury or a necessity depending on the level of expenditure.\(^{18}\)

### 2.3 Equilibrium and Structural Change

After having introduced household preferences, we use the market clearing condition to close our model and decompose the evolution of sectoral value added (assuming a closed economy). The total expenditure of the representative household is given by the total labor income of the economy,

\[
E_t = \sum_{s \in S} \sum_{j \in \{H,M,L\}} w_{jt} X_{jst}. \tag{13}
\]

Using the demand for good \( s \), Equation (9), nominal value added in sector \( s \) is

\[
VA_{st} = p_{st} c_{st} = \xi_s p_{st}^{1-\sigma} E_t^{\sigma - \varepsilon_s} P_t^{-\varepsilon_s}. \tag{14}
\]

Equation (14) illustrates that, in our model, the evolution of sectoral value added is driven by two forces: changes in aggregate expenditures, \( E_t \), and changes in sectoral prices, \( \{p_{st}\} \) (note from Equation (10) that \( P_t \) is itself a function of aggregate expenditure and prices).

Supply-side drivers of polarization and inequality such as skill-biased technical change, de-unionization or offshoring affect the sectoral composition of value added through their impact in relative sectoral prices. Indeed, in a model with homothetic preferences, changes in relative prices are the only source of sectoral reallocation in value added. To see this, consider the ratio

---

\(^{18}\)See also Hanoch (1975) and Comin et al. (2015) for further discussion on the properties of these preferences.
of Equation (14) for two sectors, $s$ and $s'$ when preferences are homothetic (e.g., $\varepsilon_s = 1 - \sigma$ for all $s$),

$$\frac{VA_{st}}{VA_{s't}} = \frac{\xi_s}{\xi_{s'}} \left( \frac{p_{st}}{p_{s't}} \right)^{1-\sigma}.$$ 

Changes in the relative sectoral composition depend on the evolution of relative prices and are independent of the overall level of expenditure, $E_t$.

Nonhomotheticities introduce a distinct, demand-driven, mechanism that affects the evolution of the sectoral composition of value added: aggregate expenditures, $E_t$. As expenditure increases, consumers shift the composition of expenditure from low income-elastic (low $\varepsilon_s$) to high income-elastic sectors (high $\varepsilon_s$). Even if relative prices were constant, our demand system would imply changes in the sectoral composition of the economy driven by aggregate expenditure,

$$\frac{VA_{st}}{VA_{s't}} = \frac{\xi_s}{\xi_{s'}} \left( \frac{E_t}{P_t} \right)^{\varepsilon_s - \varepsilon_{s'}},$$

where we have normalized relative prices to one.

### 2.4 Model Quantification: Structural Change and the Wage Bill

We now proceed to quantify our model. To do so, we first discuss how we estimate the income and price elasticities using household data. We then discuss our calibration strategy for the rest of the model: we match the US economy in 1980 and then shock this economy with 2016 values for expenditure, factor intensities and prices. We show that our calibrated model accounts well for the structural change and evolution of wage bills across occupations in the 1980-2016. We use our calibrated model to investigate the drivers of the evolution of the wage bill and assess the quantitative importance of our demand-driven mechanism.

#### 2.4.1 Estimation of income and price elasticities

As a first step towards quantifying our model, we estimate the demand parameters governing the income and price elasticities. Our baseline estimates use data from the Consumption Expenditure survey (CEX) for the 2000-2007 period. We follow Aguiar and Bils (2015) and use their sample selection for CEX households and their estimating strategy of expenditure elasticities. In particular, we focus on urban households with ages of the reference person between 25 and 64. The key difference with Aguiar and Bils is that we estimate our demand in terms of value added rather than final expenditures, as in Herrendorf et al. (2013) and Buera et al. (2015, 2018). We follow the procedure described in Buera et al. (2015) and map each expenditure category appearing in the CEX to one of the four-hundred and five industries appearing in the 2007 US input-output table, and compute household expenditures in terms of value added (see Appendix B.2 for further details on sample restrictions and data construction).
Denoting a household by \( n \), Aguiar and Bils (2015) propose to estimate expenditure elasticity of sector \( s \), \( \eta_s \), as

\[
\ln \left( \frac{x_{st}^n}{\bar{x}_{st}} \right) = \alpha_{st} + \eta_s \ln E_{tn}^n + \Gamma_s Z^n + u_{st}^n
\]  

(15)

where \( x_{st}^n \) is the expenditure of household \( n \) in sector \( s \) goods in quarter \( t \), \( \bar{x}_{st} \) denotes the average expenditure in \( s \) in quarter \( t \) across households, \( E_{tn}^n \) denotes expenditure of household \( n \), \( Z^n \) is a vector of demographic controls (dummies for age bins, number of earners and household size) and \( u_{st}^n \) is an error term. Aguiar and Bils (2015) argue that this specification “provides a tractable framework to address for the mis-measurement in the CEX” in which “respondent’s errors (…) are scaled up by their level of expenditures.”

To address potential measurement error in sectoral expenditures that would accumulate in the measure of total expenditure, \( E_t^n \), we follow Aguiar and Bils and instrument total expenditures with dummy quintiles for the household’s income group. The rationale is that “total expenditure reflects permanent income and thus will be correlated with current income.”

Column (3) in Table 3 reports the estimated expenditure elasticities. We find that education and health is the most expenditure-elastic sector, followed by arts and entertainment. Conversely, construction and agriculture appear to be the least expenditure-elastic. The range of expenditure elasticities that we find is very similar to the range that Aguiar and Bils (2015) find with demand specified over final expenditures. Similar to us, they find that the most expenditure-elastic sector is education and that food at home is the second least expenditure-elastic sector (after tobacco). These estimates correspond to the expenditure elasticities reported in the figures discussed in the Introduction.

Next, we describe how we estimate our demand system with the same CEX data. Our estimation strategy is based on the generalized method of moments proposed in Comin et al. (2015). To write our estimating equations, we leverage on the log-linear nature of the demand system and use it to invert the demand for one sector \( s \) to obtain an expression for \( C_t^n \) of household \( n \),

\[
\ln C_t^n = \frac{1}{\varepsilon_s} \left[ \ln x_{st}^n - \ln \bar{z}_s + (1 - \sigma) \ln \left( \frac{E_t^n}{p_{st}} \right) \right]
\]  

(16)

We can then substitute out \( C_t \) in the expenditure share equations for all \( s \neq \$ \)

\[
\ln x_{st}^n = \ln \bar{z}_s - \frac{\varepsilon_s}{\varepsilon_s} \ln \bar{z}_s + (1 - \sigma) \ln \left( \frac{p_{st}}{p_{st'}^n} \right) + (1 - \sigma) \left( \frac{\varepsilon_s}{\varepsilon_s} - 1 \right) \ln \left( \frac{E_t^n}{E_t^{n'}} \right) + \frac{\varepsilon_s}{\varepsilon_s} \ln x_{st}^n.
\]  

(17)

---

19In particular, this differencing strategy cancels out log-linearly additive errors that are good- and time-specific. Note also that the term \( \alpha_{st} \) controls for the effect of changing prices.

20Aguiar and Bils (2015) also propose to use a continuous measure of log-after tax income as instrument and instrumenting the same response of quarterly expenditure with responses in other interviews. All these methods yield comparable estimates of \( \eta_s \).

21We supplement our data with time series for sectoral urban prices across different US regions from the BLS as in Comin et al. (2015).
Table 3: Estimation of Expenditure and Price Elasticities

<table>
<thead>
<tr>
<th>Sectors</th>
<th>Nonhomothetic CES(^2)</th>
<th>Aguiar-Bils(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\sigma)</td>
<td>(\varepsilon_s)</td>
</tr>
<tr>
<td>Education and Health Care</td>
<td>3.14</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Arts, Entertainment,</td>
<td>1.92</td>
<td>1.57</td>
</tr>
<tr>
<td>Recreation and Food Services</td>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Government(^1)</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>(-)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Finance, Professional, Information,</td>
<td>0.90</td>
<td>1.06</td>
</tr>
<tr>
<td>other services (excl. gov’t)</td>
<td>(0.00)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.67</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Retail, Wholesale Trade and</td>
<td>0.74</td>
<td>0.91</td>
</tr>
<tr>
<td>Transportation</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Construction</td>
<td>0.32</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Agriculture, Mining and</td>
<td>0.38</td>
<td>0.71</td>
</tr>
<tr>
<td>Utilities</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Price Elasticity</td>
<td>0.51</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard errors clustered at the household level shown in parentheses. Total number of households is 60,630. The BEA sector codes in the Input-Output table that corresponds to our groupings are: 6 for Education and Health Care, 7 for Arts, etc., G for Gov’t, FIRE, PROF, 51, 81 for Finance etc., 31G for Manufacturing, 42, 44RT, 48T for Retail etc., 23 for Construction and 11,21,22 for Agriculture etc. (1): Government sector is normalized to 1. (2): \(\eta_s\) is the average expenditure elasticity in the 2000-2007 CEX data using the estimated parameters for the nonhomothetic CES parameters \(\{\varepsilon_s, \eta\}\). (3): \(\eta_s\) is the expenditure elasticity estimated using the Aguiar and Bils (2015) specification.

This procedure generates a system of equations for each household \(n\) and all \(s \neq \hat{n}\). Equation (17) makes clear that the system is identified up to a normalization of \(\varepsilon_s\) and \(\zeta_s\) for one sector. We proceed by normalizing \(\varepsilon_s = \zeta_s = 1\). We parametrize the constant taste parameter as \(\ln \zeta_s = \Gamma_s Z^n + u_{st}^n\), where \(Z^n\) denote the same household controls as in the previous estimation (dummies for age, number of earners and family size) and \(u_{st}^n\) is a measurement error term. Equation (17) becomes

\[
\ln x_{st}^n = \Gamma_s Z^n + (1 - \sigma) \ln \left( \frac{p_{st}^n}{p_{st}^m} \right) + (1 - \sigma) (\varepsilon_s - 1) \ln \left( \frac{E_{t}^n}{p_{st}^m} \right) + \varepsilon_s \ln x_{st}^n + u_{st}^n. \tag{18}
\]
The resulting system of equations for $s \neq s$ define moments in terms of observables that we use in our estimation. Since a priori we can take any sector as a base, we use in our estimation the moments implied when we use each sector $s \in \mathcal{S}$ as a base. Finally, we also use the reduced-form expenditure elasticities we estimated using the Aguiar and Bils (2015) methodology as an additional set of moments to be satisfied for the average household in the sample. In particular, using the expression in Equation (12) for expenditure elasticity, we have that

$$\hat{\eta}_s = \sigma + (1 - \sigma) \frac{\epsilon_s}{\sum_i \bar{x}_{st} \epsilon_i} + \nu_s,$$

where $\hat{\eta}_s$ is the expenditure elasticity estimated using the Aguiar and Bils (2015) method (reported in column (3) of Table 3), $\bar{x}_{st}$ is the sample average expenditure share in sector $s$ and $\nu_s$ denotes an error term.

To deal with potential measurement error and endogeneity concerns, we use instruments for the observed measures of household expenditures and relative prices. As in Aguiar and Bils (2015), we use household (after-taxes) income quintiles as instruments for quarterly expenditures. The instruments capture the permanent household income and are therefore correlated with household expenditures without being affected by transitory measurement error in total expenditures. We instrument household relative prices with a “Hausman” relative-price instrument. Each of the prices used in the relative-price instrument is constructed in two steps. First, for each sub-component of a sector, we compute the average price across regions excluding the own region. Then, the sectoral price for a region is constructed using the average region expenditure shares in each sub-component as weights. These price instruments capture the common trend in U.S. prices while alleviating endogeneity concerns due to regional shocks (and measurement error of expenditure).\(^{22}\)\(^{22}\)

Table 3 reports our estimates of the income and price elasticity preference parameters in the first and second columns. We find a value of the price elasticity of 0.51, implying that the eight sectors are complements. The third column reports the implied expenditure elasticity for each sector for the average household in our sample. The expenditure elasticities from our model estimates and from the Aguiar-Bils reduced form approach yield very similar results.\(^{23}\)\(^{23}\)

One limitation of the CEX data is that it only measures out-of-pocket expenditures. Thus, it provides an underestimate of true consumption expenditures for certain items, especially those related to education and health services. Given that we are using cross-sectional household variation to estimate expenditure elasticities, to the extent that out-of-pocket expenditures for a given expenditure item are correlated with household total expenditure, we may obtain

\(^{22}\)Using the average price in the U.S. excluding the own region addresses the concern of regional shocks, while capturing the common component of prices across regions. Using average expenditures in the region addresses the concern of mismeasurement of household expenditure shares in that region to the extent that the mismeasurement averages out in the aggregate.

\(^{23}\)We have also performed the same estimation including quarter-year and region fixed effects, obtaining very similar estimates.
biased estimates. For example, consider higher education (which has important government subsidies). If richer households are more likely to have their children attending college, we may have a systematic under-reporting of education expenditures for richer households, which would bias downwards the estimated expenditure elasticity of education.

To partially alleviate this concern, we also estimate expenditure elasticities using aggregate cross-country panel data for OECD countries from KLEMS. In particular, we run an analogous version to the Aguiar-Bils specification, where we use oil shocks and interact them with sectoral energy intensity to instrument for total value added and sectoral prices. We find a correlation between the estimated elasticities in the aggregate data and the CEX of 0.73. Moreover, the point estimate for health and education is 2.45, very similar to the 2.28 estimated in the micro data. Appendix C.1 provides further details on this exercise.

Quantification of the drivers of structural transformation A natural question to ask at this point is what share of the observed sectoral variation in value added growth between 1980 and 2016 is the result of the increase in aggregate expenditure and how much is due to the increase in sectoral prices. To answer this question we calibrate \( \{e_s\}_{s=1}^S \) to match the initial sectoral distribution of private value added,\(^{24}\) and use the estimates of the income elasticities of demand \( \{\varepsilon_s\}_{s=1}^S \) and the elasticity of substitution across sectors \( \sigma \) reported in Table 3.

We set the initial values of \( E_t, P_t \) and \( p_{st} \) to match the values in 1980 of personal consumption expenditures (pce) per capita, the pce deflator, and the sectoral value added deflator for each of the eight sectors.

Then, we use equations (10) and (14) to conduct three different exercises. The first consists in computing the price level and the sectoral value added levels in 2016 if there was a neutral increase in expenditure of the same magnitude as the one we observed between 1980 and 2016. A neutral increase in expenditure is an increase in \( E_t \) equal to the observed increase in pce per capita accompanied by an increase in sectoral prices by the same constant by which the pce deflator has increased, so that relative prices do not change. The second exercise consists in computing the sectoral value added and the price index for 2016 if we change the relative sectoral prices by the magnitudes observed in the data.\(^{25}\) The third exercise simulates the effect on sectoral value added growth of conducting simultaneously the neutral increase in expenditure and the change in relative sectoral prices.

Figure 5 plots the sectoral growth rates induced by each of these exercises vs. the actual growth rates observed in the data. Table 4 reports the covariance between the sectoral growth rates observed in the data and those generated by each of these exercises (relative to the variance of actual sectoral growth). These exhibits yield two conclusions. The first is that the model

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\(^{24}\)By using information on sectoral value added (instead of expenditure) we take into account the input-output linkages from personal consumption.

\(^{25}\)To be precise, we change the price of each sector by the factor observed in the data relative to the factor by which the pce deflator increases. In this way relative prices change by the same amount as in the data.
accounts quite well for the changes in sectoral composition in US from 1980-2016. The covariance between total predicted changes in sectoral value added and those observed in the data from 1980 to 2016 is 106% of the variance of sectoral growth. The second lesson is that while the neutral increase in expenditures accounts for 73% of the actual sectoral growth, changes in sectoral relative prices account for only 6% of the actual variation in sectoral growth.

2.5 Drivers of the relative wage bill

Substituting (14) into equation (4), we obtain the following expression for the wage bill accrued by workers in occupation \( j \) as a function of total expenditure and prices,

\[
w_{jt}X_{jt} = \sum_{s \in S} \alpha_{jst}VA_{st} + \sum_{s \in S} \alpha_{jst} \beta_s E_{st}^{\sigma+\epsilon_s} p_{st}^{1-\sigma} p_t^{-\epsilon_s}.
\]
Expression (20) allows us to study the role of non-homotheticities in the evolution of the wage bill across occupations. Because our model takes the factor intensity, $α_{jst}$, as given, we first focus on how the model affects the wage bill through the reallocation of value added across sectors. That is, Term 1 in the decomposition in Equation (6). Additionally, non-homotheticities also affect wage bills through the covariance between the growth in factor intensities and sectoral growth in value added (Term 3). To capture this effect, we also conduct a second analysis where we study the role of non-homotheticities in the evolution of the all the wage bill for each occupation.

**The Drivers of Term 1** We first explore how well the model accounts for the observed contribution of sectoral reallocation to the growth in the wage bills across occupations. To this end, we simulate the evolution of value added in the model after feeding in the observed increase in personal consumption expenditures and the observed sectoral prices from 1980 to 2016 and multiply it by the initial intensities of each occupation in each sector, $α_{j1980}$. This comparison is presented in the first two rows of table 5. The model accounts for a majority of the observed growth in wage bills induced by sectoral value added growth. The share accounted by the model ranges from 78% for the medium skill occupation to 101% for the low-skilled occupations. The model more than fully accounts for the growth in relative wage bills induced by sectoral growth.

The second exercise we conduct consists in exploring how much of this variation is induced by the neutral increase in personal expenditures and how much by the relative growth in sectoral prices. To this end, we compute the growth in the wage bills of the different occupations after simulating (alternatively) the neutral increase in expenditure we have observed or the increase in relative sectoral prices. Rows 3 and 4 of table 5 report the results. As one would expect from our findings on the relative importance for expenditures and prices for sectoral growth, the former is much more important than the former also for the growth in wage bills and for the evolution of the relative wage bills. Splitting evenly the interaction, we find that

\[26\text{By interaction, here, we refer to the differential growth in the wage bill between the case when we simultane-} \]
the contribution of the neutral expenditure for the growth in the wage bill ranges from 92% for the low skilled occupation to 105% for the medium skill. The neutral increase in aggregate expenditures generates 80% of the growth in the relative wage bills generated by the model. Therefore, we conclude that non-homotheticities have played a first order role in the observed evolution of relative wage bills across occupations in the US between 1980 and 2016.

The Drivers of Occupational Wage Bill  We now move beyond Term 1 and use expression (20) to study the drivers of growth the (entire) wage bill for each occupation. In addition to the neutral increase in expenditures and the change in relative sectoral prices, we consider the change in occupational intensities, $\alpha_{jst}$. We first simulate the evolution of the wage bills after feeding in the observed increase in neutral expenditures, relative prices and occupational intensities. This calculation is reported in row 2 of Table XX. This simulation shows that the model does a good job in accounting for the patterns of wage bill growth observed in the data (row 1), both in terms of the absolute and relative growth of teh wage bills across occupations. Rows 3 and 4 presents the simulated growth in the wage bills after feeding in the increase in neutral expenditures (row 3) and the change in relative sectoral prices and the occupational intensities (row 4). Row 5 reports the contribution of the neutral increase in expenditure to the overall growth in the wage bill produced.\footnote{When computing the contribution, we evenly split the combined effect of both changes on the wage bill growth as described in Appendix C.} The key take away from this exercise is that roughly the neutral increase in expenditures explains roughly 50% of the change in the relative wage bill generated by the model comes from the neutral increase in expenditure. Therefore, even in a simplified context with a unitary household assumption and with exogenous sectoral prices, the direct effect of the increase in aggregate expenditures through non-homotheticities

<table>
<thead>
<tr>
<th>Wage Bill Growth from actual sectoral Growth (Term 1)</th>
<th>H</th>
<th>M</th>
<th>L</th>
<th>H-M</th>
<th>L-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage Bill Growth from Demand Model</td>
<td>7.05</td>
<td>4.66</td>
<td>7.09</td>
<td>2.39</td>
<td>2.43</td>
</tr>
<tr>
<td>Neutral Expenditure</td>
<td>6.2</td>
<td>3.63</td>
<td>7.16</td>
<td>2.57</td>
<td>3.53</td>
</tr>
<tr>
<td>Growth in sectoral prices</td>
<td>5.64</td>
<td>3.91</td>
<td>6.24</td>
<td>1.73</td>
<td>2.33</td>
</tr>
<tr>
<td>Model/Data Contriution</td>
<td>0.08</td>
<td>-0.06</td>
<td>0.16</td>
<td>0.14</td>
<td>0.21</td>
</tr>
<tr>
<td>Neutral increase in Expenditure</td>
<td>0.88</td>
<td>0.78</td>
<td>1.01</td>
<td>1.08</td>
<td>1.46</td>
</tr>
<tr>
<td>Contribution on Expenditure</td>
<td>0.95</td>
<td>1.05</td>
<td>0.92</td>
<td>0.81</td>
<td>0.8</td>
</tr>
</tbody>
</table>
in demand is a major driver of the relative wage bill. Next, we extend our analysis to our full-blown model that in addition to being more realistic incorporates new channels and predictions for the evolution of relative wages and hours worked.

3 Model with Endogenous Labor Supply and Prices

To further investigate the role of non-homotheticities in the polarization of labor markets, it would be desirable to go beyond wage bills and to study the differential effects of non-homotheticities in the number of jobs and the wage rates earned by workers. To pursue these goals, we extend our simple model in several directions. First, we explicitly model the labor supply of workers for the different occupations so that our model has predictions not only for the evolution of the wage bill but also for the relative wages and employment across occupations. Second, we allow for an exogenous varying labor share. Third, we introduce a non-unitary treatment of household consumption decisions.

3.1 Environment

3.1.1 Production

There is a representative firm in each sector that produces final output according to

\[ Y_{st} = A_{st} K_{st}^{1-\beta_{st}} \prod_{j \in \{H,M,L\}} \tilde{X}_{jst}^{\alpha_{jst}}, \]  

(21)

where \( \tilde{X}_{jst} \) denotes the number of efficiency units of labor are employed in occupation \( j \) in sector \( s \) in year \( t \) and \( 0 < \beta_{st} < 1 \) and \( \sum_{j} \alpha_{jst} = 1 \). The key difference is that we have now introduced capital in our model. As a result the labor share can vary over time and across sectors. Optimal demand from the representative firm implies

\[ \tilde{w}_{jt} \tilde{X}_{jst} = \beta_{st} \alpha_{jst} p_{st} Y_{st} \]  

(22)

\[ r_{t} K_{st} = (1 - \beta_{st}) p_{st} Y_{st} \]  

(23)

The total wage bill in sector \( s \) is

\[ \sum_{j=1}^{J} \tilde{w}_{jt} \tilde{X}_{jst} = \beta_{st} p_{st} Y_{st} \sum_{j=1}^{J} \alpha_{jst}, \]  

(24)

\[ \beta_{st} = \frac{\sum_{j=1}^{J} \tilde{w}_{jt} \tilde{X}_{jst}}{p_{st} Y_{st}}. \]  

(25)
Therefore, $\beta_{st}$ is the labor share in sector $s$.

The total number of efficiency units demanded for occupation $j$ are

$$X_{jt} \equiv \sum_{s=1}^{S} \bar{X}_{jst} = \sum_{s=1}^{S} \beta_{st} \alpha_{jst} p_{st} Y_{st} \overline{\bar{w}}_{jt}$$

(26)

and the wage bill required to compensate workers employed in occupation $j$ is

$$\bar{w}_{jt} \bar{X}_{jt} = \sum_{s=1}^{S} \beta_{st} \alpha_{jst} p_{st} Y_{st}$$

(27)

The total revenue accrued by capital owners is

$$r_{t} K_{t} = \sum_{s=1}^{S} (1 - \beta_{st}) p_{st} Y_{st}.$$  (28)

where we have allowed for sectoral heterogeneity in the rental rate of capital, $r_{st}$. Finally, the price of the sectoral output is

$$p_{st} = \left(\prod_{j=1}^{J} \frac{\alpha_{jst}}{\beta_{st}} \right)^{\frac{\beta_{s}}{\beta_{st}}} \left(\frac{r_{t}}{1 - \beta_{st}}\right)^{1 - \beta_{st}}.$$  (29)

### 3.1.2 Household preferences

There is a continuum of households indexed by $h$ from (0,1). Each household inelastically supplies a unit of labor to one of the three occupations. Household income is composed of the labor income plus the rental income accrued from the capital it owns ($K_{ht}$). We assume that capital is evenly distributed across households, and that every period household expenditure, $E_{ht}$, equals household income.

The household demand for each of the $S$ good is such that each household maximizes its utility level $U_{ht}$ subject to the constraint that total expenditure cannot exceed its income $E_{ht}$, where $U_{ht}$ is implicitly defined by

$$\sum_{s \in S} \left( U_{ht}^{c_{ht}^{s}} \right)^{\frac{1}{\sigma}} c_{ht}^{s, -1} = 1.$$  (30)

Given a set of prices $\{p_{st}\}$ and total expenditure $E_{ht}$, household $h$ chooses

$$c_{ht} = \zeta \left( \frac{E_{ht}}{p_{st}} \right)^{\sigma} U_{ht}^{c_{ht}}$$  (31)
where \( E_{ht} \) is the total expenditure given by
\[
E_{ht}^{1-\sigma} = \sum_{s \in S} \zeta_s U_{ht}^{\sigma} E_{ht}^{1-\sigma}.
\]

As with homothetic CES, we can normalize one taste parameter \( \zeta_s = 1 \) and one income elasticity parameter \( \varepsilon_s = 1 \) for some \( s \). This uniquely defines a price index \( P_t \) and a real consumption index \( C_t \) for each household
\[
U_{ht} = \frac{E_{ht}}{P_{ht}} \equiv C_{ht}
\]
with
\[
P_{ht} = \left[ \sum_{s \in S} \left( \zeta_s p_{st}^{1-\sigma} \right)^{\chi_s} \left( x_{jst} E_{ht}^{1-\sigma} \right)^{1-\chi_s} \right]^{\frac{1}{\sigma}}
\]
where \( x_{jst} = p_{st} c_{jst} / E_{ht} \) denotes the expenditure share in sector \( s \), and \( \chi_s = (1 - \sigma) / \varepsilon_s \).

Given these household-level consumption decisions, the aggregate demand for sectoral output is
\[
C_{st} = \int \zeta_s E_{ht}^{\varepsilon + \varepsilon_s} p_{st}^{-\varepsilon} P_{ht}^{-\varepsilon} dh.
\]

### 3.1.3 Occupational Choice

A key difference with the simple model is that now we allow each household to choose an occupation. Each household draws a vector \((\eta_H, \eta_M, \eta_L)\) of efficiency units in each occupation. Given the vector of market prices per efficiency unit for the three occupations \((\bar{w}_H, \bar{w}_M, \bar{w}_L)\), households choose optimally their occupations by maximizing their earnings. Formally, \( \max \{ \eta_j \bar{w}_j \} \)
\[
W_{ht} = \frac{E_{ht}}{P_{ht}} \equiv C_{ht}
\]
with
\[
P_{ht} = \left[ \sum_{s \in S} \left( \zeta_s p_{st}^{1-\sigma} \right)^{\chi_s} \left( x_{jst} E_{ht}^{1-\sigma} \right)^{1-\chi_s} \right]^{\frac{1}{\sigma}}
\]
where \( x_{jst} = p_{st} c_{jst} / E_{ht} \) denotes the expenditure share in sector \( s \), and \( \chi_s = (1 - \sigma) / \varepsilon_s \).

Let \( F(\eta_H, \eta_M, \eta_L) \) be the CDF of the joint distribution of the efficiency units across occupations. We have that the density of a household choosing occupation \( H \) is
\[
\frac{\partial F(\eta_H, \eta_M, \eta_L)}{\partial \eta_H}
\]
where \( F_H = \frac{\partial F(\eta_H, \eta_M, \eta_L)}{\partial \eta_H} \). Thus, the share of households choosing occupation \( H \) is
\[
\pi_H = \int_{y \in Y} F_H \left( \frac{\bar{w}_H y}{\bar{w}_M y}, \ldots, \frac{\bar{w}_H y}{\bar{w}_L y} \right) dy
\]
where $\mathcal{Y}$ denotes the support of the distribution for $\eta_H$.

The supply of efficiency units in occupation $H$ is

$$X_{jt} = \int_{y \in \mathcal{Y}} y F_H \left( \frac{\bar{w}_H}{\bar{w}_M} y, \ldots, \frac{\bar{w}_H}{\bar{w}_L} y \right) dy$$

(38)

and the wage bill accrued by workers in occupation $H$ is

$$\bar{w}_H X_{jt} = \bar{w}_H \int_{y \in \mathcal{Y}} y F_H \left( \frac{\bar{w}_H}{\bar{w}_M} y, \ldots, \frac{\bar{w}_H}{\bar{w}_L} y \right) dy$$

(39)

Analogous expressions to (37), (38) and (39) hold for the other occupations.

To evaluate quantitatively the model, we consider the particular case where the $\eta_j$'s are independent and distributed log-Normal with the mean and standard deviation of log $\eta_j$ being $\mu_j$ and $\sigma_j$.\textsuperscript{28}

### 3.2 Competitive Equilibrium

We focus our analysis on the competitive equilibrium of this economy. A competitive equilibrium is defined by a sequence of prices $\{\{p_{st}\}_{s \in S}, \bar{w}_{lt}, \bar{w}_{mt}, \bar{w}_{ht}\}_{t=0}^T$, allocations $\{\{c_{hst}\}_{s \in S, k \in H}\}$ and household occupational choices such that

1. Each household chooses the occupation that maximizes labor income, (35).

2. Household income equals household expenditure $E_{ht}$ period by period. Income is equal to labor income plus the return to capital (which is assumed to be uniform across households).

3. Each household maximizes utility (8) subject to the budget constraint $\sum_{s \in S} p_{st} c_{hst} = E_{ht}$.

4. Firms maximize profits taking prices as given, $\max p_{st} A_{st} K_{st}^{1-\beta_{st}} \prod_{j \in \{H,M,L\}} X_{jst}^{\alpha_{jst}} - \bar{w}_{lt} X_{lst} - \bar{w}_{mt} X_{mst} - \bar{w}_{ht} X_{hst}$.

5. Aggregate effective labor supply in an occupation (equation 38) equals aggregate demand (equation 26).

6. All goods markets clear at any point in time, $\int \ell_{hst} dh = Y_{st}$.

\textsuperscript{28}We note that assuming a Fréchet distribution (or a multi-variate Fréchet in the max-stable family as described in Lind and Ramondo, 2018) in this setting would have the counterfactual prediction that average wage per worker is equalized across occupations. Authors that have used the Fréchet distribution in this setting need to resort to unobserved costs or worker attributes.
3.3 Calibration

To study the drivers of the relative evolution of employment and wages across occupations we must calibrate two types of parameters: those that are fixed, and those that vary exogenously. To calibrate the first, we use data moments from 1980, while for the latter, we use data from 1980 and 2016 to determine their initial and final values, respectively.

Let’s start by discussing the calibration of the distributions of the productivity parameters \( \eta_j \). First note that the definition of an efficiency unit is arbitrary, therefore, without loss of generality, we can set \( \mu_j = 1 \). To calibrate \( \sigma_j \), we take advantage of the fact that our model is modular in the sense that conditional on the relative wage bill across occupations, the distribution of productivities determines the relative wages per worker across occupations, independently of the rest of parameters in the model. To better understand this property, note that equations (38) and (39) determine the wage bill for occupation \( j \) supplied by workers. Additionally, equation (37) determines the share of workers employed in each occupation. These three equations depend only on the distribution of productivities and on the equilibrium efficiency wages (\( \bar{w}_j \)). Therefore, we can proceed as follows. For any given \( \sigma_j \), the requirement that the relative wage bill supplied at each occupation matches the distribution of wage bills in 1980 pins down the equilibrium efficiency wages (\( \{ \bar{w}_j \} \)). The dispersion parameters \( \sigma_j \) can be calibrated by additionally requiring that the average wage per capita for \( h \) and \( l \) occupations relative to the average wage per capita for \( m \) occupations match the equivalent ratios observed in 1980.

There are two relevant observations worth noting. The first is that because we match the 1980 relative wage bill and average wage per workers across occupations, we will also match the employment shares across occupations in 1980. The second is that because we only use information on relative wage bills and relative wage per worker across occupations, without loss of generality, we can normalize \( \sigma_l \) to 1.

We calibrate the occupation intensities, \( \{ q_{jst} \} \), by matching the relative wage bill in each sector (equation 3) computed with wage information from the CPS and occupation information from the ACS. We calibrate \( \{ \beta_s \} \) by matching the sectoral labor share (equation 25) computed from the CPS, ACS and KLEMS. The preference parameters \( \{ \varepsilon_s \} \) and \( \sigma \) are estimated from the CEX as described in the Appendix. Given those, we calibrate the taste parameters, \( \{ \xi_s \} \), to match the aggregate value added share of each sector in 1980.

To study the drivers of the evolution of relative wages and employment shares, we consider exogenous changes in three parameters. The changes in the factor intensities, \( \{ a_{jst} \} \), and

---

29 We compute labor shares by computing wage bills using hours worked from the ACS and wages from the CPS. We calibrate the relative values of \( \{ \beta_s \} \) from the ratio of total wage bill taken from the ACS and CPS to nominal VA in each sector. While these values correctly capture the relative share of employment in each sector, the resulting level is too small. We then adjust the level of \( \{ \beta_s \} \) by multiplying all values of by \( \min(\text{labor compensation share in KLEMS}/\beta) \). To calibrate \( r \) we use the private sector lending rate from the IMF International Financial Statistics. For our baseline model we use the average value between 1980-2016. We then run robustness checks were we let it vary over time.
labor shares, \( \{\beta_{st}\} \), are directly measured in the data. Most importantly, we would like to implement the equivalent in this full-blown model to the neutral increase in aggregate nominal expenditure we have introduced in section 2. This requires the use of two of the parameters to induce the increases in aggregate expenditures and in the price level we have observed in the data.\(^{30}\) Since the model pins down the equilibrium value of relative wages, the level of wages is a free parameter. This allows us to exogenously induce changes in nominal variables by altering directly one of the wage rates, (e.g., \( \bar{w}_{lt} \)). The second instrument we use to induce the neutral increase in expenditure is aggregate TFP. Consequently, we set the increases in TFP and \( \bar{w}_{lt} \) to simultaneously match the increase in nominal personal consumption expenditure per capita and in the pce deflator observed in the US from 1980 to 2016. Note that these two variables have different effects on real and nominal expenditure because for given inputs, TFP only affects real income while \( \bar{w}_{lt} \) affects both. In our baseline calibration, we set the rental rate of capital equal to the average private sector lending rate from 1980-2016 as reported in the IMF International Financial Statistics.\(^{31}\)

3.4 Results

The results from our baseline analysis are reported in Table 6. The first two rows report the actual values of the key variables in the data. As before, we study the wage bills of the three occupations but also the relative wages and the share of hours worked in each of the three occupations. Row 3 reports the model simulations for 1980 which, by design, match the data. Rows 4 to 6 report the values of the variables of interest for 2016 produced by the model in different simulations. We consider three exercises. The neutral increase in expenditure (row 4), a simultaneous increase in the sectoral factor intensities and in the sectoral labor shares as the one observed in the data (row 5), and the effect of conducting simultaneously both exercises (row 6). Row 7 reports the fraction of the observed change in each of the variables from 1980 to 2016 that the model produces. Rows 8 and 9 report the contributions of the neutral increase in expenditure and the change in factor intensities and labor shares in the evolution of each variable using the approach detailed in section (D) in the appendix.\(^{32}\)

There are two key findings of the quantitative evaluation of our model. The first is that the model does a good job in generating the polarization of the labor markets both in terms of the evolution of the relative wages of high vs. medium and low vs. medium skill occupations,

\(^{30}\)To construct the Fisher price index we use as a target for our model we use nominal value added and price deflators for each sector provided by the BEA. We use the values from the baseline year 1980 and each target year to first construct the Laspeyres and Paasche index between the two periods. Then we construct the Fisher index as the geometric mean of the two. The key difference between this index and the one reported by the BEA is that we do not chain it over the years. We construct this alternative price index since our model simulation does not run for each of the years between the two periods we are interested in.

\(^{31}\)We conduct robustness checks using the initial and final values as well as allowing the rental rate to vary over time.

\(^{32}\)Note that these contributions are relative to the change induced by the model when all the exogenous variables change.
Table 6: Full Quantitative Model

<table>
<thead>
<tr>
<th>Year</th>
<th>$W_{L}/W_{M}$</th>
<th>$W_{H}/W_{M}$</th>
<th>$L_s$</th>
<th>$M_s$</th>
<th>$H_s$</th>
<th>$W_{L}/\sum W_{k}$</th>
<th>$W_{H}/\sum W_{k}$</th>
<th>$W_{M}/\sum W_{k}$</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td>0.068</td>
<td>0.630</td>
<td>0.302</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>0.80</td>
<td>1.49</td>
<td>0.129</td>
<td>0.488</td>
<td>0.383</td>
<td>0.088</td>
<td>0.421</td>
<td>0.491</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td>0.068</td>
<td>0.630</td>
<td>0.302</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>0.87</td>
<td>1.47</td>
<td>0.137</td>
<td>0.531</td>
<td>0.332</td>
<td>0.105</td>
<td>0.467</td>
<td>0.428 E</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>0.77</td>
<td>1.41</td>
<td>0.095</td>
<td>0.580</td>
<td>0.324</td>
<td>0.066</td>
<td>0.522</td>
<td>0.413 $\alpha+\beta$</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>0.88</td>
<td>1.58</td>
<td>0.128</td>
<td>0.494</td>
<td>0.378</td>
<td>0.093</td>
<td>0.410</td>
<td>0.497 E+\beta+\alpha</td>
<td></td>
</tr>
<tr>
<td>Fraction of observed change$^1$</td>
<td>2.33</td>
<td>1.36</td>
<td>0.97</td>
<td>0.96</td>
<td>0.96</td>
<td>1.25</td>
<td>1.05</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>Contribution of $E$</td>
<td>0.86</td>
<td>0.59</td>
<td>1.14</td>
<td>0.65</td>
<td>0.53</td>
<td>1.28</td>
<td>0.63</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>Contribution of $\alpha+\beta$</td>
<td>0.14</td>
<td>0.41</td>
<td>-0.14</td>
<td>0.35</td>
<td>0.47</td>
<td>-0.28</td>
<td>0.38</td>
<td>0.46</td>
<td></td>
</tr>
</tbody>
</table>

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

and in terms of the hollowing out of medium skill hours worked increasing the share of hours worked in high and low-skill occupations. Specifically, the model slightly over-predicts the 2016 relative wages of low vs. medium (0.87 vs. 0.8 in the data) and high vs. medium (1.57 vs. 1.49 in the data). It virtually nails the increase in the share of hours worked by each occupation (0.125 vs. 0.129 for low, 0.499 vs. 0.488 for medium and 0.376 vs. 0.383 for high-skill occupations). Consequently, the model virtually nails the 2016 wage bill distribution across occupations (0.091 vs. 0.088 for low, 0.416 vs. 0.421 for medium and 0.493 vs. 0.491 for high-skill occupations).

The second key finding from Table 6 is that the neutral increase in aggregate expenditures accounts for a remarkable share of the polarization in labor markets generated by our model both in terms of the evolution of relative wages and hours worked across all occupations. In particular, the neutral increase in expenditure accounts, respectively, for 85% and 55% of the increases in the low vs. medium and high vs. medium relative wages generated by the model. The neutral increase in expenditure is responsible for a share of the change in the share of hours worked generated by the model that ranges from 50% for high-skill occupations to 113% for low-skill occupations. Finally, the neutral increase in expenditures is responsible for a share of the change in the wage bill shares generated by the model that ranges from 51% for the high-skill occupation to 126% for the low skill occupation.

The relevance of the neutral increase in aggregate expenditure for labor market polariza-
tion is quite intuitive in the light of our findings in section 2. As we have documented, non-homotheticities in demand induce income elastic sectors to grow more in response to the increase in aggregate expenditure. These sectors are more intensive in high- and low-skill occupations hence the reallocation of the wage bill. The increase in labor demand for high and low-skill workers translates both into higher relative wages and a higher share of employment in high and low-skill occupations. The log-Normal distribution of productivity draws does a good job in splitting the increase in relative labor demand between changes in relative wages and in relative hours across occupations.

In addition to the direct effect through aggregate expenditure, non-homotheticities also induce labor market polarization indirectly through relative sectoral prices. In particular, because non-homotheticities induce relative increases in the wage per-efficiency unit of high and low-skill occupations, they lead to relative increases in the sectoral prices of sectors that are more intensive in these occupations. Since the elasticity of substitution of demand across sectors is smaller than 1, the increase in relative sectoral prices leads to an additional increase in the (nominal) GDP share of sectors that are intensive in high and low-skill occupations. Hence amplifying the direct effect of non-homotheticities.

3.4.1 Evolution of Polarization: Simulation by Decades

We conclude our analysis of the drivers of the wage polarization in the US from 1980 to 2016 by exploring the drivers in different subperiods. To this end, we simulate the evolution of model after gradually adjusting the exogenous variables. Table 7 reports the resulting evolution of the labor market variables and the contribution of the neutral increase in expenditure to the change of each variable for each of the two subperiods. This exercise allows us to better understand the relevance of both driving forces during each of the two subperiods. The key take away is that the neutral increase in expenditure was very relevant in the labor market polarization during both subperiods, however it was somewhat less important for low-skilled labor in the 2000-2016 period. More specifically, we find that the neutral increase in expenditure observed during the 1980-2000 period more than accounts for the evolution of relative wage and the share of hours for low-skill occupations, and it accounts for 70% and 61%, respectively, of the change in the model simulations for relative wages and the share in hours worked by high-skill occupations. In the 2000s, the contribution of the neutral expenditure decrease but it is still key for the model success in explaining the observed polarization of labor markets. It generates between 50% and 60% of the change in the relative wage, share of hours worked and wage bill for high and medium-skill occupations and around 35% for the low-skill occupations.

There are several reasons for the stronger role of the neutral increase in expenditures on the model polarization during the 1980-2000 period. First, both nominal and real expenditure per capita increased more during the 1980-2000 period than during the 2000-2016 period. Second, the intensity of low-skill occupations (in the sectors that make up most of their wage bill) was
Table 7: Model Simulation by Subperiods

<table>
<thead>
<tr>
<th>Year</th>
<th>$W_L/W_M$</th>
<th>$W_H/W_M$</th>
<th>$L_s$</th>
<th>$M_s$</th>
<th>$H_s$</th>
<th>$\sum W_L/\sum W_K$</th>
<th>$W_M/M$</th>
<th>$W_H/H$</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td>0.068</td>
<td>0.630</td>
<td>0.302</td>
</tr>
<tr>
<td>Data</td>
<td>2000</td>
<td>0.79</td>
<td>1.44</td>
<td>0.100</td>
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<td>0.324</td>
<td>0.070</td>
<td>0.514</td>
<td>0.416</td>
</tr>
<tr>
<td>Model</td>
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<td>0.84</td>
<td>1.41</td>
<td>0.128</td>
<td>0.559</td>
<td>0.313</td>
<td>0.097</td>
<td>0.504</td>
<td>0.399</td>
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<tr>
<td></td>
<td>2000</td>
<td>0.72</td>
<td>1.33</td>
<td>0.080</td>
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<td>0.79</td>
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<tr>
<td>Data</td>
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<td>0.129</td>
<td>0.488</td>
<td>0.383</td>
<td>0.088</td>
<td>0.421</td>
<td>0.491</td>
</tr>
<tr>
<td>Model</td>
<td>2016</td>
<td>0.82</td>
<td>1.48</td>
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<td>0.543</td>
<td>0.348</td>
<td>0.077</td>
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<tr>
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<td>2016</td>
<td>0.85</td>
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<td>0.499</td>
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Contribution of $E$

<table>
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<th>of $E$</th>
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<th>0.70</th>
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<th>5.88</th>
<th>0.82</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>00-16</td>
<td>0.33</td>
<td>0.30</td>
<td>0.28</td>
<td>0.30</td>
<td>0.33</td>
<td>0.25</td>
<td>0.31</td>
<td>0.32</td>
<td></td>
</tr>
</tbody>
</table>

higher in the 1980s than in the 2000s. As a result, the increase in expenditure during the 2000s did not lead to as large of an increase in demand for low-skill workers.\textsuperscript{33}

3.5 Extensions

After showing the relevance of non-homotheticities for wage polarization in our baseline calibration, next we show that the quantification we have just conducted is robust to reasonable alternatives in some of the choices we have made. First we bring into our analysis the gap between value added and expenditure that arises as a result of international trade. Second, we consider alternative distributions of capital across households. Finally, we explore in the Appendix the consequences of allowing for time-varying interest rates as another driving force for household income.

3.5.1 Accounting for International Trade

So far, our analysis has assumed a closed economy equilibrium. This section extends our analysis to account for the possibility to import and export goods in some sectors. We account for this possibility in a parsimonious way through the use of sector specific TFP shocks. (rather than fully modeling a world economy with multiple countries)

We start from the identity that aggregate production equals domestic consumption plus net

\textsuperscript{33}Additionally, the fact that the expenditure increase generates an excessive increase in the low-skill occupation wage induces a mechanical mean reversion in the contribution of aggregate expenditure.
exports, \( p_{st} Y_{st} = p_{st} C_{st} + NX_{st} \). Rearranging we have that

\[
p_{st} C_{st} = (1 - \tau_{st}) p_{st} Y_{st},
\]

where \( \tau_{st} \equiv \frac{NX_{st}}{p_{st} Y_{st}} \) captures the wedge between domestic production and consumption. If net exports are positive, we have a magnifying effect of trade on domestic production (and labor demand), while if they are negative they dampen domestic aggregate demand.

To obtain our measures of \( \{ \tau_{st} \} \), we proceed by computing US sectoral net exports in terms of value added over US value-added production (see details in Appendix E). We note that this wedge correction only affects two of our eight sectors: Agriculture, Mining and Utilities and Manufacturing. Quantitatively most of the action is on the manufacturing sector. We find that the wedge for manufacturing in 1980 is a small and positive number. The wedge becomes increasingly negative over the period 1980-2016, reflecting the increasing US trade deficit in manufactured products, reaching a value of -0.15 in 2016.

We re-do our baseline analysis imposing the observed wedges in the data. The wedges in the initial period \( \{ \tau_{s,1980} \} \) affect our calibration of \( \{ \zeta_s \} \) since now sectoral value-added shares are amended by the trade wedges as implied by Equation (40). We allow trade also to affect the effective change in demand in the final period of our simulations (see Appendix E for further details). Table 8 reports our simulation results. The main take away is that the key findings from our baseline calculations about the drivers of labor market polarization are completely unaffected by taking into account the effect of international trade. The neutral increase in aggregate expenditures still accounts for a remarkable share of the polarization in labor markets generated by our mode. In particular, it accounts, respectively, for 75% and 51% of the increases in the low vs. medium and high vs. medium relative wages generated by the model and between 47% to 93% of the change in the share of hours worked. Furthermore, the neutral increase in expenditures is responsible for a share of the change in the wage bill shares generated by the model that ranges from 51% for the high-skill occupation to 104% for the low skill occupation, which is strikingly close to the results from our closed economy model.

### 3.5.2 Allowing for sector-specific TFP growth

In our baseline production function we have not allow for sector-specific TFP growth. Note that, by affecting the evolution of relative prices, sectoral TFP growth can affect the composition of the economy and the evolution of relative wage bills. We now explore the sensitivity of our findings to including this channel in the analysis. In particular, we generalize the production function by introducing a sector-specific TFP growth rate that we calibrate by matching the observed growth in relative prices in each sector between 1980 and 2016. Table 9 contains the results. Row 6 reports the simulated variables after feeding in the neutral increase in expenditures, the changes in factor intensities (\( \alpha_{jst} \) and \( \beta_{jst} \)), and the evolution of sectoral TFP.
The model continues to do well in reproducing the evolution of the occupational wage bill and its components. Rows 4 and 5 report the simulations after feeding in the neutral increase in expenditure (row 4) and the changes in factor intensities and sectoral TFP (row 5). Row 8 reports the contribution of the neutral increase in expenditure to the model generated changes in occupational wage bill and its components. The main take away from this exercise is that allowing for sector-specific TFP growth does not affect in any way the role that non-homotheticities have in the reallocation of the wage bill and its components across occupations.

3.5.3 Assigning all capital income to high-skill households

Next, we study the robustness of our findings to alternative assumption about the ownership of capital. Given that most capital is owned by the richest households, a more realistic assumption about the capital ownership distribution is that capital is fully owned by the workers in high-skill occupations.34

Table 10 reports our simulation results and shows that our key findings are robust to alternative assumptions about the ownership of capital in the economy. Both the total change in labor market polarization generated by the model and the contribution by the neutral increase in expenditures do not differ much from those in the baseline calibration. Overall, the model

### Table 8: Model Simulation with Trade

<table>
<thead>
<tr>
<th>Year</th>
<th>( \frac{W_L}{W_M} )</th>
<th>( \frac{W_H}{W_M} )</th>
<th>( L_s )</th>
<th>( M_s )</th>
<th>( H_s )</th>
<th>( \frac{\sum W_i}{W_M} )</th>
<th>( \frac{\sum W_i}{W_M} )</th>
<th>( \frac{\sum W_i}{W_M} )</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td>0.068</td>
<td>0.630</td>
<td>0.302</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>0.80</td>
<td>1.49</td>
<td>0.129</td>
<td>0.488</td>
<td>0.383</td>
<td>0.088</td>
<td>0.421</td>
<td>0.491</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>0.88</td>
<td>1.48</td>
<td>0.139</td>
<td>0.524</td>
<td>0.337</td>
<td>0.107</td>
<td>0.457</td>
<td>0.436</td>
<td>( E )</td>
</tr>
<tr>
<td>2016</td>
<td>0.79</td>
<td>1.43</td>
<td>0.101</td>
<td>0.571</td>
<td>0.328</td>
<td>0.071</td>
<td>0.510</td>
<td>0.419</td>
<td>( \alpha + \beta + \tau )</td>
</tr>
<tr>
<td>2016</td>
<td>0.90</td>
<td>1.60</td>
<td>0.135</td>
<td>0.482</td>
<td>0.384</td>
<td>0.099</td>
<td>0.396</td>
<td>0.505</td>
<td>( \alpha + \beta + \tau )</td>
</tr>
<tr>
<td>Fraction of observed change(^1)</td>
<td>2.67</td>
<td>1.44</td>
<td>1.18</td>
<td>1.04</td>
<td>1.01</td>
<td>1.55</td>
<td>1.12</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td>Contribution of ( E )</td>
<td>0.78</td>
<td>0.57</td>
<td>0.98</td>
<td>0.64</td>
<td>0.53</td>
<td>1.08</td>
<td>0.61</td>
<td>0.54</td>
<td></td>
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<tr>
<td>Contribution of ( \alpha + \beta )</td>
<td>0.22</td>
<td>0.43</td>
<td>0.03</td>
<td>0.36</td>
<td>0.47</td>
<td>-0.08</td>
<td>0.39</td>
<td>0.46</td>
<td></td>
</tr>
</tbody>
</table>

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

34Naturally, we assume that workers do not take into account the assignment of capital to high-skill occupations when making their occupational choice.
Table 9: Model Simulation with Sector Specific TFP Growth

<table>
<thead>
<tr>
<th>Year</th>
<th>WL</th>
<th>WM</th>
<th>WH</th>
<th>Ls</th>
<th>Ms</th>
<th>Hs</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.80</td>
<td>1.49</td>
<td>0.129</td>
<td>0.488</td>
<td>0.383</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.83</td>
<td>1.40</td>
<td>0.125</td>
<td>0.565</td>
<td>0.310</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.83</td>
<td>1.49</td>
<td>0.114</td>
<td>0.539</td>
<td>0.347</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.90</td>
<td>1.62</td>
<td>0.136</td>
<td>0.476</td>
<td>0.388</td>
<td></td>
</tr>
<tr>
<td>Fraction of observed change</td>
<td>2.67</td>
<td>1.52</td>
<td>1.21</td>
<td>1.07</td>
<td>1.04</td>
<td>1.60</td>
<td>1.15</td>
</tr>
</tbody>
</table>

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

Table 10: Model Simulation with Capital Ownership by High-Skilled Households

<table>
<thead>
<tr>
<th>Year</th>
<th>WL</th>
<th>WM</th>
<th>WH</th>
<th>Ls</th>
<th>Ms</th>
<th>Hs</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.80</td>
<td>1.49</td>
<td>0.129</td>
<td>0.488</td>
<td>0.383</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.84</td>
<td>1.41</td>
<td>0.126</td>
<td>0.562</td>
<td>0.312</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.76</td>
<td>1.40</td>
<td>0.091</td>
<td>0.590</td>
<td>0.319</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.83</td>
<td>1.51</td>
<td>0.115</td>
<td>0.529</td>
<td>0.356</td>
<td></td>
</tr>
<tr>
<td>Fraction of observed change</td>
<td>1.67</td>
<td>1.08</td>
<td>0.65</td>
<td>0.76</td>
<td>0.79</td>
<td>0.80</td>
<td>0.85</td>
</tr>
</tbody>
</table>

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.
generates a slightly smaller polarization in labor markets because by assigning all capital income to the workers in high-skill occupations, their share in expenditure increases, and we need a smaller increase in $\tilde{w}_H$ to match the observed increase in the pce deflator from 1980 to 2016. Despite this, the neutral increase in expenditures accounts, respectively, for 89% and 50% of the increases in the low vs. medium and high vs. medium relative wages generated by the model and between 45% to 133% of the change in the share of hours worked. Furthermore, the neutral increase in expenditures is responsible for a fraction of the change in the wage bill shares generated by the model that ranges from 46% for the high-skill occupation to 164% for the low skill occupation, which is quite close to the results from our closed economy model.

4 Additional Exercises

The forces introduced by our model – non-homotheticities that induce changes in the sectoral composition of the economy and labor demand – have surely been relevant in other periods of history and in other geographies. In this section we explore this possibility. Specifically, we study the role of non-homotheticities in explaining labor market dynamics in the US during the period 1950-1980 and the polarization of labor markets in other advanced economies during the period 1980-2016. We conclude our analysis by using our framework to look into the evolution of distributional labor market outcomes over the next 15-20 years.

4.1 Back-tracking 1950-1980

Now that we have used our model to better understand the drivers of US labor market polarization from 1980 to 2016, a natural question to ask is what drove the evolution of wages and hours worked across occupations during period 1950-1980. This exercise is particularly interesting because much of the job polarization debate concerns the post-1980 period with the implicit assumption that some new developments triggered the polarization in labor markets. Yet, as documented by Siegel and Barany (), the labor market polarization dates back, at least to 1950 (See rows 1 and 2 in Table 11). Therefore, by conducting the back-tracking simulation we can study whether the same forces that have been important in the polarization wave from 1980 to 2016 may be responsible for the rise of the middle class we observed from 1950 to 1980 in the US.

To conduct our simulation, we use the same values for the preference parameters that we have used in section 3.4. As in our baseline calculation, we measure directly the sectoral intensity of each occupation and the labor shares. We calibrate the neutral increase in expenditure so that the model’s level of aggregate expenditure per capita and the Fisher price index match the levels observed in 1950.

Rows 3-5 of Table 11 contain the labor market outcomes generated by the model for 1950 in response to the neutral decline in expenditure (row 3), the change in factor intensities and
### Table 11: Model Simulation 1950-1980

<table>
<thead>
<tr>
<th>Year</th>
<th>Data</th>
<th>Model</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ls</td>
<td>Ms</td>
<td>Hs</td>
</tr>
<tr>
<td>1950</td>
<td>0.07</td>
<td>0.70</td>
<td>0.106</td>
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<tr>
<td>1950</td>
<td>0.79</td>
<td>1.17</td>
<td>0.075</td>
</tr>
<tr>
<td>1950</td>
<td>0.72</td>
<td>1.06</td>
<td>0.075</td>
</tr>
<tr>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
</tr>
</tbody>
</table>

| Fraction of observed change\(^\text{1}\) | 0.50 | 2.00 | 0.64 | 0.96 | 0.92 | 1.00 | 1.06 | 1.05 |
| Contribution of E | 3.25 | 0.53 | -2.86 | 0.76 | 0.45 | -2.64 | 0.67 | 0.47 |
| Contribution of α+β | -2.25 | 0.47 | 3.86 | 0.24 | 0.55 | 3.64 | 0.33 | 0.53 |

\(^1\) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

Intuitively, between 1950 and 1980 there was significant sectoral variation in value added labor shares (row 4), and simultaneously conducting both types of changes in the exogenous variables (row 5). The first observation we can draw is that the model accounts quite well for the change in labor market outcomes from 1950 to 1980 (row 7). In particular, it over predicts the change in the relative wage of high-skill occupations but under predicts the relative wage of low-skill occupations. It accounts for a significant share of the change in the share of hours worked in middle and high-skill occupation with values of 107% and 104% respectively, and somewhat less for low-skill occupations with a value of 45%. The model also provides a good account of the changes in the occupational wage bills which represent from 71% of the observed change for low-skill to 107% for medium skill occupations. Looking specifically at the contribution to the model performance of the neutral change in aggregate expenditure, we find that it plays an important role in all the dimensions other than for the evolution of the share of hours worked and wage bill accrued by low-skill workers. In particular, for high-skill occupations, it accounts for approximately 50% of the model’s increase in the relative wage, share of hours worked and wage bill from 1950 to 1980. For medium skill occupations, the neutral increase in expenditure accounts for 77% of the model generated change in hours worked and 69% of the wage bill. For the low-skill occupations, the model accounts for all the increase in relative wages, but it does not account for any of the decline in the share of hours worked and in the wage bill.
growth. As in our baseline period, there is a strong correlation between sectoral growth and the sectoral share in employment for high and low-skill occupations. Additionally, the model does a great job in predicting sector level growth from 1950 to 1980 (See panel A in Figure x). This is largely due to the differential effect of the neutral increase in expenditure on value added across sectors. In particular, the covariance between actual nominal sectoral growth and sectoral growth produced by the model in response to the neutral increase in expenditure represents 97% of the variance of actual sectoral growth between 1950 and 1980. Hence, the model’s ability to capture the evolution of the wage bill distribution across occupations. As in the post 1980- period, our model would split the evolution of the wage bill between relative wages and the distribution of hours worked. This works for medium and high-skill occupations but not for low-skill occupations that experience an increase in relative wages but a decline in the share of hours worked. Given the simplicity of our job assignment model, no single driving force can explain this negative co-movement between hours worked and wages of an occupation. So, how is it that the model does a great job in explaining both? The explanation is that while the neutral expenditure force induces an increase in the wage bill of low-skill occupations, during the 1950-80 period, the intensity in production of low skill workers declined causing a reduction in their demand. This second force tends be dominant for hours worked and the wage bill but not for the relative wages of low-skill occupations. Hence the model ability to explain the 1950-80 changes in labor outcomes for all occupational groups.

4.2 Polarization in Other Economies

European countries have experienced labor market polarization processes similar to the U.S. Goos et al., 2014 have documented the polarization of hours worked from middle-skilled to low- and high-skilled occupations in various European countries during the period 1990-2010. In this subsection we explore the relevance of non-homotheticities in demand in the evolution of distributional outcomes in European labor markets. Prior to studying the model implications, we extend Goos et al., 2014 by documenting the polarization of hourly wages and in the wage bill. Due to data consistency considerations, we focus our analysis in five large countries (Germany, France, UK, Italy and the Netherlands) and the period 2005 to 2015.35 Our data analysis shows a strong polarization in hourly wages in Netherlands, Germany and France and to a smaller extent in Italy and the UK (See Tables 14-18). As in Goos et al., 2014, we find a polarization of hours worked for our sample of countries and period with the exception of high-skilled occupations in Italy whose share in the total hours worked declined slightly from 38.8.

To study the importance of non-homotheticities, we calibrate the occupation intensities \( \{ q_{ijstc} = \{j,m,k\} \} \) by matching the relative wage bill in each country computed with wages taken from the SILC and hours worked from the LFS microdata. We calibrate

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35See Appendix B.3 for details on the data construction and sources.
by matching the sectoral labor share computed from the SILC, LFS and KLEMS
as before. For each country we re-calibrate the taste parameters \( \{ \zeta_s \}_{s=1} \) to match the aggregate value added shares of each sector in 2005, where value added shares are taken from KLEMS. Similarly, we re-calibrate the productivity dispersion in the job assignment model for each country to match the 2005 relative hourly wages.

Findings

4.3 Looking into the future of US labor markets: 2016-2035

We conclude our analysis by looking at the future and using the model to forecast the state of labor markets around 2035. To this end, we simulate our model economy if starting in 2016 there was a neutral increase in aggregate expenditures similar to half of that observed from 1980 to 2016. Note that in our simulations we assume that factor intensities and labor shares remain constant at the 2016 levels. Our model predicts a continuation of the labor market polarization. The relative of low-skill occupations increases from 0.8 to 0.93, while the relative wage of high-skill occupations increases from 1.49 to 1.7. The share of hours worked for low and high-skill occupations increases, respectively from 0.12 to 0.142 and from 0.384 to 0.414, while the share of hours worked in medium skill occupations declines by 5 percentage points. These changes in wages and hours worked imply a reallocation of the wage bill from medium skill to low- and especially to high-skill occupations. Specifically, the wage bill of medium skill occupations declines by 8 percentage points, while the wage bill of low- and high-skill occupations increases, respectively, by 2 and almost 6 percentage points.

The magnitude of these changes are quite significant both in absolute terms and compared to the polarization observed during the 1980-2016 period. This is especially the case for relative wages which our model predicts will increase by proportionately more than in the 1980-2016 period. The share of hours worked and the wage bill for low-skill occupations will also increase by proportionately more from 2016-2035 than in 1980-2016. However, the shares of hours worked and the wage bills for medium and high-skill occupations will increase by between 22% and 38% of the overall increase in 1980-2016. Though these are very significant increases, they suggest that the pace at which the wage bill has been reallocated from medium to high-skill occupations will slow down slightly.

5 Conclusion

This paper makes two contributions. First, it documents a positive correlation between sectoral income elasticity and low- and high-skill occupation intensity. Based on this fact, we propose a demand-driven labor market polarization mechanism: as income grows and demand shifts to high-income-elastic sectors, the relative demand of high- and low-skilled workers increases.
We quantify this mechanism in a multi-sector general equilibrium framework and find that it accounts for a substantial part of the US labor market polarization.
References


### A Tables and Figures

Table 13: Wage Bill Decomposition by Decades

<table>
<thead>
<tr>
<th></th>
<th>H-M</th>
<th>L-M</th>
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</thead>
<tbody>
<tr>
<td>1980-2000: Overall</td>
<td>3.26</td>
<td>1.10</td>
</tr>
<tr>
<td>Incr Sect Shares</td>
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<td>0.73</td>
</tr>
<tr>
<td>Incr alpha</td>
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<td>0.09</td>
</tr>
<tr>
<td>Cov</td>
<td>2.09</td>
<td>0.28</td>
</tr>
<tr>
<td>2000-2016: Overall</td>
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<td>2.33</td>
</tr>
<tr>
<td>Incr Sect Shares</td>
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<tr>
<td>Incr alpha</td>
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</tr>
<tr>
<td>Cov</td>
<td>1.86</td>
<td>0.51</td>
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</table>

### B Data Description

In this section we briefly detail our data sources. We take it from previous work and, thus, we provide a relatively brief description and relegate the interested reader to the original papers for further details.

#### B.1 Labor-Market Outcomes

We follow Acemoglu and Autor (2011) in the construction of the baseline data on occupations, wages and employment shares. Here we provide a brief overview and refer the reader to the original work by Acemoglu and Autor for the details. The data for employment comes from IPUMS USA and it includes the decennial censuses between 1980-2000 (with 10 years intervals) and annual data from the American Community Survey (ACS) between 2000-2007. The sample is restricted to individuals aged 16-64 who were employed in the previous year and are assigned to a known occupation (i.e., not n/a or unemployed). We further restrict the sample to exclude the top and bottom 5% of the hourly wage distribution. Wage data comes from the Current Population Survey (CPS) to compute wages per occupation. We follow Acemoglu and Autor on this choice because the data in the ACS is not consistent across years.

36 Occupations and industries are classified based on the 1990 Census Bureau classification scheme that is consistent for all the sample years. These industries are mapped to the BEA industry classification through mutual mapping to the NAICS codes. For each BEA industry, we compute the share of individuals within each occupation.

36 Specifically, weeks worked last year has only intervals starting in 2007.
Our occupation classification is also taken from Acemoglu and Autor (2011). They divide the 382 original occupations into 4 broader categories that are characterized by their skill level: (1) managerial, professional and technical occupations; (2) sales, clerical and administrative support occupations; (3) production, craft, repair and operative occupations; and (4) service occupations. The first group is characterized by high skill occupations, the second and third groups are characterized by middle skill occupations and the last group is characterized by low skill occupations. They measure skill by the average hourly wage of individuals in the occupation in 1980 where the mean wage in each occupation is calculated using workers’ hours of annual labor supply times the Census sampling weights.\textsuperscript{37}

\textbf{B.2 Household Expenditure Data}

We use cross-sectional data on household expenditure from the Consumer Expenditure Survey (CEX) to estimate the elasticity parameters of our nonhomothetic CES demand system, \(\{\sigma, \epsilon_s\}_s \in S\). We use data from the 2000-2007 period.\textsuperscript{38} We follow the procedure described in

\textsuperscript{37}Our results on the negative correlation between occupation shares in middle-skill workers and income elasticity parameters are robust to decomposing middle-skill between groups (2) and (3). We report these correlations in Table ?? in the appendix.

\textsuperscript{38}We have experimented with different time frame periods, between 1999 and 2007. The estimates are very stable across subsamples. Aguiar and Bils (2015) also report a similar finding in their estimated income elasticities.
Aguiar and Bils (2015) to clean the data. In particular, we restrict our sample to urban households with ages of the reference person between 25 and 64. We drop households if they report spending less than 100 dollars in food in 3 months per individual in the household, they have negative total or food consumption expenditure, total income is reported incomplete, they have not responded all (four quarterly) interviews, income is below 50% of minimum wage or if they earn money but do not work. To mitigate measurement error concerns, we drop the top and bottom 5% richest households according to their total income (after taxes) and we winsorize top and bottom 5% sectoral expenditures. We then follow the procedure described in Buera et al. (2015) and convert the final good expenditures reported in the CEX into value added expenditures using the BEA’s 2000 input-output tables. We do so by matching the finest level of expenditure categories in the CEX (called UCCs) to each sector in the BEA table. Following Comin et al. (2015), we also use sectoral, regional urban price series provided by the BLS for our estimation of price elasticities.

**B.3 European Countries Data**

We use microdata from the LFS and SILC in order to estimate the wages, hours worked and implied $\alpha_{jstc}$. Specifically we use the LFS data to calculate the hours worked by each skill level in each sector. In order to do so we map the NACE Rev 1.1 and 2 to our sector classification using NACE- NAICS correspondence tables provided by Eurostat. For occupation classification we use the one provided by Goos et al., 2014. This classification differs from the US classification by including Laborers in mining, construction, manufacturing, and transport as well as Models, salespersons, demonstrators and elementary sales occupations in the low skill employees definition. This classification also leaves out the following occupations: legislators and senior officials, teaching professionals and teaching associate professionals, skilled agricultural and fishery workers and agricultural, fishery and related laborers. We focus in individuals that are employed, are not family workers between the ages 16-64. Next, we use the SILC data to calculate the mean wage for each skill type across all industries. We use the same sample restrictions and skill classification. We calculate the wage per hour by dividing the monthly or annual labor income, depending on data availability, by the hours worked in the relevant period. For this purpose we use usual weekly hours worked, multiplied by number of months worked in a year and assuming individuals worked 4 weeks in each month since this data is not directly provided. To make sure the annual labor income was

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39 Total income after taxes is computed as in Aguiar and Bils (2015).
40 We use the correspondence in Buera et al. (2015). We extend the correspondence for the few cases in which there are UCCs from 2000-2007 missing from their original list.
41 When possible, we create a household-specific Stone price index for each sector from more disaggregated possible price series categories that belong to each sector. We then also convert final expenditure prices to value added prices by assuming a Cobb-Douglas production function and perfect competition, such that the log price of a sector is the input-share weighed mean of log-prices.
earned while working in the current occupation we further restrict the sample to individuals that did not switch work since the previous year. For total labor compensation share and value added share in each sector we use EU KLEMS data.

C  Estimation of Demand Elasticities

C.1  Cross-Country Regressions

We also estimate expenditure elasticities using a cross-country panel for OECD countries from KLEMS. In particular, we run an analogous version to the Aguiar-Bils specification,

\[ \ln \frac{x^n_{st}}{\bar{x}_{st}} = \alpha_{st} + \eta_s \ln E^n_t + \Gamma_s Z^n_t + \delta_n + \delta_s + u^n_{st} \]  

(41)

where \( x^n_{st} \) denotes value-added in sector \( s \), country \( c \) and year \( t \), \( \bar{x}_{st} \) denotes the average across \( n \), \( E^n_t \) is total value added in \( n \) and year \( t \), \( Z^n_t \) are country controls (log employment compensation and log total number of employees in the sector) and \( u^n_{st} \) is an error term. Finally, \( \alpha_{st} \) absorbs prices in the first order approximation in Aguiar and Bils. We use the log-price deflator of sector \( s \), and the CES price index with time-varying expenditure shares in the sector and country as weights and price elasticity of 0.5 (which corresponds to the micro elasticity).\(^{42}\)

We merge the KLEMS data with a time series for oil price shocks from Ramey and Vine (2011). The resulting dataset spans 1995-2012. We construct a price and total value-added instruments by using the time series of oil prices, oil prices squared and interacting the oil price time series with sectoral factor intensity in energy and sectoral factor intensity squared.\(^{43}\) We instrument total value added in a country with the oil price shocks from Ramey and Vine. Table 14 reports our estimated expenditure elasticities. We find the same broad ranking and a similar range of the expenditure elasticities to our household estimates. The correlation with the household estimates is 0.73. The only outlier is FIRE and other services which have a negative point estimate but appear to be very imprecisely estimated.

D  Calculation of contribution of different exogenous factors to the evolution of an endogenous variable

In this section we briefly describe how we calculate the contribution to the evolution of a given variable when there are multiple factors that impact it. For simplicity, consider the case where there are two exogenous drivers \( (a \) and \( b \)) for the endogenous variable \( v \). Our model provides a\(^{42}\)

\[^{42}\text{That is, the price index we use is } \left( \sum_s x^n_{st} \left( \frac{p^n_{st}}{p^n_{st}} \right)^{1-\sigma} \right)^{1/\sigma}. \text{ We have also experimented with using other price indices, such as the Stone or log of a linear aggregator and obtain very similar results.}\]

\[^{43}\text{We use the US input-output table for this and use the share of sector 211 (oil and gas extraction) in 1997.}\]
mapping between the exogenous vector \((a, b)\) and \(v\). In a slight abuse of notation, we denote this mapping also by \(v(.)\). The change in \(v\) between times 0 and \(F\) can be expressed as

\[
\Delta v = v(a_F, b_F) - v(a_0, b_0),
\]

where we have used subscripts \(F\) and 0 to denote the times.

We are interested in decomposing the change in \(v\) between the contribution from \(a\) and \(b\). Note that \(\Delta v\) can be decomposed in two different ways:

\[
v(a_F, b_F) - v(a_0, b_0) = v(a_F, b_F) - v(a_0, b_F) + v(a_0, b_F) - v(a_0, b_0) = \Delta v \frac{\Delta v}{\Delta a}\big|_{b_F} + \Delta v \frac{\Delta v}{\Delta b}\big|_{a_0} \\
v(a_F, b_F) - v(a_0, b_0) = v(a_F, b_F) - v(a_F, b_0) + v(a_F, b_0) - v(a_0, b_0) = \Delta v \frac{\Delta v}{\Delta b}\big|_{a_F} + \Delta v \frac{\Delta v}{\Delta a}\big|_{b_0}
\]

where we have used the notation \(\frac{\Delta v}{\Delta a}\big|_{b_F}\) to denote the change in \(v\) when \(a\) changes keeping \(b\) at its value in the final period, for example. Taking the average of both expressions, it follows that

\[
v(a_F, b_F) - v(a_0, b_0) = \frac{1}{2} \left( \frac{\Delta v}{\Delta a}\big|_{b_F} + \Delta v \frac{\Delta v}{\Delta b}\big|_{a_0} \right) + \frac{1}{2} \left( \frac{\Delta v}{\Delta b}\big|_{a_F} + \Delta v \frac{\Delta v}{\Delta a}\big|_{b_0} \right) \quad (43)
\]

Dividing throughout by the LHS we obtain the following expression for the contributions
of changes in $a$ and $b$ to $v$:

$$1 = \frac{\left[ \frac{\partial v}{\partial a} \right]_{a_0, b_0} + \left[ \frac{\partial v}{\partial a} \right]_{a_0, b_F}}{v(a_F, b_F) - v(a_0, b_0)} + \frac{\left[ \frac{\partial v}{\partial b} \right]_{a_0, b_0} + \left[ \frac{\partial v}{\partial b} \right]_{a_F, b_0}}{v(a_F, b_F) - v(a_0, b_0)}$$  \hspace{0.5cm} (44)$$

By construction, the sum of the contributions adds up to 1. Implicitly, this computation evenly assigns the effect of jointly changing $a$ and $b$ on $v$ between the two drivers.

### E Construction of the Value-Added Trade Data

We use the consolidated Input-Output table for the US from the World Input Output Database (available at http://www.wiod.org) to compute the share of value-added relative to total gross inputs by sector, $\alpha_s$, $j = 1, \ldots, S$. We compute the average across all years available for the 2013 WIOD release (1995-2011).\footnote{We have checked that there are no significant trends in value-added shares for agriculture and manufacturing. If we regress value-added shares on year and a constant we find a non-significant coefficient on time for agriculture and a significant but economically very small coefficient of 0.18% for manufacturing (this coefficient implies an increase over 16 years of 2.9% over a base of 34.6%).} Armed with the sectoral value-added shares $\{\alpha_s\}$, we compute the value-added content of net exports by sector and year. We use COMTRADE data on sectoral trade flows for 1980 and 2016 (since the WIOD input output table does not span a sufficiently long horizon). We also map sectoral trade flows and value-added shares into our eight sectors. The only sectors with positive trade flows are: Agriculture, Mining and Utilities and Manufacturing.

Note that we are imputing the US value-added shares to US imports (in addition to exports). The reason is that we are interested in understanding the effects of trade diversion on the US economy. Thus, a reduction in demand to US producers due to increased imports translates into a decline in labor demand of US producers. In order to capture this effect appropriately we need to use US value-added shares for imports.

### Calibration Details

To account for international trade we calibrate $\{\zeta_s\}$ and a sector specific TFP terms. We calibrate $\{\zeta_s\}$ so that the domestic aggregate demand in the model matches the domestic VA shares in each sector observed in 1980. We calibrate the sector specific TFP terms that so that the domestic demand augmented by the factor $(1 - \tau_{s,1980})^{-1}$ as discussed in equation (40) matches the total VA share in each sector observed in 1980. The calibration of the distribution parameters of effective units are done to match relative average wages and employment shares. They are done as in the baseline calibration since this part is independent from the trade module. In our main exercise for 2016 we augment each sector specific TFP term by a factor of $(1 - \tau_{s,2016})$ as well as adjust factor and labor intensity parameters $\alpha_{st}, \beta_{st}$.
Table 15: Model Simulation with Time Varying Interest Rate

<table>
<thead>
<tr>
<th>Year</th>
<th>( \frac{W_L}{W_M} )</th>
<th>( \frac{W_{LH}}{W_M} )</th>
<th>( L_s )</th>
<th>( M_s )</th>
<th>( H_s )</th>
<th>( \frac{W_{Ll}}{\sum W_{lK}} )</th>
<th>( \frac{W_{LH}}{\sum W_{lK}} )</th>
<th>( \frac{W_{LH}}{\sum W_{lK}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td>0.068</td>
<td>0.630</td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.80</td>
<td>1.49</td>
<td>0.129</td>
<td>0.488</td>
<td>0.383</td>
<td>0.088</td>
<td>0.421</td>
</tr>
<tr>
<td>Model</td>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td>0.068</td>
<td>0.630</td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.90</td>
<td>1.53</td>
<td>0.144</td>
<td>0.504</td>
<td>0.352</td>
<td>0.111</td>
<td>0.431</td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.85</td>
<td>1.53</td>
<td>0.119</td>
<td>0.519</td>
<td>0.362</td>
<td>0.086</td>
<td>0.442</td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.90</td>
<td>1.63</td>
<td>0.133</td>
<td>0.474</td>
<td>0.393</td>
<td>0.097</td>
<td>0.384</td>
</tr>
<tr>
<td>Exercise</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \sum W_{lK} )</td>
<td>( \sum W_{lK} )</td>
<td>( \sum W_{lK} )</td>
</tr>
</tbody>
</table>

| Fraction of observed change^1 | 2.67 | 1.56 | 1.12 | 1.08 | 1.08 | 1.45 | 1.18 | 1.15 |
| Contribution of E | 0.66 | 0.50 | 0.83 | 0.54 | 0.46 | 0.93 | 0.52 | 0.47 |
| Contribution of \( \alpha + \beta \) | 0.34 | 0.50 | 0.17 | 0.46 | 0.54 | 0.07 | 0.48 | 0.53 |

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

and then re-calibrate the change in \( \tilde{w}_{lt} \) and a TFP shock that is common to all sectors to match the increase in nominal personal consumption expenditures per capita and the price index.

\section{F Further robustness checks}

\subsection{F.1 Time-varying interest rates}

\section{G Discussion on the use of Log-Normal}

Consider low types first \( l \). The terms appearing in the integrals to the three moments \( m_{l,i} \) for \( i = 0, 1, 2 \) are

\[
m_{l,i} = \int \left( \frac{1}{z \sigma l \sqrt{2\pi}} \exp \left( -\frac{(\ln z - \mu l)^2}{2\sigma l^2} \right) \right) \cdot F \left( \frac{(\ln (w_l z/w_m) - \mu m)^2}{2\sigma m^2} \right) \cdot F \left( \frac{(\ln (w_l z/w_h) - \mu h)^2}{2\sigma h^2} \right) \, dz
\]

where \( F \) is a function related to the CDF of the log-normal distribution. From here we see that changing \( \mu \) or \( \sigma \) is equivalent if we do not pin down wages separately. Thus, since we target relative wages, we can normalize all means and one variance.
Table 16: Model Simulation - Germany

<table>
<thead>
<tr>
<th>Year</th>
<th>$W_L/W_M$</th>
<th>$W_L/K$</th>
<th>$L_s$</th>
<th>$M_s$</th>
<th>$H_s$</th>
<th>$\Sigma L/W_{L}W_M$</th>
<th>$\Sigma W_{L}W_K$</th>
<th>$\Sigma W_{M}W_K$</th>
<th>$\Sigma W_{H}W_K$</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>2005</td>
<td>0.79</td>
<td>1.29</td>
<td>0.166</td>
<td>0.379</td>
<td>0.455</td>
<td>0.12</td>
<td>0.345</td>
<td>0.535</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.85</td>
<td>1.43</td>
<td>0.189</td>
<td>0.339</td>
<td>0.472</td>
<td>0.137</td>
<td>0.289</td>
<td>0.574</td>
<td>$\alpha + \beta$</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.9</td>
<td>1.47</td>
<td>0.197</td>
<td>0.295</td>
<td>0.508</td>
<td>0.146</td>
<td>0.242</td>
<td>0.613</td>
<td>$E$</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.81</td>
<td>1.26</td>
<td>0.185</td>
<td>0.382</td>
<td>0.432</td>
<td>0.14</td>
<td>0.354</td>
<td>0.506</td>
<td>$\alpha + \beta$</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.86</td>
<td>1.41</td>
<td>0.186</td>
<td>0.321</td>
<td>0.493</td>
<td>0.136</td>
<td>0.272</td>
<td>0.592</td>
<td>$E + \beta + \alpha$</td>
</tr>
</tbody>
</table>

Fraction of observed change:

1.17 0.86 0.87 1.45 2.24 0.94 1.30 1.46

Contribution of E:

1.14 1.38 0.80 1.25 1.50 0.69 1.27 1.44

Contribution of $\alpha + \beta$:

-0.14 -0.38 0.20 -0.25 -0.50 0.31 -0.27 -0.44

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

Table 17: Model Simulation - France

<table>
<thead>
<tr>
<th>Year</th>
<th>$W_L/W_M$</th>
<th>$W_H/W_M$</th>
<th>$L_s$</th>
<th>$M_s$</th>
<th>$H_s$</th>
<th>$\Sigma L/W_{L}W_M$</th>
<th>$\Sigma W_{L}W_K$</th>
<th>$\Sigma W_{H}W_K$</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>2005</td>
<td>0.81</td>
<td>1.36</td>
<td>0.206</td>
<td>0.359</td>
<td>0.434</td>
<td>0.15</td>
<td>0.321</td>
<td>0.529</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.92</td>
<td>1.43</td>
<td>0.242</td>
<td>0.262</td>
<td>0.496</td>
<td>0.186</td>
<td>0.219</td>
<td>0.595</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.91</td>
<td>1.47</td>
<td>0.242</td>
<td>0.304</td>
<td>0.454</td>
<td>0.184</td>
<td>0.256</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.87</td>
<td>1.47</td>
<td>0.221</td>
<td>0.312</td>
<td>0.466</td>
<td>0.162</td>
<td>0.262</td>
<td>0.576</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.94</td>
<td>1.55</td>
<td>0.245</td>
<td>0.276</td>
<td>0.478</td>
<td>0.185</td>
<td>0.221</td>
<td>0.594</td>
</tr>
</tbody>
</table>

Fraction of observed change:

1.18 2.71 1.08 0.86 0.71 0.97 0.98 0.98

Contribution of E:

0.65 0.50 0.77 0.55 0.36 0.81 0.53 0.38

Contribution of $\alpha + \beta$:

0.35 0.50 0.23 0.45 0.64 0.19 0.47 0.62

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.
Table 18: Model Simulation - UK

<table>
<thead>
<tr>
<th>Year</th>
<th>Data</th>
<th>Model</th>
<th>Fraction of observed change&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Contribution of E</th>
<th>Contribution of α + β</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>0.85</td>
<td>0.85</td>
<td>7.00</td>
<td>0.57</td>
<td>0.43</td>
</tr>
<tr>
<td>2015</td>
<td>0.86</td>
<td>0.89</td>
<td>21.00</td>
<td>0.33</td>
<td>0.67</td>
</tr>
</tbody>
</table>

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

Table 19: Model Simulation - Italy

<table>
<thead>
<tr>
<th>Year</th>
<th>Data</th>
<th>Model</th>
<th>Fraction of observed change&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Contribution of E</th>
<th>Contribution of α + β</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>0.84</td>
<td>0.84</td>
<td>3.00</td>
<td>0.44</td>
<td>0.56</td>
</tr>
<tr>
<td>2015</td>
<td>0.87</td>
<td>0.89</td>
<td>2.00</td>
<td>1.75</td>
<td>-0.75</td>
</tr>
</tbody>
</table>

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.
Table 20: Model Simulation - Netherlands

<table>
<thead>
<tr>
<th>Year</th>
<th>$\frac{W_L}{W_M}$</th>
<th>$\frac{W_H}{W_M}$</th>
<th>$L_s$</th>
<th>$M_s$</th>
<th>$H_s$</th>
<th>$\frac{W_L}{\sum_i W_K}$</th>
<th>$\frac{W_M}{\sum_i W_K}$</th>
<th>$\frac{W_H}{\sum_i W_K}$</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>2005</td>
<td>0.85</td>
<td>1.26</td>
<td>0.166</td>
<td>0.309</td>
<td>0.525</td>
<td>0.127</td>
<td>0.277</td>
<td>0.595</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.99</td>
<td>1.34</td>
<td>0.215</td>
<td>0.256</td>
<td>0.529</td>
<td>0.18</td>
<td>0.217</td>
<td>0.602</td>
</tr>
<tr>
<td>Model</td>
<td>2005</td>
<td>0.85</td>
<td>1.26</td>
<td>0.166</td>
<td>0.309</td>
<td>0.525</td>
<td>0.127</td>
<td>0.277</td>
<td>0.595</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.89</td>
<td>1.33</td>
<td>0.171</td>
<td>0.278</td>
<td>0.551</td>
<td>0.131</td>
<td>0.239</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.92</td>
<td>1.27</td>
<td>0.204</td>
<td>0.29</td>
<td>0.506</td>
<td>0.168</td>
<td>0.259</td>
<td>0.573</td>
</tr>
<tr>
<td></td>
<td>2015</td>
<td>0.96</td>
<td>1.33</td>
<td>0.21</td>
<td>0.263</td>
<td>0.527</td>
<td>0.173</td>
<td>0.226</td>
<td>0.601</td>
</tr>
</tbody>
</table>

Fraction of observed change$^1$

|                  | 0.79 | 0.88 | 0.90 | 0.87 | 0.50 | 0.87 | 0.85 | 0.86 |

Contribution of $E$

|                  | 0.36 | 0.93 | 0.13 | 0.63 | 11.75 | 0.10 | 0.70 | 5.25 |

Contribution of $\alpha + \beta$

|                  | 0.64 | 0.07 | 0.88 | 0.37 | -10.75 | 0.90 | 0.30 | -4.25 |

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

To elaborate a little more on this note that:

\[
\frac{(\ln(w_lz/w_h) - \mu_h)^2}{2\sigma_h^2} = \left( \ln z \sqrt{\frac{1}{\sigma_h}} + \frac{1}{\sqrt{2\sigma_h}} \ln \frac{w_l}{2w_h\sigma_h} - \mu_h \right)^2
\]

\[
= (\ln \frac{z}{\sqrt{\sigma_h}w_h} - \bar{\mu})^2
\]

So it is clear that the average changes with both $\mu$ and $\sigma$. Since we do not observe $z$ we can always renormalize things as in the last line and only use $\mu$ or $\sigma$ for the calibration.