Firm Dispersion and Business Cycles: Estimating Aggregate Shocks Using Panel Data*

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Abstract

Are fluctuations in firm-level dispersion a cause or an effect of the business cycle? To answer this question, we build a general equilibrium rich enough to jointly explain characteristics of the firm distribution and the dynamics of macroeconomic aggregates. The model includes frictions that generate movements in dispersion following standard macroeconomic shocks such as aggregate productivity, as well as a direct shock to the dispersion of firm level productivity growth. This type of general equilibrium model with heterogeneous agents and aggregate shocks is computationally difficult to solve, which typically keeps likelihood-based estimation out of reach. We exploit recent advances in solution techniques to obtain a characterization for which estimation is feasible. To answer our question, we estimate the model using time series of macroeconomic aggregates and newly constructed cross-sectional time series, which reflect movements in the firm distribution over time. Now able to account for firm dispersion and the business cycle, we find that (i) standard macroeconomic aggregate shocks explain almost all variation in macroeconomic aggregates, (ii) an uncertainty shock explains almost all variation in firm-level dispersion.

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1 Introduction

Recently it has been well-documented that measures of firm-level dispersion are cyclical: the cross-sectional dispersion of firm-level output growth, profit growth, employment growth, stock returns and prices are countercyclical and the dispersion of investment rates is procyclical.\(^1\) It is less clear why this is the case. Increased dispersion of firm-level outcomes may be the result of shocks to the dispersion of productivities, as argued by Bloom (2009), Bloom et al. (2012), Gilchrist et al. (2014) and Christiano et al. (2014), or may be the result of heterogeneous responses across firms to first-moment shocks, as argued by Bachmann and Moscarini (2012) and Berger and Vavra (2013). In particular, Bloom et al. (2012) document that firm-level output growth and a revenue-based measure of firm-level total factor productivity (TFP) are countercyclical, and argue that this consistent with an “uncertainty” shock—a shock to the volatility of idiosyncratic productivities—which simultaneously increases the dispersion of firm-level sales growth growth, and, by inducing wait-and-see effects in firm-level investment policies, lowers investment and output. Gilchrist et al. (2014) and Christiano et al. (2014) also argue for the presence of an uncertainty shock—they show that financial frictions can amplify shocks to the volatility of firm-level productivities and generate large drops in output. On the other hand, Bachmann and Bayer (2011) document that the dispersion of investment rates is procyclical, seemingly at odds with an uncertainty-driven recession, and argue that the data is best explained by a joint process for TFP and uncertainty which is negatively correlated.\(^2\)

In this paper, we contribute to the question by bringing new information and methods to bear. We build a general equilibrium rich enough to jointly explain characteristics of the firm distribution and the dynamics of macroeconomic aggregates. We subject the model to uncertainty shocks as well as to standard macroeconomic shocks used widely in the literature: aggregate productivity, discount rate, financial and labor disutility shocks.\(^3\) We estimate the model using Bayesian methods, using as observables times series of both aggregates and moments of the distribution of firms in Compustat data. This allows us to answer two questions in a precise likelihood sense. Which shocks account for fluctuations in moments that reflect the dispersion of firm-level outcomes? Do direct shocks to the dispersion of firm level productivity growth (uncertainty shocks) account for significant movements in output?

\(^1\)See Bloom (2009), Bloom et al. (2012), Vavra (2014), Basu and Bundick (2012) and Bachmann and Bayer (2011).

\(^2\)Bachmann and Moscarini (2012) show that increased dispersion can be the result of an endogenous response of firms to a drop in aggregate TFP

\(^3\)See Chari et al. (2007) for a description of how aggregate productivity, discount rate and labor disutility shocks map into the three key ‘wedges’ in an RBC model that are necessary for the explanation of aggregate output, consumption and hours worked
We find that almost all of the variation in macroeconomic aggregates is explained by aggregate shocks rather than the uncertainty shock. The reason for this is twofold. First, while an uncertainty shock under our estimated parameterization generates a fall in output and investment on impact, it also generates a rebound and persistent overshoot in output and investment after three to four quarters. This is the result of realized productivities being more dispersed. The estimation finds this rebound and overshoot hard to reconcile with the persistence of the fall in output observed in the data. Second, for parameterizations under which an uncertainty shock generates a large drop in output, the uncertainty shock generates a counterfactually large increase in firm dispersion. When we treat the uncertainty shock more literally—beliefs regarding the distribution of future productivity become more dispersed but are not realized—this overshooting disappears, the second problem resurfaces: the size of shocks required to generate large movements in output generate counterfactually large movements in firm dispersion.

We also find that almost all of the variation in firm-level dispersion is explained by uncertainty shocks. For parameterizations that are consistent with the stationary distribution of firm-level investment rates, level shocks do not generate significant movements in cross-sectional dispersion. In particular, the model generates large responses of dispersion to level shocks only when adjustment frictions are very large. But large values of the parameters that determine the size of these frictions generate counterfactually large rates of inaction at the firm level. Although investment is lumpy it is not lumpy enough to generate sizeable movements in firm dispersion from first moment macroeconomic shocks. Finding a middle ground between these observations remains a task for future research.

To allow for macroeconomic shocks to generate movements in firm level dispersion we include two frictions in the firm problem. First we include a fixed cost of investment as in Khan and Thomas (2008) which can lead to investment booms on the extensive margin, increasing the dispersion of investment rates and sales growth in response to a positive macroeconomic shocks. Second we include a positive cost of raising external finance which can lead to a bunching up of firm investment rates following a negative shock as the marginal cost of funds increases for constrained firms. Both frictions also allow us to capture key features of the data: firm investment is lumpy and total debt/equity issuance is negative in the aggregate but positive at many firms. Moreover, as pointed out by Bloom (2009) and Bachmann and Bayer (2011) firm-level adjustment frictions are key amplifiers of uncertainty shocks. Our model, therefore, contains a number of mechanisms to both transmit macroeconomic shocks to dispersion outcomes and amplify uncertainty shocks.

A key advantage of our approach is that it allows us to clearly ask how much of movements in the dispersion in sales growth or investment rates is left to be explained by an uncertainty
shock once we have accounted for the role of standard macroeconomic shocks. Rather than studying shocks in isolation we match the relative volatility and comovement of output, consumption and investment using three macroeconomic shocks—a shock to households’ preference for labor supply, a shock to the rate of time preferences, and a shock to aggregate TFP—and treating these series as observables in the likelihood based estimation of the model.\footnote{These shock are commonly used in the literature to capture movements in aggregates. See Smets and Wouters (2007), Christiano et al. (2001) and Justiniano et al. (2011).}

Since the distribution of firms across idiosyncratic capital and productivity is itself a state of the economy, solving for the equilibrium of the model is challenging. The principal method used in the literature for models of this type is the method of Krusell and Smith (1998).\footnote{See Khan and Thomas (2008), Bachmann and Bayer (2011), Bloom et al. (2012) for applications of the Krusell and Smith (1998) method.} This method is computationally intensive (see Algan et al. (2014) for a discussion) and quantitative work that uses this method typically rely on calibration and simulated method of moments. Likelihood-based estimation has so far remained out of reach due to the computational infeasibility. Likelihood-based estimation is also a key tool in accounting for business cycles, which is the exercise at the core of this paper.

We present a novel approach to estimating heterogeneous agents models with aggregate shocks. We exploit the method of Reiter (2009), which combines elements of the perturbation methods and projection methods to produce a first-order approximate solution.\footnote{See Fernández-Villaverde et al. (2016). For recent applications of this method, see Winberry (2016a), Winberry (2016b), McKay and Reis (2013).} Given a first-order approximate solution, we can then apply the standard toolbox of estimation and analysis familiar from the study of linearized DSGE models.

The Reiter (2009) method works as follows. First, use projection methods to solve for the model’s recursive stationary equilibrium in the absence of aggregate shocks. Second, we then construct a finite representation of the equilibrium. We construct a finite approximation of the firm distribution and policy functions and a corresponding discretization of the law of motion of the distribution. Thirdly, we compute the solution in the presence of aggregate shocks by perturbing elements of the finite representation of the equilibrium around the no-aggregate-shock steady state.

An advantage of the Reiter (2009) method is that the model solution is in linear state space form, which lends itself conveniently to estimation. The perturbation techniques that the Reiter method draws on have long been used in the literature to solve representation-agent models, and there is an extensive literature on estimating model in linear state-space form.\footnote{See Fernández-Villaverde et al. (2016) and An and Schorfheide (2007) for summaries.} In particular, we use Bayesian estimation techniques, described in detail in An and
Schorfheide (2007). To the best of our knowledge, this is the first full-information estimation of a firm dynamics model.

As mentioned above we can use aggregate time series as observables in the estimation, but since our model has predictions for movements in the firm distribution we also include newly-constructed cross-sectional time series. In particular, we include in the set of observable time series the cross-sectional standard deviation of sales growth, and the cross-sectional standard deviation of investment rates. This allows us to ask, say, whether an uncertainty shock can jointly explain movements in output and sales growth dispersion in the presence of other standard shocks that would move output.

What is an uncertainty shock?

In our baseline treatment, we adopt the approach of Bloom (2009) and Gilchrist et al. (2014), in using the expression “uncertainty shock” to describe a shock to the volatility of idiosyncratic shocks. In particular, firms in our model are subject to an autoregressive productivity process with Gaussian innovations. All firms are subject to the same process, although of course, individual firms will have different realizations. An “uncertainty shock” refers to a change in the volatility of innovations to that process.

The timing of the shock is important. Our baseline assumption is that agents observe their current realization of idiosyncratic productivity $z$ and the volatility of the next draw. That is, an uncertainty shock today reflects news about next period’s productivity distribution.

“Uncertainty” shock is somewhat of a misnomer, since the shock reflects changes in the actual volatility process affecting firms. Some elements of the literature have referred to these as “risk shocks” (see Christiano et al. (2014)). The shock certainly has an uncertainty interpretation, since in the face of a more volatile productivity process, agents are indeed more uncertain about future outcomes, but it also has a second effect, that realized productivities will be more dispersed in future. We use the terminology “uncertainty shock” for the sake of consistency with previous literature.

The remainder of this paper is organized as follows. Section 2 presents our model of heterogeneous firms facing investment and financing frictions and a number of macroeconomic shocks. Section 3 discusses the solution method, which makes estimation feasible. In Section 4 we discuss the Compustat data used to estimate the model and, in stages, estimated the unknown parameters of the model. Section 5 presents our main results, which consist of forecast error variance decompositions and discussion of the impulse response properties of the model. Section 6 concludes.
2 Model

The model is set in discrete time with an infinite horizon. There are two sets of agents. A continuum of firms own the capital stock and produce the consumption good. Households supply labor, consume the consumption good and own the firms. We first describe the environment facing each of these agents before writing down their optimization problems in detail and then define the equilibrium of the model.

2.1 Firms

Production technology  A fixed unit mass of firms, indexed by \( i \in [0, 1] \), produce the consumption good every period \( t \), using installed capital stock \( k_{it} \) and labor \( n_{it} \), which they hire at the prevailing wage \( W_t \). Firms produce the consumption good according to a decreasing returns to scale production function

\[
y_{it} = X_t^Z z_{it} \left( k_{it}^{\alpha} n_{it}^{1-\alpha} \right)^\kappa
\]

where \( \kappa < 1 \) is the coefficient of decreasing returns.

Firms are subject to fluctuations in idiosyncratic productivity \( z_{it} \) and aggregate productivity (TFP) \( X_t^Z \). The aggregate TFP shock \( X_t^Z \) is common to all firms and follows an AR(1) process in logs:

\[
\log X_{t+1}^Z = \rho^Z \log X_t^Z + \varepsilon_{t+1}^Z
\]

where \( \varepsilon_{t+1}^Z \sim N(0, \sigma^Z) \). The idiosyncratic shock \( z_{it} \) is independent across firms. Each firm’s productivity follows an AR(1) process in logs:

\[
\log z_{it} = \rho^z \log z_{it} + \varepsilon_{it+1}^z
\]

where \( \varepsilon_{it+1}^z \sim N(0, X_t^Z \bar{\sigma}_z) \). The volatility of this process \( X_t^Z \bar{\sigma}_z \) has two components, a permanent component \( \bar{\sigma}_z \), which reflects average volatility over time, and a time-varying aggregate component \( X_t^\sigma \), which is common across firms, and follows an AR(1) process in levels:

\[
X_{t+1}^\sigma = \rho^\sigma X_t^\sigma + \varepsilon_{t+1}^\sigma
\]

where \( \varepsilon_{t+1}^\sigma \sim N(0, \sigma^\sigma) \). We refer to fluctuations in \( X_t^\sigma \) as uncertainty shocks. As (1) makes clear, the timing of the shock is such that the standard deviation of the innovations to \( t+1 \) idiosyncratic productivity, \( \varepsilon_{it+1}^z \), are known to firms at the beginning of period \( t \).

In order to produce, firms must hire labor \( n_{it} \), which they hire in a frictionless labor market at the prevailing wage \( W_t \). After producing, firms decide on next period’s capital
stock, \( k_{it+1} \). Capital depreciates at a rate \( \delta \) every period, so to enter period \( t+1 \) with capital stock \( k_{it+1} \), a firm must invest \( x_{it} = k_{it+1} - (1 - \delta) k_{it} \).

**Investment frictions** Large adjustments of the capital stock are subject to a fixed cost of investment. In particular, if investment \( x_{it} \) lies outside the interval \([-bk_{it}, bk_{it}]\), the firm must pay a cost \( \xi_{it} \) in units of labor, where \( \xi_{it} \) is stochastic. The fixed cost \( \xi_{it} \) is uniformly distributed on the interval \([0, \bar{\xi}]\) and is independently distributed across firms and time.

**Financing frictions** Any excess revenue from production is paid out to households as a dividend payout \( d_{it} \). If costs are in excess of revenues, the firm payout is negative, or in other words, the firm raises external finance from households. Raising external finance is costly: if dividends are negative, then the firm must pay an additional cost \( \varphi (d_{it}) \) which is quadratic in the quantity of external finance raised:

\[
\varphi (d_{it}) = 1 \{d_{it} < 0\} \bar{\phi} X_t^\phi d_{it}^2
\]

The coefficient on the cost of external finance has two components, a fixed component \( \bar{\phi} \), which reflects the time series average of the cost, and a stochastic component \( X_t^\phi \), which follows an AR(1) process in logs:

\[
X_{t+1}^\phi = \rho^\phi X_t^\phi + \epsilon_{t+1}^\phi,
\]

where \( \epsilon_{t+1}^\phi \sim N \left( 0, (\sigma^\phi)^2 \right) \).

We also assume that firms must pay a fixed cost of operating \( \chi \) every period. We include this feature in order to match the data on external financing flows. Without a fixed cost of operating (i.e. with \( \chi = 0 \)), too few firms would access external finance relative to the data. We wish to investigate whether external financing frictions are a source of transmission of first order shocks through to firm level dispersion, so capturing this aspect of the data is important to us.

### 2.2 Households

**Preferences** We assume a unit measure of identical households, which value consumption and leisure, supply labor and own the firms. The household’s utility function is given by

\[
E_0 \left\{ \sum_{t=0}^{\infty} (X_t^\beta)^t \left[ \frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - X_t^{\psi} \psi \frac{N_t^{1+\eta}}{1+\eta} \right] \right\}
\]
where $C_t$ denotes consumption at period $t$, $H_t$ is the habit stock, $N_t$ denotes labor supplied in period $t$, $X^\beta_t$ is the household’s discount factor. The household discount factor $X^\beta_t$ has two components, a permanent component $\beta$ and a stochastic component $X^\beta_t$, which follows an AR(1) process in logs:

$$\log X^\beta_{t+1} = \rho^\beta \log X_t + \varepsilon_{t+1}^\beta$$

where $\varepsilon_{t+1}^\beta \sim N\left(0, \sigma^\beta\right)$. Household consumption preferences also feature habit formation, of the form described in Campbell and Cochrane (1999). That is, households value consumption in excess of the habit stock $C_t - H_t$, where $H_t$ responds slowly to past consumption. Campbell and Cochrane (1999) show that preferences of this form can be written in terms of the surplus consumption ratio $S_t = (C_t - H_t) / C_t$ and define the law of motion for $S_t$ as

$$\log S_{t+1} = \left(1 - \rho^S\right) \log \bar{S} + \rho^S \log S_t + \lambda^S \log \left(\frac{C_{t+1}}{C_t}\right).$$

Winberry (2016b) shows that habit preferences of this form ensure that the correlation of the real interest with output is not counterfactually large.

Households preferences over leisure feature a constant Frisch elasticity $1/\eta$. The coefficient on the disutility of leisure has two components, a fixed component $\psi$ and a stochastic component $X^\psi_t$, which follow an AR(1) process in logs:

$$\log X^\psi_{t+1} = \rho^\psi \log X^\psi_t + \varepsilon_{t+1}^\psi$$

where $\varepsilon_{t+1}^\psi \sim N\left(0, \sigma^\psi\right)$. An increase $X^\psi_t$ increases the disutility that households incur from supplying labor.

### 2.3 Optimization

We now describe the firm and household problems recursively. Let $S$ denote the aggregate state, which consists of the distribution of firms over idiosyncratic states $\mu(k, z, \xi)$, the aggregate shocks $X = (X^Z, X^\psi, X^\beta, X^\phi, X^\sigma)$ and level of habit stock $S_t$.

#### Firm optimization

Let $v(k, z, \xi; S)$ be the present discounted value of dividends (net of external financing costs) of the firm, as valued by the household’s discount factor, given realizations of the current aggregate state $S$ and idiosyncratic states $k, z, \xi$. Since the firm must decide whether to adjust its capital or not, it is convenient to consider separately the value of adjusting, the value of not adjusting, and the adjustment decision.
Value of adjusting Let $v^{\text{adj}}(k, z; S)$ be the value of adjusting, conditional on $(k, z)$ and aggregate state $S$:

$$v^{\text{adj}}(k, z; S) = \max_{k' \geq 0, n} \left\{ d + 1_{\{d < 0\}} \varphi(d) + \mathbb{E} \left[ M(S, S') v(k', z', \xi'; S') \right] \right\},$$

subject to

$$d = \pi(k, zk; S) + (k' - (1 - \delta) k),$$

$$\varphi(d) = X^\phi \tilde{d}^2,$$

$$S' = \Gamma(S'|S),$$

where $M(S, S')$ is the household’s one period stochastic discount factor defined in (16), $\Gamma$ is the firm’s perceived law of motion of the aggregate state. Operating profits and $\pi(z, k; S)$ are the outcome of the firm’s static labor demand choice

$$\pi(k, z; S) = \max_n X^Z z \left( k^{\alpha n \lambda - 1} \right)^\kappa - W(S)n - \chi.$$  \hfill (7)

We can express the firms labor demand $n(z, k; S)$ which solves this problem in closed form

$$n(k, z; S) = \left( X^Z z \nu \kappa k^{\nu(1 - \nu)} W(S)^{-1} \right)^{\frac{1}{1 - \nu \kappa}}.$$  \hfill (8)

The solution to this problem is the firm’s optimal level of capital next period which we denote $k^{\text{adj}}(k, z; S)$.

Value of not adjusting The value of not adjusting, $v^{\text{stay}}(k, z; S)$ is the same as (3) to (6) above, subject to the additional constraint that the investment rate of the firm is constrained within a small interval

$$\frac{k' - (1 - \delta) k}{k} \in [-bk, bk].$$

Let $k^{\text{stay}}(k, z; S)$ denote the firm’s optimal choice of capital conditional on not adjusting. Note that the labor demand decision $n$ and operating profits $\pi$ are the same for adjusting and non-adjusting firms.

Adjustment After observing $k, z, \xi$ and the aggregate state $S$ the firm chooses whether to adjust or not. If the firm adjusts, it pays a fixed cost $\xi$ in units of labor. The value of the firm at the start of the period can therefore be expressed as

$$v(k, z, \xi; S) = \max \left\{ -\xi W(S) + v^{\text{adj}}(k, z; S), v^{\text{stay}}(k, z; S) \right\}.$$
The firm chooses to adjust if and only if the value of adjusting is greater than its cost, that is, if and only if
\[ v^{\text{adj}}(k, z; S) - \xi W(S) \geq v^{\text{stay}}(k, z; S). \] (10)

For every \((k, z; S)\) there is a threshold value of \(\xi\), which we denote \(\xi^*(k, z; S)\), at which the firm is indifferent between adjusting and not adjusting. The firm adjusts if \(\xi \leq \xi^*(k, z; S)\) and does not adjust if \(\xi > \xi^*(k, z; S)\). We can see from (10) that the threshold is given by
\[ \xi^*(k, z; S) = \frac{v^{\text{adj}}(k, z; S) - v^{\text{stay}}(k, z; S)}{W(S)}. \] (11)

Let \(k'(k, z, \xi; S)\) denote the capital choice of the firm conditional on \(k, z, \xi\) and \(S\):
\[ k'(k, z, \xi; S) = \begin{cases} 
  k^{\text{adj}}(k, z; S) & \text{if } \xi < \xi^*(k, z; S), \\
  k^{\text{stay}}(k, z; S) & \text{if } \xi \geq \xi^*(k, z; S).
\end{cases} \] (12)

Finally, let \(d(k, z, \xi; S)\) denote the net payout of firm conditional on \(k, z, \xi\) and \(S\),
\[ d(k, z, \xi; S) = \pi(k, z; S) - [k'(k, z, \xi; S) - (1-\delta)k], \]
recalling that if \(d(k, z, \xi; S) < 0\) then the firm must pay an additional cost of external finance.

**Household optimization**

The household takes the evolution of the habit stock as given, rendering the household’s problem static. In every period, households solve the following static problem: taking \(W(S)\) and \(\Pi(S)\) as given,
\[
\max_{C,N} \quad \frac{(C - H)^{1-\sigma}}{1 - \sigma} - \psi X \psi(S) \frac{N^{1+\eta}}{1+\eta} \\
\text{subject to} \quad C = W^*(S) N + \Pi(S) \] (13) (14)

where \(W^*(S)\) is the wage facing the households and \(\Pi(S)\) is net payouts from firms.

The solution to this problem is the household’s labor supply condition
\[ W^*(S) = \frac{\psi X \psi(S) N(S)^{\eta}}{[C(S) - H(S)]^{-\sigma}}. \] (15)
The household intertemporal marginal rate of substitution is given by

\[ M(S, S') = \beta X^\beta \frac{(C(S') - H(S'))^{-\sigma}}{(C(S) - H(S))^{-\sigma}}, \]  
(16)

which—since markets are complete—is the stochastic discount factor used by firms to price future payoffs. Note that we can rewrite \( C(S) - H(S) \) more conveniently in terms of the surplus consumption ratio:

\[ C(S) - H(S) = C(S) S(S), \]

which allows us to write the household’s labor supply condition and discount factor

\[ W^*(S) = \frac{\psi X^\psi (S) N^\psi}{(C(S) S(S))^{-\sigma}}, \quad M(S, S') = \beta X^\beta \left( \frac{C(S)}{C(S')} \right)^\sigma \left( \frac{S(S)}{S(S')} \right)^\sigma. \]  
(17)

This makes clear how habit formation affects the interest rate in the economy. Even if consumption is expected to grow quickly, the habit stock moves slowly, leading to a muted response of interest rates.

We allow for some rigidity in the evolution of the wage, in order to capture the fact that wages appear to adjust slowly over the cycle. In the interests of tractability, we adopt the parsimonious specification used in Beraja et al. (2016). Let \( W^*_t \) be the wage that would determine labor supply from the household’s intratemporal first order condition (15). We assume that the market wage \( W_t \) which is paid by firms has an autoregressive component and adjusts slowly towards \( W^*_t \):

\[ \log W_t = \omega \log W^*_t + (1 - \omega) \log W_{t-1} \]  
(18)

We therefore assume that the quantity of labor traded is determined by labor demand. That is, firms choose labor demand optimally given \( W_t \), and households must supply whatever labor is demanded of them. Note that if \( \omega = 1 \), wages are fully flexible and the labor market clears.

### 2.4 Equilibrium

A recursive competitive equilibrium of the model is a firm value function \( v(k, z, \xi; S) \) and associated policy functions \( k'(k, z, \xi; S) \), \( n(k, z; S) \) and \( \xi^*(k, z; S) \), household policy func-

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As noted in Beraja et al. (2016), this specification can be thought of as a reduced form that stands in for the endogenous wage stickiness that arises from the wage bargaining model of Hall and Milgrom (2008) or the model of monopsonistic competition discussed in Gali (2011) under the assumption of myopia on the part of the agents.
tions $C(S)$ and $N(S)$ and associated stochastic discount factor $M(S, S')$, wage $W(S)$ and flexible wage $W^*(S)$, surplus consumption $S(S)$, lagged wage $W_{-1}(S)$, lagged consumption $C_{-1}(S)$ and lagged surplus consumption $S_{-1}(S)$, firm payouts $\Pi(S)$, a distribution of firms $\mu(k, z, \xi; S)$ and a law of motion for the distribution of firms $\Gamma(\mu, \mu'; S)$, such that

1. Taking $W(S), \mu(S), M(S, S')$ and $\Gamma(\mu, \mu'; S)$ as given, $k'(k, z, \xi; S), n(k, z; S)$ and $\xi^*(k, z; S)$ solve the firm’s problem (3)-(6), $v(k, z, \xi; S)$ is the associated value function and $\Pi(S)$ is the corresponding aggregate net payout of firms:

$$
\Pi(S) = \int \pi(k, z; S) d\mu(k, z, \xi; S) - W(S) \int 1\{\xi < \xi^*(k, z; S)\} \xi d\mu(k, z, \xi; S) - \int (k'(k, z, \xi; S) - (1 - \delta) k) d\mu(k, z; S)
$$

2. Taking $W(S), \Pi(S)$ and $S(S)$ as given, $C(S)$ solves the household problem (13)-(14), and $M(S, S')$ is the corresponding stochastic discount factor.

3. Taking $N(S)$ as given, $W^*(S)$ satisfies the household’s labor supply condition (15), where $N(S)$ is aggregate labor demand:

$$
N(S) = \int n(k, z; S) d\mu(k, z, \xi; S) + \int \xi 1\{\xi \leq \xi^*(k, z; S)\} d\mu(k, z; S)
$$

4. The surplus consumption ratio evolves according to (2).

5. The law of motion of the distribution $\Gamma$ is consistent with the firm’s policies. For all measurable sets $\mathcal{K} \times \mathcal{Z} \times \mathcal{X}$,

$$
\mu'(\mathcal{K} \times \mathcal{Z} \times \mathcal{X}) = Q(\mathcal{K}, \mathcal{Z}, \mathcal{X}, k, z, \xi; S) \times \mu(dk, dz, d\xi)
$$

where

$$
Q(\mathcal{K}, \mathcal{Z}, \mathcal{X}, k, z, \xi) = \int P(z' \in \mathcal{Z}|z) dz' \times 1\{k'(k, z, \xi; S) \in \mathcal{K}\} \times G(\mathcal{X})
$$

6. The aggregate shocks $X$ evolve according to the exogenous process:

$$
\log(X^j)' = \rho^j \log X^j + (\varepsilon^j)', \quad \varepsilon^j \sim N\left(0, (\sigma^j)^2\right), \quad \forall j \in \{Z, \psi, \beta, \phi\}
$$

$$
(X^\sigma)' = \rho^\sigma X^\sigma + \varepsilon^\sigma, \quad \varepsilon^\sigma \sim N\left(0, (\sigma^\sigma)^2\right)
$$
3 Solution method

In general, it is difficult to solve for the recursive competitive equilibrium for models of this type, since the firm’s policies depend on firm’s forecasts of the aggregate consumption and wage, and next period’s aggregate consumption and wage depend on next period’s distribution of firms, which is an infinite-dimensional object $\mu$. The pre-eminent method in the literature is the method of Krusell and Smith (1998). However, given the large number of aggregate shocks in our model, this approach is infeasible for computational reasons. We turn to a second approach used in the literature, pioneered by Reiter (2009), which involves solving firm’s policies globally at the deterministic steady state, and then perturbing the solution with respect to aggregate shocks.

Finite representation We first construct a finite representation of the value function $\tilde{v}(k, z; S)$ using cubic splines. (Note that we work with $\tilde{v}(k, z; S) = \int v(k, z, \xi; S) dG(\xi)$ rather than $v(k, z, \xi; S)$ from this point onwards. Since the adjustment cost $\xi$ is iid, it is convenient to integrate it out and work with the expected value function.) Denote by $\theta_{ij}^V$ the coefficients of the cubic spline representation, where $i \in (1, \ldots, n_k)$ and $j \in (1, \ldots, n_z)$.

We approximate the distribution $\mu(k, z; S)$ with a histogram, parameterized by $\lambda_{ij}$. Denote by $X$ the set of aggregate shocks, $X = (X^Z, X^\psi, X^\beta, X^\phi, X^\sigma)$.

Linearizing The finite representation above can be written as a function of the set of variables $\Theta_t$,

$$\Theta_t = \left[ \left( \theta_{ij}^V \right)_t, (\lambda_{ij})_t, X_t, C_t, S_t, W_t, W^*_t, N_t, Y_t, I_t, \sigma_{t}^{sg}, \sigma_{t}^{ir}, ef_t \right].$$

where $\Theta_t$ contains the state and jump variables, as well as variables which will be used as observables in the estimation step. We first solve for the equilibrium value of $\Theta_t$ when the aggregate shocks $X_t$ are zero, using nonlinear global methods. We call the value of $\Theta_t$ when the aggregate shocks are zero the deterministic steady state, denoted by $\bar{\Theta}$. We then express the finite equilibrium conditions in terms of log deviations from steady state, $\hat{\Theta}_t = \log \Theta_t - \log \bar{\Theta}$ and take a first-order Taylor expansion. This delivers a linear system of equations:

$$\Gamma_0 \hat{\Theta}_{t+1} = \Gamma_1 \hat{\Theta}_t + \Psi \varepsilon_{t+1}$$

where $\varepsilon_{t+1}$ is a $n_\varepsilon \times 1$ vector of Gaussian disturbances. The matrices $\Gamma_0$ and $\Gamma_1$ contain first-order partial derivatives of the equilibrium conditions with respect to the elements of $\Theta_t$, which are computed numerically. To understand the composition of $\Gamma_0$ and $\Gamma_1$, consider
the example of the equilibrium condition which defines aggregate output $Y$:

$$Y = \int X^Z y(k, z, \xi) d \mu(k, z, \xi).$$

Following a shock to $X^Z$ output responds for three reasons: (1) there is a direct effect holding firm decisions and the distribution of firms constant, (2) firm policies respond—which can be separated into direct responses to the shock and indirect responses due to the movement of prices—and (3) the distribution of firms shifts. Numerically differentiating this condition to compute one element of $\Gamma_0$ requires perturbing $X^Z$ and computing these responses. Moreover, other elements of these matrices pick up how prices respond to shocks, how policies respond to prices, and how the distribution responds to changes in firm policies.

Given this finite, linear formulation of the model’s equilibrium conditions we can use standard methods, such as Sims (2002), to obtain a linear Gaussian state-space representation

$$\hat{\Theta}_{t+1} = A\hat{\Theta}_t + B\varepsilon_{t+1}$$

where $A$ is a $n_x \times n_x$ matrix and and $B$ is a $n_x \times n_\xi$ matrix. With this representation in hand we can easily compute the likelihood of any given sequence of $\Xi_t$.

4 Estimation

We estimate the model in three stages. First, we externally calibrate a number of parameters. These include preference parameters, depreciation rate $\delta$, and labor output elasticity $\alpha$. Second, we estimate a subset of the model parameters by simulated method of moments. These are the remaining parameters needed to solve the steady state of the model: the cost of equity issuance $\bar{\phi}$, upper bound on the uniform distribution of fixed adjustment costs $\bar{\xi}$, parameters of the firm productivity process $\rho_z, \sigma_z$, fixed operating cost $\chi$ and decreasing returns to scale parameter $\kappa$. The moments used in the estimation capture key properties of firm-level investment and finance. Third, we estimate the parameters of the aggregate shock processes using Bayesian methods and time series data for aggregates and moments of the distribution.

4.1 Data

For moments of the distribution of firms we use Compustat data, which collects accounting data for the universe publicly listed firms in the United States. For aggregate time series we
Table 1: Externally calibrated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>Curvature of utility function</td>
<td>$\sigma$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>$\eta$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Output elasticity of labor</td>
<td>$\alpha$</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Wage flexibility</td>
<td>$1 - \omega$</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>Average surplus ratio</td>
<td>$\tilde{S}$</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Surplus autocorrelation</td>
<td>$\rho^S$</td>
<td>0.95</td>
<td></td>
</tr>
</tbody>
</table>

use NIPA data. In both cases data is quarterly and for the time period 1985:I to 2014:IV.

4.2 Externally calibrated parameters

Externally calibrated parameters are reported in Table 1. The model is quarterly, so we set the discount factor $\beta$ to 0.99. This results in an average real interest of 4% annually. We set the depreciation rate $\delta$ to 0.03, which gives an aggregate investment rate of 12 percent annually, consistent with aggregate US data. The output elasticity of labor $\alpha$ is chosen to match a labor share of 0.65. For wage rigidity, we take the estimate of the autoregressive coefficient of 0.31 from Beraja et al. (2016). The inverse Frisch elasticity is set to 2, consistent with Chetty et al. (2011). For the habit process, we follow Winberry (2016a), who shows that a surplus consumption ratio $\tilde{S} = 0.65$ and surplus autocorrelation $\rho^S = 0.95$ deliver a correlation of interest rates and output close to zero, as is the case in the data. We verify in Section 4 that this holds in our model.

4.3 Simulated method of moments

The vector of parameters to be estimated is

$$\theta_{SMM} = (\tilde{\xi}, \rho_z, \sigma_z, \kappa, \chi, \tilde{\phi}).$$

To estimate these parameters we proceed by simulated method of moments. We specify a vector of moments $g_t$ which we compute for each quarter in our sample. Our estimate $\hat{\theta}_{SMM}$ minimizes the minimum distance criterion function

$$Q(\theta) = \left(g \left(\tilde{S}; \theta\right) - T^{-1} \sum_{t=1}^{T} g_t\right)\prime W \left(g \left(\tilde{S}; \theta\right) - T^{-1} \sum_{t=1}^{T} g_t\right),$$
where \( \mathbf{g}(\bar{\mathbf{S}}; \theta) \) is the corresponding vector of moments computed from the model when aggregate shocks are all set to zero. Note that given our linear solution of the aggregate dynamics of the model, these also correspond to the time-series average of simulations of the model with aggregate shocks turned on, that is \( \mathbf{g}(\bar{\mathbf{S}}) = \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} \mathbf{g}(\mathbf{S}_t) \). The weighting matrix is diagonal with entries equal to \( 1/g_{mt}^2 \) for each moment \( m \).

We need to estimate six parameters. The six moments are chosen to provide power in identifying \( \theta_{SMM} \) and capture key properties of both firm level investment and finance. Where most papers in the literature consider these separately we make sure the model is jointly accountable to the investment behavior of firms and how this is financed. Investment moments include the fraction of non-adjusting firms and dispersion of investment rates. Finance moments include the rate of debt and equity issuance of firms that raise finance and the overall rate of financing. We also include the ratio of dividends to output which is informative for decreasing returns to scale. Finally we include the dispersion of sales growth. This last moment and a number of the others will also be used as observables in the Bayesian estimation step. As such it is important that they are included and matched in the steady state of the model. We now detail how we construct these moments.

**Investment**  The investment rate of firm \( j \) in period \( t \), \( \mathbf{ir}_t \), is defined as the ratio of gross investment of the firm between periods \( t \) and \( t + 1 \) to the average capital stock in the two periods:

\[
\mathbf{ir}_jt = \frac{(k_{jt+1} - k_{jt})}{\frac{1}{2} (k_{jt+1} + k_{jt})}.
\]

In the model, the equivalent variable is:

\[
\mathbf{ir}(k, z, \xi; \mathbf{S}) = \frac{k'(k, z, \xi; \mathbf{S}) - k}{\frac{1}{2} (k'(k, z, \xi; \mathbf{S}) + k)}.
\]

The investment inaction rate in period \( t \), \( \text{inaction}_t \), is the fraction of firms with investment rate \( \mathbf{ir}_jt \) less than 1 percent in absolute terms:

\[
\text{inaction}_t = \frac{1}{N_t} \sum_{j \in J_t} 1 \{|\mathbf{ir}_jt| < 0.01\}.
\]

where \( J_t \) is the set of firms of firms in the economy at time \( t \) and \( N_t \) is the number of firms at time \( t \). In the model, the corresponding moment is

\[
\text{inaction}(\mathbf{S}) = \int 1 \{|\mathbf{ir}(k, z, \xi; \mathbf{S})| < 0.01\} d\mu(k, z, \xi; \mathbf{S}).
\]
The standard deviation of investment rates, conditional on adjustment, at time $t$, is the standard deviation of $ir_{jt}$ across adjusting firms:

$$
\sigma^r_t = \frac{\sum_{j \in J_t} (ir_{jt} - \bar{ir}_t)^2 1 \{ |ir_{jt}| > 0.01 \}}{\sum_{j \in J_t} 1 \{ |ir_{jt}| > 0.01 \}}
$$

where $\bar{ir}_t$ is the mean investment rate. In the model, the corresponding moment is

$$
\sigma^r (S) = \int \frac{[ir (k, z, \xi; S) - \bar{ir} (S)]^2 1 \{ |ir (k, z, \xi; S)| < 0.01 \} d\mu (k, z, \xi; S)}{1 \{ |ir (k, z, \xi; S)| < 0.01 \} d\mu (k, z, \xi; S)}
$$

where $\bar{ir} (S)$ is the mean investment rate, conditional on adjustment.

**Finance** The positive issuance rate is defined as the sum of all positive issuance (i.e. negative dividends) by firms, divided by aggregated capital stock of the issuing firms. In the data this is computed

$$
ef_t = \sum_{j \in J_t} \frac{ef_{jt}}{k_{jt}} 1 \{ ef_{jt} > 0 \},
$$

where $ef_{jt}$ and $k_{jt}$ are the net external financing and capital stock, respectively, of firm $j$ in period $t$. The construction of these variables is detailed in Section B.1. The corresponding moment in the model is

$$
ef (S) = \int \frac{-d (k, z, \xi; S)}{k} 1 \{ d (k, z, \xi; S) \} d\mu (k, z, \xi; S).
$$

The average net issuance rate is defined as the negative of net payouts by firms, divided by the capital stock. In the model, this is

$$
nef(S) = \int \frac{-d (k, z, \xi)}{k} d\mu (k, z, \xi; S)
$$

and in the data,

$$
nef_t = \sum_{j \in J_t} \frac{ef_{jt}}{k_{jt}}.
$$

Sample selection and the removal of quarterly seasonality and industry fixed effects are described in Section B.1.
**Sales and output** The sales growth of a firm \( j \) at time \( t \) is the change in the firm’s sales between \( t \) and \( t - 1 \), as a fraction of average sales in the two periods:

\[
sg_{jt} = \frac{s_{jt} - s_{jt-1}}{\frac{1}{2} (s_{jt} + s_{jt-1})},
\]

where \( s_{jt} \) is sales for firm \( j \) in period \( t \) (see Section B.1 for details). The corresponding quantity in the model is the sales growth of a firm with state \((k, z, \xi)\) and previous state \((k_{-1}, z_{-1}, \xi_{-1})\), when the aggregate state is \( S \) and last period’s state was \( S_{-1} \):

\[
sg(k, z, \xi, k_{-1}, z_{-1}, \xi_{-1}; S, S_{-1}) = \frac{y(k, z, \xi; S) - y(k_{-1}, z_{-1}, \xi_{-1}; S_{-1})}{\frac{1}{2} (y(k', z', \xi'; S) + y(k, z, \xi; S_{-1}))},
\]

where \( y(k, z, \xi; S) = X^Z(S) z (n(k, z; S)^V k^{1-V})^\kappa \). At each period \( t \), the cross-sectional standard deviation of sales growth, \( \sigma_{sg}^t \), is the standard deviation of \( sg_{jt} \) across all firms:

\[
\sigma_{sg}^t = \frac{1}{N_t} \sum_{j \in J_t} (sg_{jt} - \frac{1}{N} \sum sg_{jt}).
\]

The corresponding moment in the model is

\[
\sigma_{sg}^t(S, S_{-1}) = \int \left\{ \int [sg(k, z, \xi, k_{-1}, z_{-1}, \xi_{-1}; S, S_{-1}) - \overline{sg}(S, S_{-1})]^2 d\mu(k, z, \xi; S) \right\} d\mu(k_{-1}, z_{-1}, \xi_{-1}; S_{-1}).
\]

Finally, the ratio of dividends to sales in the data and model are computed

\[
dy_t = \frac{\sum_{j \in J_t} d_{jt}}{\sum_{j \in J_t} s_{jt}}, \quad dy(S) = \frac{\int d(k, z, \xi) d\mu(k, z, \xi; S)}{\int y(k, z, \xi) d\mu(k, z, \xi; S)}.
\]

**Cleaning** In practice we compute these moments within sectors and remove seasonal and sectoral effects. Again, more details can be found in B.1. The vectors of moments are

\[
g_t = \left( \text{inaction}_t, \sigma_{ir}^t, \sigma_{sg}^t, \text{ef}_t, \text{nef}_t, dy_t \right), \quad g(S) = \left( \text{inaction} \left( \overline{S} \right), \sigma_{ir} \left( \overline{S} \right), \sigma_{sg} \left( \overline{S} \right), \text{ef} \left( \overline{S} \right), \text{nef} \left( \overline{S} \right), dy \left( \overline{S} \right) \right).
\]

**Parameter estimates** Table 2 summarizes the estimated parameters and vector of moments in the data and model. At these parameters the model very closely matches the data. The adjustment cost parameter \( \overline{\xi} \) implies that adjustment costs are 0.4% of output, which is close to the 1% reported by Khan and Thomas (2008). The estimated parameters for the idiosyncratic productivity process imply a cross-sectional dispersion of productivity in steady state of 0.25. This is slightly lower than the average dispersion of productivity.
Table 2: Targeted moments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_{SMM}$</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper bound of adjustment costs</td>
<td>$\bar{\xi}$</td>
<td>0.0009</td>
<td>Investment inaction rate</td>
<td>0.263</td>
<td>0.246</td>
</tr>
<tr>
<td>Persistence of firm-level shocks</td>
<td>$\rho_z$</td>
<td>0.955</td>
<td>SD sales growth</td>
<td>0.285</td>
<td>0.279</td>
</tr>
<tr>
<td>Average volatility of firm-level shocks</td>
<td>$\bar{\sigma}_z$</td>
<td>0.071</td>
<td>SD investment rates</td>
<td>0.154</td>
<td>0.153</td>
</tr>
<tr>
<td>Fixed operating cost</td>
<td>$\chi$</td>
<td>0.073</td>
<td>Dividends / sales</td>
<td>0.066</td>
<td>0.069</td>
</tr>
<tr>
<td>Decreasing returns to scale</td>
<td>$\kappa$</td>
<td>0.839</td>
<td>Positive issuance rate</td>
<td>0.070</td>
<td>0.073</td>
</tr>
<tr>
<td>Cost of external finance</td>
<td>$\bar{\phi}$</td>
<td>0.067</td>
<td>Net issuance rate</td>
<td>-0.015</td>
<td>-0.015</td>
</tr>
</tbody>
</table>

of 0.375 estimated by Imrohoroglu and Tuzel (2014) for the US, but substantially larger than that considered in other papers in the firm level investment literature.\(^9\) The degree of decreasing returns to scale is 0.839, which lies within the range of values in the literature: Bachmann and Bayer (2014) estimate decreasing returns to scale of 0.75, while Khan and Thomas (2008) calibrated to 0.9.

4.4 Bayesian estimation

We estimate the parameters of the shock processes $\theta_{MLE} = (\rho^Z, \sigma^Z, \rho^\psi, \sigma^\psi, \rho^\beta, \sigma^\beta, \rho^\sigma, \sigma^\sigma)$ using Bayesian methods, surveyed in An and Schorfheide (2007). For expositional purposes we estimate only four of the shocks, turning off the aggregate shock to the cost of external finance. We detail results for the model with all five shocks in the online appendix to this paper.

4.4.1 Computing the likelihood

As described in Section 3, for a given set of parameters $\theta$, we obtain a model solution of the form

$$\hat{\Theta}_t = A(\theta) \hat{\Theta}_{t-1} + B(\theta) \mathcal{E}_t$$

(20)

where $\Xi_t$ is a $n_\xi \times 1$ vector of latent states, and $\mathcal{E}_t$ is a $n_\varepsilon \times 1$ vector of standard normal Gaussian innovations

$$\mathcal{E}_t \sim N(0, \mathbb{I}_{n_\varepsilon \times n_\varepsilon})$$

\(^9\)Although unreported, the dispersion in productivity due to idiosyncratic shocks in these papers is as follows: Bloom et al. (2012) - 0.12, Winberry Winberry (2016a) - 0.07, Khan and Thomas (2008) - 0.04. These appear counterfactually small.
We define an observation equation

\[ \Upsilon_t = D\Theta_t + C\eta_t \]  

(21)

where \( \Upsilon_t \) is an \( n_y \times 1 \) vector of observables, \( D \) is an \( n_y \times n_x \) matrix which selects elements of \( \Theta_t \), \( \eta_t \) is an \( n_y \times 1 \) vector of measurement errors and \( C \) is an \( n_x \times n_x \) covariance matrix.

Given a set of time series data corresponding to the elements of \( \Upsilon_t \), the likelihood of a model in the form (20)-(21) is easily computed using the Kalman filter.\(^{10}\)

### 4.4.2 Observable time series

In order to estimate the parameters associated with four aggregate shocks we must specify four observable time-series, that is \( n_y = 4 \). We include three variables that are standard in the literature—output, consumption, and hours worked—and one variable which we construct from the Compustat microdata: the standard deviation of sales growth. We take the aggregate time series from NIPA, take logs, and filter using a one-sided HP filter. The resulting time series are plotted in Figure 2.

For the standard deviation of sales growth—and other cross sectional time series that we compare in the model and data—we remove seasonal effects at a quarterly frequency and industry fixed effects. Each series is then de-trended by removing a cubic time trend. See Appendix B.1 for details. The variable definitions are given in Section 4.3. The resulting time series are plotted in Figure 2-5.

Note that the approach we have taken allows movements in the distribution of firms to identify aggregate shocks. In this version of the paper we use only the standard deviation of sales growth as a proxy for the various measures of dispersion which have been documented to be countercyclical (labor growth, profit growth, prices, credit spreads, stock returns).

Since we have four time series and four shocks, we could proceed by setting all elements of the measurement equation matrix \( C \) to zero. However, we find in preliminary calculations that, for some set of parameters, this gives likelihoods which are indistinguishable from zero to machine precision. In order to avoid this artefact of the numerical estimation, we allow for some measurement error in the estimation: in particular, we allow for error in the measurement of the cross-sectional variable \( \sigma_{t}^{sg} \), since we conjecture that this is measured with more error than the aggregate variables. We set all elements of \( C \) to zero except for the diagonal element corresponding to \( \sigma_{t}^{sg} \), which we denote \( \eta_{\sigma} \), and include as a parameter.

\(^{10}\)Initializing the filter requires that we specify a prior mean and variance for the state in the initial period. We pick the prior distribution to have zero mean (since the state is expressed as deviations from steady state), and variance \( \Sigma \), where \( \Sigma \) is the unconditional variance-covariance matrix of \( \Theta_t \), which solves \( \Sigma = A\Sigma A' + BB' \), which we obtain by simulation.
Table 3: Shock process parameters.

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter</th>
<th>Type</th>
<th>Mean</th>
<th>SD</th>
<th>Mode</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation, TFP shock</td>
<td>$\rho^Z$</td>
<td>Beta</td>
<td>0.600</td>
<td>0.260</td>
<td>0.982</td>
<td>0.032</td>
</tr>
<tr>
<td>Autocorrelation, labor supply shock</td>
<td>$\rho^\psi$</td>
<td>Beta</td>
<td>0.600</td>
<td>0.260</td>
<td>0.989</td>
<td>0.006</td>
</tr>
<tr>
<td>Autocorrelation, time preference shock</td>
<td>$\rho^\beta$</td>
<td>Beta</td>
<td>0.600</td>
<td>0.260</td>
<td>0.628</td>
<td>0.143</td>
</tr>
<tr>
<td>Autocorrelation, uncertainty shock</td>
<td>$\rho^\sigma$</td>
<td>Beta</td>
<td>0.600</td>
<td>0.260</td>
<td>0.945</td>
<td>0.177</td>
</tr>
<tr>
<td>Standard deviation, TFP shock</td>
<td>$\sigma^Z$</td>
<td>Exp</td>
<td>0.100</td>
<td>0.100</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>Standard deviation, labor supply shock</td>
<td>$\sigma^\psi$</td>
<td>Exp</td>
<td>0.100</td>
<td>0.100</td>
<td>0.011</td>
<td>0.006</td>
</tr>
<tr>
<td>Standard deviation, time preference shock</td>
<td>$\sigma^\beta$</td>
<td>Exp</td>
<td>0.100</td>
<td>0.100</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>Standard deviation, uncertainty shock</td>
<td>$\sigma^\sigma$</td>
<td>Exp</td>
<td>0.100</td>
<td>0.100</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>

We compute the mode and standard deviation of the posterior by drawing from the posterior distribution using the Metropolis-Hastings algorithm, as described in . We use 10 chains of 100,000 draws each.

in our estimation.

4.4.3 Priors and posteriors

We estimate the parameters using Bayesian methods following An and Schorfheide (2007). The independent prior distributions for each of the estimated parameters are given in the first three columns of Table 3. For all the autocorrelation parameters ($\rho^Z, \rho^\psi, \rho^\beta, \rho^\sigma$) we set a Beta prior with a mean of 0.6 and standard deviation of 0.26. This is a dispersed prior which restricts the parameters to be between 0 and 1, as autocorrelation parameters must be for the process to be stationary, and is close to the prior chosen by Smets and Wouters (2007). For the standard deviation parameters ($\sigma^Z, \sigma^\psi, \sigma^\beta, \sigma^\sigma$), we set an exponential prior with mean 0.1. The conventional choice in the literature is to set priors that follow the inverse Gamma distribution for standard deviation parameters. However, this is not an appropriate choice for us, since the inverse Gamma function tends to 0 as the argument approaches zero and therefore tends to move the standard deviation away from zero.\(^ {11}\) Given prior densities we compute the posterior using the Metropolis-Hastings algorithm. This is implemented by using 10 parallel chains of 100,000 draws, of which we discard the first 10,000 draws of each chain and splice together the remaining draws. We calculate convergence diagnostics as described in Gelman and Rubin (1992) and verify that the $\sqrt{R}$ statistic is less than 1.05 for each parameter separately.

The prior and posterior densities for each parameter are plotted in Figure 1. The estimated persistence for the TFP shock, $\rho^Z$, and for the labor supply shock, $\rho^\psi$, are very high and reasonably well-identified, at 0.982 and 0.989, respectively, whereas the estimated

\(^{11}\)See Gelman (2006) for a discussion of the inverse Gamma distribution as a prior.
Table 4: Forecast error variance decomposition, 1 quarter ahead

<table>
<thead>
<tr>
<th>Shock</th>
<th>Output</th>
<th>Consumption</th>
<th>Labor</th>
<th>SD sales growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0.36</td>
<td>0.23</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Labor supply</td>
<td>0.56</td>
<td>0.36</td>
<td>0.85</td>
<td>0.08</td>
</tr>
<tr>
<td>Time preference</td>
<td>0.08</td>
<td>0.40</td>
<td>0.12</td>
<td>0.86</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The persistence of the time preference shock is lower, at 0.628. We see from the high standard deviation of the posterior distribution. The posterior modes of the persistence parameters are broadly consistent with previous estimates in RBC and New Keynesian models: Smets and Wouters (2007), for instance, find high persistence in the TFP shock and wage markup shocks (0.95 and 0.97, respectively), which are similar to our TFP and labor supply shocks, whereas they find a low persistence (0.22) for their risk premium shock, which, like our time preference factor shock, appears as the a wedge in the household’s Euler equation. The posterior standard deviations are smaller than the prior standard deviations in all cases. The two parameters which seem to be the least well identified are the persistence parameters of the preference and uncertainty shocks. We return to explain these following our discussion of our results.

5 Results

With the model estimated we can quantitatively approach our main question: Are movements in firm-level dispersion a source of or a response to business cycle fluctuations? This can be answered by studying variance decompositions of the model. These attribute the percentage of the variance of a given model-series due to each shock. We compute these for a number of different horizons. To be clear, we would conclude that firm-level dispersion is a source of business cycle fluctuations if the variance decomposition attributed a large fraction of fluctuations in output to the uncertainty shock. While firm-level dispersion is a response to business cycle fluctuations if the variance decomposition attributing a large fraction of fluctuations in sales growth dispersion to the productivity, labor disutility and preference shocks.

We find that neither of these cases jumps out from our results. Figures 6-7 show the forecast error variance decomposition of the four observables used in the estimation. The decompositions for one quarter ahead and two quarters ahead are summarized in Tables 4 and 5.
We see from Figure 7 that, except for at a 1-quarter horizon, almost all of the fluctuations in the standard deviation of sales growth are explained by the uncertainty shock. The timing of the uncertainty shock is such that an uncertainty shock at time $t$ means that productivities will be more dispersed at time $t+1$. Therefore it is not surprising that the uncertainty shock has a very small effect on impact, especially on sales growth between $t$ and $t-1$. In period $t$, the only margin available to firms which might influence $\sigma_{t^g}$ is the hiring decision.

In Figure 8, we plot what the model implies for $\sigma_{t^g}$ when the uncertainty shock is shut down (i.e. $X^g_t$ is set to 0). This confirms the result of the forecast error variance decomposition. The RBC shocks explain a negligible portion of the movement in $\sigma_{t^g}$ across the entire time period.

Similarly, almost all of the movement in aggregates is explained by the “RBC” shocks (the TFP, labor supply and discount factor shocks). Figure 6 shows that the uncertainty shock explains 1%, 1% and 2% of the movements in output, consumption and labor, respectively, at a 1-period horizon.

Figure 6 shows that the largest relative effect of the uncertainty shock on output is in fact at a 10-quarter horizon. This surprising result is illuminated by looking at the impulse response of output to an uncertainty shock, which is plotted (for the posterior mode parameters) in Figure 10. There is an initial drop in output, due to the wait-and-see effect of an uncertainty shock, documented in Bloom (2009), but it is then followed by large and persistent rebound in output. This is due to the dispersion in productivities which accompanies the realization of the uncertainty shock. Since optimal capital is concave in productivity, a more dispersed productivity distribution results in an investment boom and an increase in output. This rebound effect is stronger than the initial downward effect of uncertainty on output. This is why the uncertainty shock affects output the most at a 10-quarter horizon. The intuition is further borne out by Figure 9, which shows the movement in output if all shocks except for the uncertainty shock are set to zero. The uncertainty shock alone induces a procyclical movement in uncertainty. At times when the standard deviation of sales growth is high, implying a high level of uncertainty, the uncertainty shock increases output, which is completely counterfactual. These observations are hard to square with the theory of an uncertainty-driven recession.

We can further compose the response to the uncertainty shock into two parts: the portion that is driven by changes in perceived volatility of future outcomes and the portion that is driven by increased realized volatility. In Figure 11, we plot the response of output and investment to a “news-only” uncertainty along with the original uncertainty shock. The “news-only” uncertainty is a shock to the beliefs of agents about future volatility without the corresponding realization of increased volatility. To compute the IRF with respect to
the “news-only” output shock, we compute how the economy evolves when we shock the dispersion of firm-level productivity shocks as before, but we restrict the firm policy function to stay at its steady state value. We see that the news component of the uncertainty shock generates a drop in investment and output, a kind of wait-and-see effect, as documented by Bloom (2009). There is no overshooting of output and investment, as there is in the baseline case. This indicates that the overshooting is driven by the increase in realized dispersion. A firm’s optimal choice of capital is a convex function of productivity, so in the absence of frictions, an increase in the dispersion of productivity would lead to an increase in investment and output.

This result indicates that a different way of modelling uncertainty shocks may be more fruitful, in terms of explaining the comovements of aggregates and firm dispersion. A definition of an uncertainty shock which entails only less information about future productivity and not also a realized increase in the dispersion of productivity might result in negative co-movement of output and firm dispersion without the rebound and overshoot that we observe in our baseline definition.

### 6 Conclusion

The cyclicality of firm dispersion suggests that firm dispersion may be a driving factor of business cycles or an endogenous response to business cycles or some combination of both. We build a general equilibrium model of firm investment and financing rich enough to quantify the relative strengths of these forces. We include investment and financing frictions so shock standard business cycle shocks (a TFP shock, a labor supply shock and a time preference shock) may have effects on the dispersion of firm-level sales growth and investment rates. We allow the model to be shocked by an uncertainty shock, i.e. a shock to the distribution of firm-level productivities.

We use a novel solution method that allows us to characterize the model solution in linear form, for which estimation is computationally feasible. We find that uncertainty

<table>
<thead>
<tr>
<th>Shock</th>
<th>Output</th>
<th>Consumption</th>
<th>Labor</th>
<th>SD sales growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0.37</td>
<td>0.31</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Labor supply</td>
<td>0.57</td>
<td>0.47</td>
<td>0.89</td>
<td>0.00</td>
</tr>
<tr>
<td>Time preference</td>
<td>0.06</td>
<td>0.22</td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
shocks account for almost all of the variation in firm-level dispersion, and that standard business cycle shocks account for all of the movement in aggregates. Part of the reason for this is that, at the estimated parameters, although output falls initially in response to a rise in uncertainty, it then rebound and overshoots, resulting in an eventual boom. This result calls into question the idea of an uncertainty-driven recession.
References


Appendix

This appendix is organized as follows Section A provides more details on the solution method. Section B provides more details on our construction of the moments used in the estimation of the model, the data used, how it is cleaned, and various adjustments made. Section C provides all figures referenced, but not included, in the main text.

A Method details

A.1 Discretized equilibrium conditions

We reproduce in this section the full set of equilibrium conditions in terms of the finite representation of the model state. We follow the Khan and Thomas (2008) trick of writing the Bellman equation in terms of an adjusted value function which is scaled by marginal utility \( \tilde{v}_t \equiv u'(C_t) \tilde{v}_t \). Let \( \Phi^V \) be the (time-invariant) basis function for the cubic spline that approximates the adjusted value function, and let \( \theta^V_t \) be the corresponding vector of approximating coefficients. Denote by \( K \) the set of approximating points for capital \( k \), and by \( Z \) the set of approximating points for productivity \( z \).

(Bellman equation.) For all \( k_t \in K \) and \( z_j \in Z \),

\[
\begin{align*}
\Phi^V (k_t, z_j) \theta^V_t &= \frac{\xi_t^*(k_t, z_j)}{\xi} \left[ V^\text{adj}_t (k_t, z_j) - \frac{1}{2} W_t \xi_t^* (k_t, z_j) \right] + \left( 1 - \frac{\xi_t^*(k_t, z_j)}{\xi} \right) V^\text{stay}_t (k_t, z_j) \\
\xi_t^* (k_t, z_j) &= \frac{V^\text{adj}_t (k_t, z_j) - V^\text{stay}_t (k_t, z_j)}{W_t} \\
V^\text{adj}_t (k_t, z_j) &= pt \left( d^\text{adj}_t (k_t, z_j) - 1 \left\{ d^\text{adj}_t (k_t, z_j) < 0 \right\} X_t^\phi (d^\text{adj}_t (k_t, z_j))^2 \right) \\
&\quad + X_t^\beta \sum_{z' \in Z} P (z, z'; X_t^\gamma) \Phi^V (k^\text{adj}_t (k_t, z_j), z') \theta^V_{t+1} \\
V^\text{stay}_t (k_t, z_j) &= pt \left( d^\text{stay}_t (k_t, z_j) - 1 \left\{ d^\text{stay}_t (k_t, z_j) < 0 \right\} X_t^\phi (d^\text{stay}_t (k_t, z_j))^2 \right) \\
&\quad + X_t^\beta \sum_{z' \in Z} P (z, z'; X_t^\gamma) \Phi^V (k^\text{stay}_t (k_t, z_j), z') \theta^V_{t+1} \\
d^\text{adj}_t (k_t, z_j) &= \pi_t (k_t, z_j) - \left( k^\text{adj}_t (k_t, z_j) - (1 - \delta) k_t \right) \\
d^\text{stay}_t (k_t, z_j) &= \pi_t (k_t, z_j) - \left( k^\text{stay}_t (k_t, z_j) - (1 - \delta) k_t \right) \\
\pi_t (k_t, z_j) &= X_t^\gamma z_j \nu \theta_t (k_t, x_j) W_t - W_t n_t (k_t, x_j) - \chi \\
n_t (k_t, z_j) &= \left( X_t^\gamma z_j (1 - \nu) W_t^{-1} \right)^{-1/n}
\end{align*}
\]

(Firm optimality.) For all \( k_t \in K \) and \( z_j \in Z \),

\[
pt \left( 1 - 2 \phi d^\text{adj}_t (k_t, z_j) 1 \left\{ d^\text{adj}_t (k_t, z_j) < 0 \right\} \right) = \beta \sum_{z' \in Z} P (z, z') \frac{\partial \Phi^V}{\partial k} (k^\text{adj}_t (k_t, z), z') \theta^V_t
\]
(Law of motion.) For all $k_{j'} \in K$ and $z_{j'} \in Z$,

$$\lambda_t (k_{j'}, z_{j'}) = \sum_{k_i \in K} \sum_{z_j \in Z} Q (k_{j'}, z_{j'}, k_i, z_j) \lambda_{t-1} (k_i, z_j)$$

$$Q (k_{j'}, z_{j'}, k_i, z_j) = P (z_{j'}, z_j) \times \begin{cases} \xi^* (k_i, z_j) \times \frac{k_{j'+1}^{k_{adj}(k_i, z_j)} - k_{j'}^{k_{adj}(k_i, z_j)}}{k_{j'+1}^{k_{adj}(k_i, z_j)} - k_{j'}^{k_{adj}(k_i, z_j)}} & \text{if } k^{adj} (k_i, z_j) \in [k_{j'}, k_{j'+1}] \\ \xi^* (k_i, z_j) \times \frac{k_{adj}(k_i, z_j) - k_{j'-1}^{k_{adj}(k_i, z_j)}}{k_{j'-1}^{k_{adj}(k_i, z_j)} - k_{j'}^{k_{adj}(k_i, z_j)}} & \text{if } k^{adj} (k_i, z_j) \in [k_{j'-1}, k_{j'}] \\ (1 - \xi^* (k_i, z_j)) \times \frac{k_{j'+1}^{k_{adj}(k_i, z_j)} - k_{j'}^{k_{adj}(k_i, z_j)}}{k_{j'+1}^{k_{adj}(k_i, z_j)} - k_{j'}^{k_{adj}(k_i, z_j)}} & \text{if } k^{adj} (k_i, z_j) \in [k_{j'} - k_{j'-1}] \\ (1 - \xi^* (k_i, z_j)) \times \frac{k_{adj}(k_i, z_j) - k_{j'-1}^{k_{adj}(k_i, z_j)}}{k_{j'-1}^{k_{adj}(k_i, z_j)} - k_{j'}^{k_{adj}(k_i, z_j)}} & \text{if } k^{adj} (k_i, z_j) \in [k_{j'} - k_{j'-1}] \end{cases}$$

(Labor market clearing.)

$$N_t = \sum_{k_i \in K} \sum_{z_j \in Z} n_t (k_i, z_j) \lambda_{t-1} (k_i, z_j) + \sum_{k_i \in K} \sum_{z_j \in Z} \xi 1 \{\xi \leq \xi^* (k_i, z_j)\} \lambda_{t-1} (k_i, z_j)$$

(Wage law of motion.)

$$W_t = \omega W_t^* + (1 - \omega) W_{t-1}$$

$$W_t^* = \frac{\psi X_t' N_t'}{\rho_t}$$

(Stochastic discount factor.)

$$\rho_t = (C_t S_t)^{-\alpha}$$

(Habit stock law of motion.)

$$\log S_t = (1 - \rho^S) \log \bar{S} + \rho^S \log S_{t-1} + \lambda^S \log (C_t / C_{t-1})$$

(Resource constraint.)

$$C_t = Y_t - I_t - A_t$$

$$Y_t = \sum_{k_i \in K} \sum_{z_j \in Z} X_t^2 z_t \{ n (k_i, z_j)^{i' \nu} k^{k_{1-i}}_{i'1} \} \lambda_{t-1} (k_i, z_j)$$

$$I_t = \sum_{k_i \in K} \sum_{z_j \in Z} \frac{\xi^* (k_i, z_j)}{\xi} \{ k^{adj} (k_i, z_j) - (1 - \delta) k_i \} \lambda_{t-1} (k_i, z_j)$$

$$+ \sum_{k_i \in K} \sum_{z_j \in Z} \{ 1 - \frac{\xi^* (k_i, z_j)}{\xi} \} \{ k^{stay} (k_i, z_j) - (1 - \delta) k_i \} \lambda_{t-1} (k_i, z_j)$$

$$A_t = \sum_{k_i \in K} \sum_{z_j \in Z} \frac{\xi^* (k_i, z_j)}{\xi} \frac{1}{2} W_t \xi^* (k_i, z_j) \lambda_{t-1} (k_i, z_j)$$

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B Data

B.1 Data

The investment and issuance data targets are computed for the sample of firms in the Compustat North American quarterly database for the period 1985:I to 2014:IV.

We construct a measure of firm capital using a perpetual the inventory method, following Clementi and Palazzo (2010). For firm \( j \), we set the initial value of the capital stock \( k_{j,t} \) to the first available entry for PPEGT\(_{j,t} \) (gross value of property, plant and equipment), and recursively construct the value for the capital stock using PPENTQ\(_{j,t} \) (net value of property, plant and equipment)

\[
k_{j,t+1} = k_{j,t} + \text{PPENT}_{j,t+1} - \text{PPENT}_{j,t},
\]

where we interpolate linearly for PPENT\(_{j,t} \) wherever PPENT\(_{j,t} \) is missing and PPENT\(_{j,t+1} \) and PPENT\(_{j,t-1} \) are available.

We define the investment rate for firm \( j \) in period \( t \), \( ir_{j,t} \), as:

\[
ir_{j,t} = \frac{k_{j,t} - k_{j,t-1}}{\frac{1}{2}(k_{j,t} + k_{j,t-1})}
\]

and sales growth \( sg_{j,t} \)

\[
sg_{j,t} = \frac{s_{j,t} - s_{j,t-1}}{\frac{1}{2}(s_{j,t} + s_{j,t-1})},
\]

where for sales \( s_{j,t} \) we use the variable \( \text{SALES}_{j,t} \).

For the external financing variables, we first construct an unadjusted measure of net cashflow from external finance. For unadjusted net cashflow from external financing, \( \hat{ef}_{j,t} \), we use FINCFY\(_{j,t} \) (net cash flow from financing activities) if it is available. If it is not available, we use the sum of its constituent parts,

\[
\hat{ef}_{j,t} = \text{STK}_{j,t} - \text{PRSTKC}_{j,t} - \text{DV}_{j,t} + \text{DLTIS}_{j,t} - \text{DLTR}_{j,t} + \text{DLCCH}_{j,t} + \text{FIAO}_{j,t},
\]

where SSTKY\(_{j,t} \) is net cash flow from sale of common and preferred stock, PRSTKCY\(_{j,t} \) is net cash flow from purchase of common and preferred stock, DVY\(_{j,t} \) is net cash flow from cash dividends, DLTIS\(_{j,t} \) is net cash flow from long-term debt issuance, DLTR\(_{j,t} \) is net cash flow from long-term debt reduction, DLCCH\(_{j,t} \) is net cash flow from changes in current debt and FIAO\(_{j,t} \) is net cash flow from other financing activities. If any of the constituent parts is missing, we replace that part with zero.

In the model, firms raise external finance in order to invest, to hire labor or to pay costs.
In the data, however, a significant portion of net financing activity involves funding financial investment (that is, buying financial assets). We adjust the measure of external financing to account for this: we define adjusted external financing, $\text{ef}_{j,t}$, as unadjusted external financing less net cashflow due to increases in financial assets:

$$
\text{ef}_{j,t} = \hat{\text{ef}}_{j,t} - \left( -\text{IVCH}_{j,t} + \text{SIV}_{j,t} + \text{IVSTCH}_{j,t} - \text{AQC}_{j,t} + \text{IVACO}_{j,t} + \text{CHECH}_{j,t} \right)
$$

where $\text{IVCH}_{j,t}$ is net cashflow from increase in investments, $\text{SIV}_{j,t}$ is net cashflow from sale of investments, $\text{IVSTCH}_{j,t}$ is net cashflow from change in short-term investments, $\text{AQC}_{j,t}$ is net cashflow from acquisitions, $\text{IVACO}_{j,t}$ is net cashflow from other investing activities, and $\text{CHECH}_{j,t}$ is increase in cash and cash equivalents.

### B.1.1 Quarterly effects

Before computing moments and constructing the time series, we remove quarter effects at the firm level. (Note that we remove the effect of a particular quarter of the year $q \in \{1, 2, 3, 4\}$ not the effect of a particular time period.)

For each of $\text{ir}_{j,t}$, $\text{sg}_{j,t}$ and $\text{ef}_{j,t}$, let $x_{ijyq}$ be the value for firm $j$ in year $y$ and quarter $q$. We first drop incomplete firm-years. We then define the firm-year average,

$$
\bar{x}_{iy} \equiv \frac{1}{4} \sum_{q=1}^{4} x_{ijyq}, \quad \forall i \in I, y \in Y,
$$

where, $I$ is the set of all firms, and $Y$ is is the set of all years in the sample. We then compute each firm-quarter’s deviation from the firm-year average

$$
h_{ijyq} \equiv x_{ijyq} - \bar{x}_{iy}, \quad \forall i \in I, q \in \{1, 2, 3, 4\}, y \in Y,
$$

We compute firm $i$’s average quarter-$q$ deviation across years,

$$
\gamma_{iq} \equiv \frac{1}{T_i} \sum_{y \in Y_i} h_{ijyq}, \quad \forall i \in I, q \in \{1, 2, 3, 4\},
$$

where $Y_i$ is the set of years for which firm $i$ has observations. We define each firm $i$’s excess deviation for a period $qy$ as its actual quarterly deviation in $qy$ less its average quarterly deviation in quarter $q$,

$$
\delta_{ijyq} = h_{ijyq} - \gamma_{iq}, \quad \forall i \in I, q \in \{1, 2, 3, 4\}, y \in Y
$$
and define the quarterly-adjusted value as the firm-year mean plus the firm-quarter’s excess deviation:

\[ \tilde{x}_{iqy} = \bar{x}_{iy} + \delta_{iqy}. \]

**B.1.2 Industry effects**

We remove industry fixed effects at the level of 2-digit SIC industries.
C Figures

Figure 1: Prior and posterior densities

Figure 2: Aggregate time series.

Figure 3: Countercyclicality of cross-sectional sales growth dispersion.
Figure 4: Procyclicality of fraction of adjusting firms.

Figure 5: Procyclicality of dispersion of investment rate (conditional on adjustment)
Figure 6: Forecast error variance decomposition for macroeconomic aggregates.
Figure 7: Forecast error variance decomposition for dispersion measures

(a) Standard deviation of sales growth

(b) Standard deviation of investment rates (conditional on adjustment)
Figure 8: Contribution of TFP, labor supply and discount factor shocks to the standard deviation of sales growth.

![Movements in SD sales growth explained by RBC shocks](image)

- Model with RBC shocks only
- Data

Figure 9: Contribution of uncertainty shock to output

![Movements in output explained by uncertainty shocks](image)

- Model with uncertainty shock only
- Data
Figure 10: Impulse responses to uncertainty shock

Figure 11: Response of aggregates to “news-only” uncertainty shock