Long-Term Finance and Economic Development

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Abstract

What are the linkages between the maturity of corporate debt, the liquidity of financial markets and the real economy? Relative to developed countries, firms on emerging economies borrow at shorter maturities and assets are traded in markets with larger frictions. This paper studies how firms choose and finance long-term investment projects in a production economy subject to an over-the-counter trading friction in financial markets. We show that the liquidity spread increases with the maturity of the asset, generating an upward sloping yield curve. As a result, improvements in the liquidity of financial markets flatten the yield curve, improve borrowing terms for long-term projects, and induce firms to invest at longer horizons. The model is calibrated to match the US corporate debt market and counterfactual exercises show that the liquidity of the secondary market can account for a significant fraction of the variations in maturity choices across countries. Finally, we propose a new policy designed to improve the liquidity of financial markets. The intervention is effective to improve the liquidity, and as a result, firms extend the maturity of projects which generate substantial welfare gains.

JEL Classifications: E44, G30, O16.

Keywords: Debt maturity, Over-the-counter market, Liquidity, Secondary markets.

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1 Introduction

Rome was not built in a day.
Li Proverbe au Vilain, 1190

Policy makers argue that access to long-term finance is an important determinant of economic development because it helps to build back-loaded projects which have higher returns than short-term investment.\footnote{For example, see World Economic Forum (2011); European Comission (2013); OECD (2013); Group of Thirty (2013); World Bank (2015) among others.} Indeed, there is a vast empirical literature documenting the lack of long-term finance in developing countries, while there is no structural framework designed for policy analysis. The aim of this paper is to build a model about the determinants of long-term finance and what policies can stimulate it. Importantly, the model captures that financial markets for corporate debt have trading frictions which are more pronounced in developing countries than in advanced ones. We show that improvements in the liquidity of financial markets reduce borrowing costs, particularly at longer horizons, because long-term securities rely more on the possibility of being traded in secondary markets than short-term assets. This is a new channel through which financial development—an increase in market’s liquidity—can promote economic development. Finally, we propose a new government intervention called \textit{Government-Sponsored Intermediaries} and show that it can be powerful to improve the liquidity of financial markets and stimulate the use of long-term finance.

Section 2 explores the empirical evidence on corporate debt maturity and the liquidity of secondary markets across countries. First, it is well-known that firms borrow at shorter maturities in developing countries. For example, the median maturity is seven years in developing countries, while it is eleven years in advance ones (Cortina Lorente, Didier, and Schmukler, 2016). Second, using data about asset trading in secondary markets for the US, Asia, and Latin America, we show that trading frictions are more severe in developing countries.

To understand the linkages between the maturity of corporate debt, the liquidity of financial markets and the real economy we study a production economy with an over-the-counter (OTC) trading friction in the corporate bonds market à la Duffie, Gárleanu, and Pedersen (2005). An investor has to choose the duration of the investment project in which longer periods of investment have higher returns but the payoff realizes at a later period. Figure 1 shows a schematic representation of financial markets. Firms finance their investments with issuances of bonds in the primary market taking as given the yield curve and choosing the maturity
structure of their liabilities. These corporate bonds are traded in an OTC secondary market. We use this framework to study how trading frictions in the secondary market affect the yield curve and financial and investment decisions of firms.

How does trading frictions affect the yield curve? In Section 3 we answer this question by analyzing the lenders’ problem and deriving two important properties. First, the credit spread due to liquidity, which is endogenous in the model, is increasing in maturity. Intuitively, an investor can exit a long position by selling the asset on the secondary market, or waiting until maturity. Hence, longer-term assets rely more on the possibility of being traded. As a result, the trading friction delivers an upward sloping yield curve which is consistent with the data. Second, liquidity is more important for long-term assets, in the sense that improvements in liquidity not only reduce the liquidity spread but, more importantly, they also reduce the slope of the credit spread with respect to maturity. Finally, we show that there is a feedback loop between default and liquidity in which the liquidity spread increases with the default probability.

Section 4 studies what determines the choice of investment projects and the financial decisions. Firms have access to a continuum of potential investment plans differentiated by their back-loaded profile: longer periods of investment have future higher returns but they need external financing for a longer horizon. We show that this life-cycle profile for investment projects is supported by alternative microfoundations. Regarding the financial decisions, firms need to issue long-term debt to finance investment. We show that the steepness of the yield curve plays a major role in investment choices. When the term premium—the difference between borrowing long- and short-term—is high, long-term projects are too costly to finance which induce firms to choose shorter-term projects.

Section 5 solves the equilibrium of lenders and borrowers decisions and studies the effect of financial development on credit spreads and maturity choices. In particular, we evaluate the
consequences of an increase in the liquidity of the OTC market. First, when the secondary market is more liquid, there is a level effect in which the credit spread diminish for all maturities. More importantly, there is also a slope effect in which the yield curve flattens because long-term assets rely more on the secondary market than shorter-term assets. Hence, financial development reduces the interest rate schedule and its slope to maturity. Regarding the maturity choice, a back-loaded profile implies that to build a more profitable firm an entrepreneur needs to borrow for a longer period. Hence, when borrowing costs at longer horizons decline, firms invest in more profitable longer-term projects. This result shows that the observed short maturity in the data can be interpreted as a substitution for the liquidity of the secondary market.

Section 6 estimates the model to match moments from the US corporate debt market. We identify the liquidity spread as the difference between the yield curve for Treasuries and the High-quality Market of corporate bonds (rated A and above). Importantly, this spread is increasing in maturity both in the model and the data. Then, we use the estimated model to assess the effects of variations in trading frictions to account for cross-country differences on credit spreads and output per capita. To identify financial system across countries, we consider variations in search costs such that we can match the observed maturity choice in the cross-country data. Using the estimation of financial systems across countries, we find that the model predicts credit spreads close to the cross-country data counterpart, and a relationship between maturity and GDP quite similar to the data for developed economies and about half of the GDP differences for developing countries. Note that we did not use any data on credit spreads and GDP across countries in the estimation of the model, which suggest that the model successfully captures some important features of maturity choices.

Section 7 explores the normative implications of a policy designed to increase the liquidity of financial markets. Government-Sponsored Enterprises are institutions created by the US Congress to enhance the availability and reduce the cost of credit to households (e.g., Fieldhouse, Mertens, and Ravn, 2017). In a similar vein, we propose the creation of an institution designed to improve the credit conditions to the corporate sector called Government-Sponsored Intermediaries (GSIs). This institution intermediates corporate securities in secondary markets by buying and selling bonds at a potentially different price than in private bilateral meetings. The government chooses the size of the institution—i.e., the measure of government traders—and buy and sell prices. This intervention can be costly for the government, so they also implement a proportional profit tax to the corporate sector to finance GSIs. Under the optimal policy, the government buys at higher prices than in private meetings to provide more gains.
from trade to private sellers and alleviate the trading frictions. The government chooses to sell securities at a lower price than in private meetings to stimulate the entry of potential buyers to the market. Note that this policy is similar to the large-scale asset purchased operation adopted in the aftermath of the 2008 financial crisis ("Quantitative easing" one, two, and three). Interestingly, one difference with those interventions is that GSIs buy and sell securities without a large increase in the balance sheet of the FED. We show that the optimal policy can increase the market’s liquidity, and, therefore flattens the yield curve and stimulates the use of long-term finance. Quantitatively, the policy generates an increase in maturity of about half a year for an economy like the US and an increase of about one-and-a-half years for an economy with a financial system as in Latin America.

In the remaining sections of the paper we perform several extensions and robustness exercises. First, in the benchmark model firms can issue bonds only at the beginning of the project; they are not allowed to rollover short-term contracts to finance long-term projects. However, one important question regarding the importance of long-term finance is why firms do not use short-term debt to finance their investment needs. Previous empirical studies have shown that firms tend to match the maturity of their assets and liabilities. They often use long-term debt to make long-term investments, such as purchases of fixed assets or equipment, while they use short-term debt to finance working capital such as payroll and inventory.\(^2\) Section 8 extends the model and allow entrepreneurs to rollover short-term contracts to finance long-term projects. In this framework, the total financial cost is composed by costs of issuance and costs associated to the illiquidity of the secondary market. We show that firms in this extension have the same trade-offs as in the benchmark model. When the secondary market is more liquid, the total financial cost reduces, firms decide to rollover less frequently, borrow at longer maturities, and start projects with longer duration.

Another potential concern is that the liquidity of the secondary market and the rollover cost can be associated. In the model, the liquidity cost is endogenous while the rollover cost is exogenous and fixed—i.e., issuance costs do not respond to changes in the market’s liquidity, and a potential Lucas critique can apply. However, when the secondary market becomes more liquid both issuance and rollover costs should diminish which implies that fixed issuance costs is a conservative assumption. In the model, both when the liquidity of the secondary market

\(^2\)Studies for developed and developing countries find evidence that firms do match the maturity of their assets and liabilities (e.g., Stohs and Mauer (1996) for the United States; Schiantarelli and Sembenelli (1997) for Italy and the United Kingdom; Schiantarelli and Srivastava (1997) for India; and Schiantarelli and Jaramillo (2002) for Ecuador). Additionally, Graham and Harvey (2001) use survey data to show that chief financial officers of US companies consider matching the maturity of their firm’s debt with the life of its assets as the most important factor affecting their choice between short- and long-term debt.
improves and when the issuance cost diminishes, the firm adopts a project of longer maturity. Hence, if the two effects are present—an increase in liquidity and a reduction of the issuance cost—we expect that the firm will borrow at longer maturities, even more than with fixed issuance costs.

Finally, in the benchmark model assets of different maturities are traded in a single secondary market. A potential concern could be that many assets of short maturities, with small gains from trade, preclude the entry of buyers to the secondary market. In Section 9 we consider an alternative specification such that secondary markets are segmented by the time to maturity of the asset. The main takeaway of this exercise is that even though the market tightness (defined as the ratio of sellers-to-buyers) for short-term bonds increases, the tightness for long-term assets remains similar to the benchmark model with a single market. Hence, the secondary market in the baseline case is effectively a market for long-term assets.

Through the paper, we assume that firms have access to corporate bonds since these assets are already well studied in the literature about trading frictions (e.g., He and Milbradt, 2014). However, similar frictions also affect other sources of external finance. For example, we can also interpret the security as a bank loan because there exists a secondary market for this type of assets (see Altman, Gande, and Saunders, 2010; Drucker and Puri, 2008). Alternatively, we can interpret the financial architecture as venture capital or private equity funds. These are investment vehicles that provide long-term finance, in particular for start-ups that need financing for back-loaded projects. However, those markets are smaller than corporate debt, and there is little empirical evidence to quantify the model. Hence, the analytical results are portable for these alternative interpretations but due to data availability we choose to focus on corporate bonds in the quantitative section of the paper.

1.1 Related literature

This paper contributes to five several strands of the literature. First, there is an extensive literature on financial frictions and development (e.g., Greenwood, Sanchez, and Wang, 2010; Buera, Kaboski, and Shin, 2011; Moll, 2014; Midrigan and Xu, 2014, among others). Most of

However, the maturity of bank loans tend to be shorter than the maturity of corporate bonds (Cortina Lorente, Didier, and Schmukler, 2016) and financial systems become market-based during the process of economic development (Demirgüç-Kunt, Feyen, and Levine, 2013). Moreover, banks need to raise capital to extend long-term loans. Hence, the frictions analyzed in this paper also affect bank’s borrowing rates which are likely to affect the loans’ rates.
these papers focus on short-term (one period) contracts, and in most of the cases they study contractual frictions between lenders and borrowers. One notable exception is Cole, Greenwood, and Sanchez (2016) that studies the optimal long-term contract in a production economy. The contribution of this paper is to consider trading frictions within lenders and focus on the choice of maturity which is absent in previous analyses. We also study the effect of policy interventions to alleviate the illiquidity generated by trading frictions.

Second, this paper is also related to the literature on OTC markets developed by Duffie, Gâteanu, and Pedersen (2005) in which some papers applied to the bonds markets (e.g., Chen, Xu, and Yang, 2012; He and Milbradt, 2014; Chen, Cui, He, and Milbradt, 2017). The contribution to this strand of the literature is to characterize the interaction between maturity and liquidity which is absent in previous analyses. We also introduce this trading friction in a production economy and analyze the impact on investment and output.

Third, many papers studied the term structure of interest rates through the lens of the consumption-based capital asset pricing model (see Grkaynak and Wright, 2012, for a recent review). These papers extend the expectation hypothesis framework which posits that long-term interest rates are expectations of future average short-term rates. This paper is closer to an older literature that attributes the shape of the yield curve to liquidity considerations and recently Geromichalos, Herrenbrueck, and Salyer (2016) propose a monetary-search model, with assets of two maturities, to rationalize the yield curve. The contribution of this paper is to use a simpler search framework, based on Duffie, Gâteanu, and Pedersen (2005), that allow us to characterize the yield curve in a continuum of maturities and take it to the data.

Fourth, some papers study the interaction between the liquidity of secondary markets and the type of assets issued in primary markets (e.g., Bruche and Segura, 2016; Arseneau, Rappoport, and Vardoulakis, 2016). This paper contributes by considering the implications of this feedback for non-financial firms and how it affects the real sector of the economy.

Finally, some papers study the maturity choice of projects (e.g. Majd and Pindyck, 1987) and how to finance it (e.g. Manuelli and Sanchez, 2016). The contribution is to study these choices in an equilibrium model subject to trading frictions.

The rest of the paper is organized as follows. Section 2 discuss the empirical facts. Sections 3 to 5 present the model and study the effect of financial development. Section 6 estimates the model and presents the quantitative results. Next, in Section 7 we perform the normative analysis of the economy. Sections 8 and 9 extend the model in several dimensions. Finally,
2 Empirical Facts

In this section we summarize two well-known empirical facts about corporate debt markets across countries which motivate the model studied in the paper. First, the maturity of corporate debt is positively correlated with the GDP per capita of the country. Second, although corporate bonds are traded in over-the-counter secondary markets both in developed and developing countries, secondary markets are less liquid in developing economies.

2.1 Corporate debt maturity

There is a vast empirical literature showing that firms in developing countries borrow at shorter maturities than in advanced economies (e.g., Caprio and Demirgüç-Kunt, 1998; Demirgüç-Kunt and Maksimovic, 1998, 1999; Booth, Aivazian, Demirguc-Kunt, and Maksimovic, 2001; Fan, Titman, and Twite, 2012, among others). These studies use firm-level balance sheet data—i.e., including different securities, not only corporate bonds—and find that the ratio of long-term debt (defined as maturity greater than one year to total liabilities) is typically lower in developing countries than in advanced ones, even after controlling for firm characteristics. Similar arguments have been made using bank-level balance sheet data, which distinguish total bank credit granted to corporations, aggregated in broad maturity buckets (World Bank, 2015).

Figure 2 shows that firms in developing countries borrow at shorter maturities than in advanced ones. We use data from Cortina Lorente, Didier, and Schmukler (2016), which compiles an extensive dataset of corporate bond issuances in domestic markets for 1991-2014 across 80 countries (41 developed and 39 developing economies). The top left panel of Figure 2 shows the empirical distribution of corporate debt maturities for advanced and developing countries. In developing countries, the median maturity is about seven years, while it is about eleven years in advanced ones. Moreover, the top right figure shows that maturity and GDP per capita relative to the US are positively correlated across regions. For example, in the US the average maturity is 12.2 years while in Latin America the average maturity is about 7.6 and GDP per capita is about 30% relative to the US. Of course, both debt maturity and GDP

\footnote{I thank the authors for sharing their data, for details see Appendix F.1.}
2.2 Efficiency of financial markets

In this section we compare the efficiency of financial markets across countries. Beck, Demirgüç-Kunt, and Levine (2000) constructs a database about financial markets and intermediaries across countries and Demirgüç-Kunt and Levine (2004) shows that financial markets are larger, more active, and more efficient in richer countries. Moreover, as countries become richer, financial markets become more active and efficient relative to banks.
Most of previous empirical studies on financial development focus on market size by looking at moments like debt-to-GDP ratios (see Levine, 2005, for a survey of the literature). However, the focus of this paper is in a different aspect of financial development. We are interested in how easy it is to reallocate securities in the corporate debt market—i.e., the liquidity of the market—because it has been shown that liquidity causes an important component of credit spreads in the US (e.g., Friewald, Jankowitsch, and Subrahmanyam, 2012; He and Milbradt, 2014).

Gyntelberg, Ma, Remolona, et al. (2006) shows that corporate bonds markets in Asia are less liquid than in the US. For example, the average turnover rate in Asia is about 70% lower than in the US (first panel of Table 1). For Latin America, there is very little data on corporate debt markets. However, Jeanneau and Tovar (2006) studies the sovereign bonds market and compare it to the US. Both turnover rates and Bid-Ask spreads indicate that secondary markets in Latin America are less liquid than in the US (second panel of Table 1).

For the US and Argentina we can compare different measures related to the liquidity of corporate debt markets. For the US, we use data from Trade Reporting and Compliance Engine (TRACE) that provides transaction-level information of corporate bonds in secondary markets and has been widely used to study the liquidity of this market (e.g. Edwards, Harris, and Piwowar, 2007; Bao, Pan, and Wang, 2011, among others). For Argentina, we use a novel data set borrowed from Mercado Abierto Electronico (MAE, the over-the-counter exchange in Argentina). We got access to transaction-level information of corporate bonds (Obligaciones Negociables) which provides similar information than the TRACE database.

The first measure of liquidity is the trading frequency, defined as the frequency of days with at least one trade for each asset. The bottom left panel of Figure 2 shows the distribution of this measure in US and Argentina. In the US, the median of the trading frequency is 47.8%—i.e., assets are traded at least once every two days. On the other hand, in Argentina, the median is 0.4%—i.e., assets are traded a bit more than once a year. Second, we measure liquidity as the

5One exception is Levine and Zervos (1998) which studies the turnover of stock market as a measure of financial development (see also De la Torre, Gozzi, and Schmukler, 2007).

6The US Treasury market is one of the most liquid secondary markets. However, under the assumption that corporate markets are less liquid than treasury markets, these moments provide information on the liquidity for corporate markets in Latin America.

7I thanks Argentina’s Central Bank to provide access to the data. To the best of my knowledge, this is the first paper to use this database. Borensztein, Cowan, Eichengreen, and Panniza (2008) documents some aggregate facts about the corporate debt market in Argentina using an aggregate version of the data. Appendix F.2 describes both datasets and their respective cleaning. We compare the liquidity of the US and Argentinean markets during the year 2012 while results are similar for different time periods.
annualized turnover ratio. The right bottom panel of Figure 2 shows that this measure is also much lower in Argentina than in the US. The median turnover is 57% in the US whereas it is 11.9% in Argentina.

The secondary market’s liquidity account for a large fraction of credit spreads in the US (e.g., He and Milbradt, 2014) while the empirical evidence presented in this section shows that secondary markets are substantially less liquid in developing countries. This suggest that we should pay attention to trading frictions for the study of credit spreads across countries.

Table 1: Liquidity of bonds markets.

<table>
<thead>
<tr>
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<th>Turnover relative to US</th>
<th>Bid-Ask Spreads bps above US</th>
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<tbody>
<tr>
<td>Corporate bonds in Asia</td>
<td></td>
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<tr>
<td>Malaysia</td>
<td>65</td>
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<tr>
<td>Japan</td>
<td>40</td>
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<tr>
<td>India</td>
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<td>New Zealand</td>
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<td>Thailand</td>
<td>20</td>
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<tr>
<td>Korea</td>
<td>5</td>
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<tr>
<td>Sovereign bonds in Latin America</td>
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<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>23</td>
<td>6</td>
</tr>
<tr>
<td>Argentina</td>
<td>9</td>
<td>29</td>
</tr>
<tr>
<td>Colombia</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Brazil</td>
<td>4</td>
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<tr>
<td>Chile</td>
<td>4</td>
<td>5</td>
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<tr>
<td>Peru</td>
<td>2</td>
<td>14</td>
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<tr>
<td>Venezuela</td>
<td>2</td>
<td>74</td>
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</tbody>
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Source: Gyntelberg, Ma, Remolona, et al. (2006); Jeanneau and Tovar (2006). Bid-Ask Spread for US treasuries is about 1.2 basis points.

3 Theory

Time is continuous, starts at $t = 0$, and goes on forever. The economy is populated by a financial sector, a production sector, and a representative household. Firms in the production sector choose a project from a menu of investments opportunities characterized by their maturity such that the return of the project increases with the duration of investment. However, firms need to borrow from the financial sector via issuances of corporate bonds to finance the projects.
These securities are the assets traded in the financial sector in the over-the-counter (OTC) secondary market.

In this section we take the issuances of corporate bonds as given and study the asset pricing implications. Section 4 analyze the borrower’s problem while Section 5 characterize the equilibrium between lenders and borrowers.

3.1 Lenders

The financial sector trades corporate bonds in an OTC market à la Duffie, Gărleanu, and Pedersen (2005). Bonds have maturity $\tau$, no coupon, and a face value of one. With Poisson arrival rate $\lambda^D$ the firm is hit by a default shock and the value of the bond goes to zero. A measure $\mu^0$ of bonds with maturity $\tau$ are issued every period. Therefore, in the ergodic distribution, the mass of assets with time-to-maturity $y$ that did not default is given by $\mu(y) = \mu^0 e^{-\lambda^D (\tau-y)}$. Note that, at issuance, all securities are identical which implies that the distribution of assets only depends on the time-to-maturity $y$.

An agent of the financial sector can have either zero or one asset. An agent without the asset has three alternatives: (i) go to the primary market and get an on-the-run bond (i.e., a newly issued bond); (ii) search in the secondary market for an off-the-run bond; or (iii) stay inactive. There is a large measure of potential agents that can enter into primary and secondary markets which implies that both markets are competitive, and the agent is indifferent between going to these markets or staying inactive with a value normalized to zero.

An agent that buys an asset either in the primary or the secondary market starts as a high valuation agent. However, he faces an idiosyncratic liquidity risk of becoming a low valuation agent. With Poisson intensity $\lambda^H$ a high valuation agent becomes low valuation which implies that he has to pay the holding cost $h$ per unit of time. This idiosyncratic risk generates differences in valuations which causes motives for trade on the secondary market. In particular, a low valuation agent has three ways of stop paying the holding cost: (i) sell the asset in the secondary market; (ii) wait until maturity; or (iii) liquidity by default.

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8The portfolio restriction is common in the literature and simplifies the tractability of the model.

9The modeling assumptions about high versus low-valuation agents is standard in the literature (e.g., Duffie, Gărleanu, and Pedersen, 2005; He and Milbradt, 2014). A low investor may have (i) high discounting (ii) high financing costs; (iii) hedging reasons; (iv) tax disadvantage, or (v) lower personal use of the asset.
Secondary market  On the one hand, low valuation agents are the sellers in the secondary market. The measure of sellers with a bond with time-to-maturity \( t \) is \( \mu^L(y) \) and the total mass of sellers is \( \mu^S = \int_0^T \mu^L(y) dy \). On the other hand, buyers are agents without an asset searching in the secondary market. The free entry condition characterizes the measure of buyers \( \mu^B \). The matching function has constant returns to scale \( M(\mu^S, \mu^B) = A (\mu^S)^\alpha (\mu^B)^{1-\alpha} \). Note that we assume random matching across different maturities—i.e., securities of different time-to-maturity are pulled together in one secondary market. However, Section 9 extends the model to study markets segmented by the time-to-maturity and shows that the main results also hold under this specification.

We define the market tightness as the ratio of sellers-to-buyers, \( \theta = \frac{\mu^S}{\mu^B} \). The matching function implies that a seller finds a counterpart at rate \( \lambda^S = A \theta^{1-\alpha} \) and a buyer finds a counterpart at rate \( \lambda^B = A \theta^\alpha \). Note that upon a match, the probability that a buyer meets with a seller of an asset with time-to-maturity \( y \) is \( \frac{\mu^L(y)}{\mu^S} \). Upon a meeting, the seller and the buyer bargain over the price of the asset, \( P^S(y) \). We assume a Nash Bargaining protocol and let \( \gamma \) be the bargaining power of the seller.

3.2 Distribution of lenders

All the agents that buy the security in the primary market start as high valuation. However, over the life-cycle, some agents receive liquidity shocks while others trade in the secondary market. Let \( \mu^H(y) \) and \( \mu^L(y) \) be the measure of high- and low-valuation agents holding an asset of time-to-maturity \( y \), respectively. The law of motions are

\[
\dot{\mu}^H(y) = \left(\lambda^H + \lambda^D\right) \mu^H(y) - \lambda^B \frac{\mu^L(t)}{\mu^S} \mu^B
\]

\[
\dot{\mu}^L(y) = -\lambda^H \mu^H(y) + \left(\lambda^D + \lambda^S\right) \mu^L(t)
\]

with boundary conditions \( \mu^H(\tau) = \mu^0 \) and \( \mu^L(\tau) = 0 \). Equations (1) and (2) show that as we move closer to maturity (lower \( y \)) a fraction \( \lambda^H \) of high-valuation agent become low-valuation agents and a fraction \( \lambda^D \) of both type of agents are holding an asset that enters into default. Moreover, a measure \( \lambda^B \frac{\mu^L(y)}{\mu^S} \mu^B \) of buyers find a counterpart in the secondary market and become high-valuation agents. Finally, a measure \( \lambda^S \) of low-valuation agents are able to sell in the secondary market. Lemma 1 characterizes the steady-state distribution of financiers.
Lemma 1. The distribution of financiers is given by

\[ \mu^H(y) = \frac{\mu^0 \lambda^H}{\lambda^H + \lambda^S} \left( e^{\lambda D(y-\tau)} \frac{\lambda^S}{\lambda^H} + e^{(\lambda^H+\lambda^S+\lambda^D)(y-\tau)} \right) \]  \hspace{1cm} (3)

\[ \mu^L(y) = \frac{\mu^0 \lambda^H}{\lambda^H + \lambda^S} \left( e^{\lambda D(y-\tau)} - e^{(\lambda^H+\lambda^S+\lambda^D)(y-\tau)} \right) \]  \hspace{1cm} (4)

Non-defaulted assets are held by high and low valuation agents. Default occurs at rate \( \lambda^D \) and is an absorbing state. At issuance—i.e., time-to-maturity equal to \( \tau \)—all assets are held by high valuation agents that bought them in the primary market. However, as time evolves, some of these agents become low valuation, and some of the assets enter into default. When the secondary market is well-functioning—the selling intensity, \( \lambda^S \), is relatively high—the mass of low valuation agents \( \mu^L(y) \) is small. However, when \( \lambda^S \) diminishes, the secondary market is more illiquid and the mass of low valuation agents increases.

### 3.3 Valuations

Let \( D^H(y) \) and \( D^L(y) \) be the value of holding an asset of time-to-maturity \( y \) for high- and low-valuation agents, respectively. The value of search in the secondary market, \( D^S \), is

\[ \rho D^S = -c + \lambda^B \int_0^\tau \frac{\mu^L(y)}{\mu^S} \left( D^H(y) - D^S - P^S(y) \right) dy, \]  \hspace{1cm} (5)

where \( \rho \) is the discount factor and \( c \) is the search cost. The discounted value of search is equal to the search cost and the expected gains from trade. With intensity \( \lambda^B \frac{\mu^L(y)}{\mu^S} \) the buyer is matched with a seller of a bond with time to maturity \( y \), and the gains from trade are \( D^H(y) - D^S - P^S(y) \). The agent becomes high valuation, with value \( D^H(y) \) and pays the price \( P^S(y) \). Note that the agent is searching across assets of maturities \( y \in [0, \tau] \). Free entry in the secondary market implies that, in equilibrium, \( D^S \leq 0 \).

The values of high- and low-valuation agents holding an asset are

\[ \rho D^H(y) = -\dot{D}^H(y) + \lambda^H \left( D^L(y) - D^H(y) \right) + \lambda^D \left( 0 - D^H(y) \right), \]  \hspace{1cm} (6)

\[ \rho D^L(y) = -h - \dot{D}^L(y) + \lambda^S \left( P^S(y) - D^L(y) \right) + \lambda^D \left( 0 - D^L(y) \right). \]  \hspace{1cm} (7)

The boundary condition at maturity, \( y = 0 \), is such that both investors recover the principal value of the asset, \( D^H(0) = D^L(0) = 1 \). Equation (6) defines the value of high-valuation agents.
The left-hand side is the required return from holding the bond. The first term on the right-hand-side represents the change in value due to being closer to maturity. The second term captures the liquidity shocks that transform the investor into a low-valuation agent, which occurs at intensity $\lambda^H$. The third term captures the risk of default of the bond. Equation (7) captures the value of low valuation agents and follows a similar intuition as the previous equation. The two differences are on the right-hand side: a low-valuation investor incurs a holding cost $h$, and the third term reflects the value of the secondary market. The investor meets a counterpart at rate $\lambda^S$ and sells his bond at price $P^S(y)$.

3.4 Asset prices

The price in the secondary market, $P^S(y)$, is the outcome of Nash Bargaining between the seller and the buyer

$$\max_{P^S(y)} \left( P^S(y) - D^L(y) \right)^\gamma \left( D^H(y) - P^S(y) \right)^{(1-\gamma)}$$

where $\gamma$ is the bargaining power of the seller. The solution is

$$P^S(y) = D^L(y) + \gamma \left( D^H(y) - D^L(y) \right). \quad (8)$$

The gains from trade are $D^H(y) - D^L(y)$ and the seller gets a fraction $\gamma$ of them.

The price in the primary market is characterized by the following break-even condition. Recall that a high-valuation agent that is not holding an asset can go to the primary market and get a newly issued bond. This implies that the price in the primary market is $P(\tau, \Lambda) = D^H(\tau)$. We define $\Lambda = \lambda^S\gamma$ as the liquidity of the secondary market. The first term, $\lambda^S$, is the selling intensity in the secondary market—an equilibrium object. If $\lambda^S$ increases, it becomes easier to sell on the secondary market. The second term, $\gamma$, is the bargaining power of the seller in the secondary market—a parameter. When $\gamma$ increases, sellers keep a larger fraction of the gains from trade. Therefore, we say that the secondary market is more liquid when $\Lambda$ increases.

Lemma 2 uses the equilibrium price in the secondary market to solve for equations (6) and (7), and characterizes the asset price in the primary market.
Lemma 2. The price in the primary market is

\[ P(\tau, \Lambda) = e^{-(\rho + \lambda D)\tau} - \mathcal{L}(\tau, \Lambda), \]  

where the illiquidity cost \( \mathcal{L}(\tau, \Lambda) \) is

\[ \mathcal{L}(\tau, \Lambda) = h \frac{\lambda^H}{\lambda^H + \Lambda} \int_0^\tau e^{-(\rho + \lambda D)y} \left( 1 - e^{-(\lambda^H + \Lambda)y} \right) dy. \]  

The illiquidity cost satisfies the following properties:

1. \( \mathcal{L}(\tau, \Lambda) \) is positive.

2. Sensitivity with respect to maturity \( \tau \):
   (a) \( \mathcal{L}(\tau, \Lambda) \) is increasing in \( \tau \);
   (b) \( \mathcal{L}(\tau, \Lambda) \) has a finite limit, \( \lim_{\tau \to \infty} \mathcal{L}(\tau, \Lambda) = h \frac{\lambda^H}{(\rho + \lambda D)(\rho + \lambda^H + \lambda^H + \gamma \lambda^S)} \).

3. Sensitivity with respect to liquidity shocks \( \lambda^H \):
   (a) If there are no liquidity shocks, \( \lambda^H = 0 \), then \( \mathcal{L}(\tau, \Lambda) = 0 \);
   (b) If \( \lambda^H \to \infty \) (i.e., always has to pay the cost \( h \)) then

   \[ \lim_{\lambda^H \to \infty} \mathcal{L}(\tau, \Lambda) = h \frac{1 - e^{-(\rho + \lambda D)t}}{\rho + \lambda D}. \]

4. Sensitivity with respect to liquidity \( \Lambda \):
   (a) \( \mathcal{L}(\tau, \Lambda) \) is decreasing in \( \Lambda \);
   (b) If there are no secondary markets, \( \Lambda = 0 \), the liquidity term only represents the expected holding costs, i.e.,

   \[ \mathcal{L}(\tau, 0) = h \int_0^\tau e^{-(\rho + \lambda D)y} \left( 1 - e^{-\lambda^H y} \right) dy; \]

   (c) If secondary markets are totally liquid (i.e., \( \Lambda \to \infty \)) then \( \mathcal{L}(\tau, \Lambda) = 0 \).

5. Liquidity is more important for long-term assets: \( \frac{\partial^2 \mathcal{L}(\tau, \Lambda)}{\partial \tau \partial \Lambda} \leq 0 \).

Lemma 2 shows that we can decompose the price \( P(\tau, \Lambda) \) in two terms. The first component represents the frictionless solution: the value of a promise to pay one unit in \( \tau \) periods when
the discount rate is $\rho$ and the default intensity is $\lambda^D$. Note that absent the second term, the expectation hypothesis holds: the long-term interest rate is equivalent to the average of short-term rates. The second term, $\mathcal{L}(\tau, \Lambda)$, represents the illiquidity cost. It is equivalent to the expected discounted time that the asset will be held by a low valuation agent times the respective holding cost $h$. When this term is different from zero the expectation hypothesis does not hold and borrowing at longer horizons becomes more expensive than the average of short-term rates.

Lemma 2 describes five properties of the illiquidity cost while the left panel Figure 3 summarizes the three most important characteristics. First, the illiquidity cost is increasing in maturity. Longer securities spend more time in the market which increases the expected time that they are held by low-valuation agents. Second, the illiquidity cost is decreasing in liquidity $\Lambda$. If the liquidity of the secondary market increases, the holders of that security spend less time paying the holding cost which implies a lower illiquidity cost. Third, the cross partial derivative to maturity and liquidity of the illiquidity cost is negative, i.e., the liquidity of the secondary market is more important for long-term assets. An investor that wants to exit a financial position on a bond can either sell it on the secondary market or wait until maturity. Hence, the role of the secondary market is more important for an agent holding a long-term asset. As a result, the cross partial derivative is negative: an increase in liquidity benefits more long- than short-term securities.
3.5 Yield curve

Define \( r(\tau, \Lambda) \) as the compound interest rate that solves \( P(\tau, \Lambda) = e^{-r(\tau, \Lambda)\tau} \), that is the value of the asset conditional on it being held to maturity without default or re-trading. The interest rate for a bond of maturity \( \tau \) is

\[
r(\tau, \Lambda) = \rho + \lambda D + \frac{1}{\tau} \log \left( \frac{1}{1 - e^{(\rho + \lambda D)\tau} \mathcal{L}(\tau, \Lambda)} \right).
\] (11)

The first term, \( \rho \), is the risk-free rate, while the remaining two terms are the credit spread. We decompose the spread into a default and a liquidity component, following He and Milbradt (2014). Consider a marginal investor with no idiosyncratic liquidity risk. Such investor requires an interest rate equal to \( \rho + \lambda D \). Therefore, the credit spread due to default is \( \lambda D \). Finally, we can define the credit spread due to liquidity by subtracting the default component. Hence, the third term corresponds to the liquidity spread

\[
cs_{\text{liq}}(\tau, \Lambda) = \frac{1}{\tau} \log \left( \frac{1}{1 - e^{(\rho + \lambda D)\tau} \mathcal{L}(\tau, \Lambda)} \right).
\] (12)

and \( r(\tau, \Lambda) = \rho + cs_{\text{def}} + cs_{\text{liq}}(\tau, \Lambda) \). Moreover, we define the term premium as the difference between the short-term rate and the long-term rate, i.e., \( \text{tp}(\tau, \Lambda) = r(\tau, \Lambda) - r(0) \). Note that in this model the term premium is equivalent to the credit spread due to liquidity. Variations in interest rates across maturities are explained by differences in the liquidity component of the security. Note that all assets are traded in the same secondary market, however the liquidity spread varies with maturity as the importance of the secondary market is different across assets.

We are ready to derive the first proposition of the paper.

**Proposition 1.** The liquidity spread is increasing in maturity and decreasing in liquidity. Moreover, the proportional bid-ask spread is increasing in maturity, and the liquidity spread increases with the default intensity.

Proposition 1 establish three important properties. First, the liquidity spread—and therefore the term premium—is increasing in maturity and decreasing in liquidity \( \Lambda \), represented in the right panel of Figure 3. Importantly, note that changes in the liquidity of the secondary market affect more the long-end of the curve. Recall that this result is also present in \( \mathcal{L}(\tau, \lambda^S) \) and the yield curve preserves the property.

Second, Proposition 1 shows that there is a feedback-loop between default and liquidity in which the liquidity spread is increasing in the default intensity \( \lambda^D \). Note that \( \lambda^D \) has the same
role as the discount factor $\rho$. An increase in the discount factor decreases the value of illiquidity at maturity which increases the liquidity spread. Section 8.1 extends the analysis of the default and liquidity interactions and their effect on maturity choices.

Finally, we define the proportional bid-ask spread as the gains from trade normalized by the mid-price

$$BA(t) = \frac{D^H(t) - D^L(t)}{\frac{1}{2}(D^H(t) + D^L(t))}$$

Proposition 1 shows that the proportional bid-ask spread is increasing in maturity. An asset of longer maturity has larger gains from trade. As a result, the bid-ask spread increases with maturity. Importantly, all the predictions of Proposition 1 are consistent with the empirical evidence (e.g., Edwards, Harris, and Piwowar, 2007).

3.6 Free entry

To solve for the entry in the secondary market we replace the price of Equation (5) and solve for the free-entry condition

$$c = \lambda^B(1 - \gamma) \int_0^\tau \frac{\mu^L(y)}{\mu^S} (D^H(y) - D^L(y)) \, dy.$$  \hspace{1cm} (13)

To characterize the equilibrium between lenders and borrowers it is useful to establish how the incentives to enter into the secondary market change with the maturity at issuance of the asset. Proposition 2 describes $\Lambda$ as a function of $\tau$.

**Proposition 2.** $\Lambda(\tau)$ is increasing in $\tau$ and $\Lambda(\tau) \mapsto [0, \overline{\Lambda}]$.

When assets are of zero maturity, there are no gains from trade which implies no entry in secondary markets, $\Lambda(0) = 0$. As $\tau$ increases, gains from trade also increase. As a result, there is more entry of potential buyers which generates an increase in liquidity. When $\tau$ goes to infinity the gains from trade are bounded which implies that $\Lambda$ converges to a finite number. Figure 4 shows the relationship between $\Lambda$ and $\tau$ under the parameters estimated in Section 6. Interestingly, the upper bound of $\Lambda$ is approached quite fast. For maturities above five years $\Lambda$ converges to its upper bound. Section 5 solves equilibrium between lenders and borrowers in which the locus $\Lambda(\tau)$ represents the lenders component. In the next section we solve for the maturity choice of borrowers.
Figure 4: Free-entry in secondary markets.

Liquidity ($\Lambda$)

Maturity ($\tau$)

Note: The figure shows how the free entry condition (13) changes with maturity $\tau$. The parameter values are discussed in Section 6.

4 Borrowers

We study how the yield curve affects the demand for long-term debt. Every period a new cohort of identical entrepreneurs enters the economy and implements a new project from a menu of potential investments differentiated by the life-cycle of returns. The qualitative results hold for a large class of models such that there are back-loaded projects that require long-term financing. However, to guide the quantitative analysis, we propose a specify microfoundation based on a simple production model in which firms invest in TFP. Appendix B.3 propose alternative specifications based on a model of the quality-ladder and a model of time-to-build capital that also generates a similar demand for long-term bonds.\(^\text{10}\)

Life-cycle There are two important empirical facts which motivate the assumptions about the life-cycle of investment projects. First, small firms grow faster than large firms (e.g., Akcigit and Kerr, 2017). Second, small firms are more financially constrained, and in particular for research and development (Midrigan and Xu, 2014; Itenberg, 2015). Based on these facts, we make a stark assumption and divide the life-cycle of firms in two periods: (i) *young* for $t \leq \tau$, and (ii) *mature* for $t > \tau$. A newborn firm chooses $\tau$, the age at which she will become mature.

\(^{10}\)Rioja and Valev (2004) finds that finance boosts growth in rich countries by speeding-up productivity growth, while finance encourages growth in poorer countries by accelerating capital accumulation. In Appendix B.3 we propose an alternative microfoundation for back-loaded projects based on time-to-build capital. These alternative formulations support the idea that the demand for long-term debt is similar across countries, irrespectively if they need to finance investment in capital, TFP or quality improvements.
Therefore, a new firm has a menu of potential projects summarized by \( \tau \geq 0 \).

**Young firm** A young firm invest in research and development to improve its productivity. Let \( z(t) \) be the productivity of the firm such that \( z(0) = 0 \) and \( \dot{z}(t) = \delta z \) if \( t \leq \tau \). The flow cost of investment is \( \kappa \) per unit of time doing R&D and at maturity productivity is \( z(\tau) = \delta z \tau \). The firm faces an exogenous exit shock such that with intensity \( \lambda^D \) the value of the project goes to zero.

**Mature firm** At age \( \tau \) the firm stops doing R&D and starts the production face. The production technology is \( y = zl^\sigma \) where \( l \) is the labor demand and \( \sigma \leq 1 \) captures the decreasing returns to scale. There is an exogenous labor supply normalized to one, and all firms are identical. This implies that static profits for mature firms are \( \pi(z) = z(1 - \sigma) \).\(^{11}\)

These assumptions imply that the net present value of a mature firm that spent \( \tau \) periods doing R&D is \( F(\tau) = \tau \frac{\delta z (1-\sigma)}{\rho + \lambda^D} \) where \( \lambda^D \) is the Poisson arrival rate of an exogenous exit shock. For the qualitative results, we can also assume a general return function \( F(\tau) \) such that it is increasing and concave in \( \tau \).

### 4.1 Financing costs

A newborn firm chooses the maturity of the project \( \tau \) and issues zero coupon bonds to finance investment costs. A bond of maturity \( y \) has a fixed cost of issuance \( \Phi \) and an interest rate \( r(y) \).\(^{12}\) Let \( J \) be the total number of issuances up to age \( \tau \) and note that the fixed issuance cost implies that \( J \) is finite. We also assume that once the firm becomes mature, she does not need to borrow.\(^{13}\)

\(^{11}\)We assume that a young firm does not produce. However, results are similar if we assume that a young firm produces and use internal funds for investment while \( \kappa \) are the external funds needed for investment.

\(^{12}\)In the data, issuance costs include management fee, selling concession, registration fee, underwriter fee, underwriter spread (the difference between the offering price and the guaranteed price to the issuer), underpricing (the difference between the market price and the offering price), and printing, legal and auditing costs. For the Eurobond market, Melnik and Nissim (2003) finds that the total issuance cost is 37 basis points. Lee, Lochhead, Ritter, and Zhao (1996) finds similar costs and reports evidence of economies of scale, reflecting that a significant fraction is a fixed cost.

\(^{13}\)The interpretation is that the project becomes liquid at \( \tau \), and the firm can repay outstanding debt with the value of the project. As the interest rate is higher than the discount factor of the borrower, a mature firm will not use the bonds market and only young firms borrow.
Consider a young firm of age \( x \) that is building a project with maturity \( \tau \geq x \), and has to rollover outstanding debt \( B \).\(^{14}\) Let \( E(\tau, x, B) \) be the value of the firm so

\[
E(\tau, x, B) = \max_{y \leq \tau - x} e^{-(\rho + \lambda D) y} E(\tau, x + y, B'(y))
\]

\[
B'(y) = e^{r(y)y} (B + \Phi + I(y))
\]

\[
E(\tau, \tau, B) = F(\tau) - B
\]

The firm chooses the maturity of the new issuance \( y \in [0, \tau - x] \) and borrows \( B' \) to rollover existing debt, \( B \), pays the fixed cost of issuance, \( \Phi \), and borrows money to finance research for the next \( y \) periods. \( I(y) \) is the money needed to fund the research costs in a window of length \( y \). The firm deposits the money in a bank account with risk-free rate \( \rho \) and withdraws \( \kappa \) per units of time, so \( I(y) = \kappa \frac{1 - e^{-\rho y}}{\rho} \).

Lemma 3 shows that the firm’s value can be decomposed in two terms: (i) the net present value of the project, and (ii) the financial cost.

**Lemma 3.** A new firm solves

\[
\max_\tau e^{-(\rho + \lambda D) \tau} F(\tau) - FIN^{COST}(\tau)
\]

where the financial cost is

\[
FIN^{COST}(\tau) = e^{-(\rho + \lambda D) \tau} \min_{J, \{y_j\}_{j=1}^J} \sum_{i=1}^J (\Phi + I(y_i)) e^{\sum_{s=1}^J r_s y_s} \quad \text{s.t.} \quad \sum_{j=1}^J y_j = \tau.
\]

Consider a project of maturity \( \tau \). The financial cost consists of choosing the number of issuances \( J \) and the maturity of each bond \( \{y_j\}_{j=1}^J \) to minimize the net present value of issuances costs \( \Phi \), and investment needs \( I(y_j) \). Hence, two frictions shape the financial decisions: (i) issuance cost, and (ii) liquidity spread.

The top panel of Figure 5 shows the number of issuances, the financial cost, and the dispersion of maturities for different issuance cost \( \Phi \). Naturally, as the cost increases, the number of issuances decreases and the financial cost increases. Note that if \( \Phi \) is sufficiently large, the firm optimally chooses to issue only one time, matching the maturity of the project and the liabilities. In the next section, we focus in this particular case in which \( J = 1 \) to derive a sharper analytical characterization of the effects of secondary markets liquidity on the project’s

\(^{14}\) A firm that has positive cash holdings will always wait until she runs out of money to issue new debt. Hence, without loss of generality, we only have to consider the choices of the firm when outstanding debt matures which coincides with the moment in which the firm runs out of money.
choice. Section 8 extends the analysis to study the interaction of rollover decisions and the choice of projects.

As the secondary market becomes more liquid—higher $\Lambda$—the credit spread due to liquidity diminish, generating a flatter yield curve (Proposition 1). As $\Lambda$ increases, firms decide to rollover less often, borrow at longer maturities, and reduce the financial cost (see bottom panel of Figure 5). Hence, for a given $\tau$, the financial cost diminish when $\lambda$ increases. In the next section, we study how this change in financial cost affects the decision of which project to implement.

The optimal maturity structure solve the trade-off between equalizing the credit spread across different issuances and decreasing future fixed cost of issuances. In equilibrium, the maturity structure is a decreasing sequence—i.e., $y_1 \geq y_2 \geq \cdots \geq y_J$ to postpone future fixed costs of issuance in which the dispersion across maturities depends on the issuance cost and the slope of the yield curve. On the one hand, figure 5 shows that, conditional on the number of issuances, when the issuance cost increases firms choose to increase the maturity dispersion. Intuitively, as $\Phi$ increases, firms want to postpone the fixed cost payments of later issuances, and, as a result, they extend the maturity of the first few issuances and decrease later ones. On the other hand, the bottom panel shows that when the secondary market becomes more illiquid (lower $\Lambda$), the dispersion of maturities diminishes to generate similar liquidity spreads across issuances.

4.2 Project’s choice

Empirical evidence shows that firms tend to match the maturity of assets and liabilities.\textsuperscript{15} Hence, from now on we assume that firms make only one issuance, $J = 1$, i.e., they match the maturity of the project and the bond. This assumption helps us to obtain a sharper characterization of the equilibrium. In section 8 we relax this assumption and show that similar results are also present when issuance costs are lower and firms issue debt more than one time.

The firm’s problem is

$$\max_{\tau} \quad e^{-(\rho+\lambda\delta)\tau} F(\tau) - (\Phi + I(\tau)) e^{\ln(\Lambda, \tau)\tau}. \quad (14)$$

Figure 5: **Financial cost with rollover.**

Issuances

Financing cost

Maturity dispersion

Note: Financial cost for a given maturity $\tau$ and different issuance cost $\Phi$ and liquidity $\Lambda$. Parameter values are discussed in Section 6.

The firm discount factor is $\rho + \lambda^D$ while the interest is $r(\tau) = \rho + \lambda^D + cs_{liq}(\Lambda, \tau)$. Note that the liquidity spread $cs_{liq}(\Lambda, \tau)$ captures the frictions on the secondary market and generates a wedge between the discount factor of lenders and borrowers. This spread increases with $\tau$ for two reasons. First, the firm has to pay the cost for a longer period. Second, Proposition 1 establishes that the liquidity spread increases with maturity. These two effects are the channel through which frictions in the secondary market affect the choice of project $\tau$.

The optimal maturity is characterized by the following trade-offs

$$\frac{\partial F(\tau)}{\partial \tau} = (\rho + \lambda^D) F(\tau) + \frac{\partial I(\tau)}{\partial \tau} e^{r(\Lambda, \tau)\tau} + (\Phi + I(\tau)) e^{r(\Lambda, \tau)\tau} cs_{liq}(\Lambda, \tau) \left(1 + \epsilon_{cs_{liq},\tau}(\Lambda, \tau)\right)$$

(15)

where $\epsilon_{cs_{liq},\tau}(\Lambda, \tau)$ is the elasticity of the liquidity spread to maturity

$$\epsilon_{cs_{liq},\tau}(\Lambda, \tau) = \frac{\partial cs_{liq}(\Lambda, \tau)}{\partial \tau} \frac{\tau}{cs_{liq}(\Lambda, \tau)}.$$

Consider a marginal increase in $\tau$. The left-hand side of Equation (15) represents the benefits of operating a firm with higher TFP and the right-hand side captures three associated costs. First, a project in which returns are more back-loaded requires more time to become profitable. This
Note: Optimal maturity \( \tau(\Lambda) \). The parameter values are discussed in Section 6.

implies a higher time-discount on future profits. Second, a larger firm requires more investment. Note that even without financial frictions (i.e. constant interest rates, \( r(\tau) = \rho + \lambda D \) and \( \Phi = 0 \)) we have an interior solution for \( \tau \).\(^{16}\) Intuitively, there is an interior solution because as the firm chooses a larger \( \tau \) it is both more costly and it takes more time to complete.

The third term of Equation (15) captures the effect of the financial cost. First, as maturity increases, the firm has to pay the liquidity spread for a longer period. Second, the liquidity spread increases with maturity (Proposition 1). These two forces generate a maturity choice shorter than in the frictionless economy.

Proposition 3. The optimal maturity is increasing in the liquidity of the secondary market and \( \tau(\Lambda) : [0, \Lambda] \mapsto [\tau, \tau^*] \) with \( 0 \leq \tau \leq \tau^* < \infty \).

Proposition 3 shows that the optimal maturity increases with the liquidity of the secondary market. By Proposition 1, the liquidity spread is lower when secondary markets are more liquid. This implies that it is cheaper to borrow and in particular at longer horizons. As a result, firms choose to increase the maturity of their projects. Figure 6 shows the locus \( \tau(\Lambda) \) which represent the borrowers allocation in the equilibrium between lenders and borrowers.

\(^{16}\)Note that this is also true even if we have no default, \( \lambda D = 0 \).
5 Equilibrium and Financial Development

We characterize the equilibrium of the model as the intersection of the entry decision of lenders in financial markets from Section 3 and the issuance choice of borrowers from Section 4. Definition 1 states the steady-state search and matching equilibrium.

**Definition 1.** A steady-state search and matching equilibrium is characterized by the liquidity of secondary markets $\Lambda$, an interest rate schedule $r(\Lambda, \tau)$, and maturity $\tau$ such that:

1. Liquidity $\Lambda$ solves the free-entry condition of financiers (13), and characterizes the yield curve $r(\Lambda, \tau)$ taking the assets as given;

2. Maturity $\tau$ solves the firm’s optimal maturity problem (14), taking the interest rate schedule $r(\Lambda, \tau)$ as given.

The equilibrium is characterized by liquidity $\Lambda$ and maturity $\tau$ given by (13) and (15). The solid red (blue) line of Figure 7 shows the lenders (borrowers) allocation. The intersection of these two curves characterizes the equilibrium of the economy. Proposition 4 states that an equilibrium exists. The proof is very simple and follows from Propositions 2 and 3.

**Proposition 4.** A steady-state search and matching equilibrium always exists.
5.1 Financial development

We interpret financial development as improving the efficiency of secondary markets, which, in the model, can be interpreted as a reduction of search costs $c$. On the one hand, the optimal maturity, characterized by the curve $\tau(\Lambda)$, depends on the equilibrium $\Lambda$, but not directly on $c$, which implies that the curve $\tau(\Lambda)$ does not move when $c$ changes. On the other hand, the curve $\Lambda(\tau)$ depends directly on $c$. For a given maturity, a lower search cost induces more potential buyers to enter the market, which reduces the tightness of the secondary market, and increases the liquidity. As a result, the curve moves to the right. The red-dashed line in Figure 7 shows the new locus for the equilibrium condition under financial development: Both liquidity and maturity increase. This exercise shows that financial development—an increase in the efficiency of secondary markets—causes an increase of debt maturity. In the next section, we evaluate this mechanism quantitatively and show that the liquidity of the secondary market can generate quantitatively large movements in maturity choices.

What is financial development? First, there is a literal interpretation as the technology to execute trades. In developed markets, there are clearing houses such as Euroclear or Clearstream while in developing countries the time to execute a trade is delayed by technological constraints. For example, in Argentina investors liquidate securities in one place, but make payments in a different bank. Second, a broader interpretation is to think about the participants in the market. In developed financial markets, there are mutual funds which are agents that trade more frequently than the rest of market’s participants. Because these funds are either small or inexistent in developing countries, this could imply lower liquidity. Finally, Bethune, Sultanum, and Trachter (2017) shows that private information about the valuation of the security creates informational rents and can reduce trading. Hence, their model predicts that markets will be more illiquid when there are larger informational rents which might be the case for developing countries, for example, due to weak credit bureaus.

6 Quantitative Analysis

The model is calibrated to match US corporate debt market’s moments. Then, we use the estimated model to show that variations in financial systems can explain cross-country differences.
in maturities, credit spreads, and output.

6.1 Calibration

Some parameters can be calibrated “externally”, while others must be calibrated “internally” from the simulation of the model. Table 2 summarize the parameter values.

Liquidity spread The most important moment to discipline is the component of the term premium that is driven by liquidity in the data. Let $r_{\text{treasury}}(t)$ and $r_{\text{corporate}}(t)$ be the zero-coupon yield curve for treasuries and corporate debt, respectively. For corporate debt, we focus on the high-quality market (bonds with rating A and above) such that we can abstract from default risk. The US Treasury provides synthetic estimates of these two yield curves.\textsuperscript{19} We define the liquidity spread as $S(t) = r_{\text{corporate}}(t) - r_{\text{treasury}}(t)$. The identification assumption is that differences in markets’ liquidity imply all the variations in the spread between corporate and treasury rates. This assumption is valid because the US treasury is a very liquid market but the corporate market is known for its illiquidity. Figure 8 shows the estimated liquidity spread which is positive and increases with maturity. We target only this spread at the equilibrium maturity, i.e., one point of this yield curve, and evaluate the performance of the model in replicating the data at different maturities. Gilchrist and Zakrajšek (2012) also finds that credit spreads are increasing in maturity after controlling for distance to default and bond-specific characteristics (amount outstanding, coupon rate, callable, industry fixed effects, and credit ratings fixed effects).

Financial sector To discipline the search cost $c$ and the holding costs $h$ we target the term premium at maturity to be equal to 1.6% and the expected time to sell of 2 weeks (He and Milbradt, 2014). We calibrate the intensity of the liquidity shocks, $\lambda^H$, to match an annual turnover rate of 57% (He and Milbradt, 2014; Chen, Cui, He, and Milbradt, 2017). Note that turnover is equal to $\left((\lambda^H)^{-1} + (\lambda^S)^{-1}\right)^{-1}$, so we can directly calibrate $\lambda^H$ without simulating the model.\textsuperscript{20}

\textsuperscript{19}Appendix F.3 provides additional details.

\textsuperscript{20}The expected time to sell of two weeks implies that $\lambda^S = 26$. 
The figure shows the spread between the yield curve for treasuries and corporate bonds. Source: US Treasury.

**Maturity** We normalize the measure of newborn firms $\mu^0 = 1$. For the maturity of corporate bonds, we target a debt maturity at issuance of 12.2 years (Cortina Lorente, Didier, and Schmukler, 2016). By inspection of equation (15), we can normalize $\kappa = 1$ and internally calibrate $\delta_z$ to match the maturity of 12.2 years. Finally, the returns to scale parameter $\sigma$ is set equal to 0.8 as it is standard in the literature.

**Matching** For the secondary market we assume a constant-return-to-scale Cobb-Douglass matching function with efficiency parameter normalized to one, and symmetric loadings on sellers and buyers. We further assume symmetric bargaining power, $\gamma = 0.5$.

Finally, we assume a discount factor of $\rho = 0.02$ (He and Milbradt, 2014). For the default/exit rate we assume $\lambda^D = 3\%$ to match the default rate of speculative grade firms (Moody, 2015).

Table 2 shows that the model is able to match the four target moments. Even though the four parameters are jointly chosen to match the four target moments, we can derive some intuition on the identification of the parameters. Appendix E.4 discusses how the moments provide useful information to estimate the parameters. We now move to evaluate the model outside the target moments.
Table 2: Parameters and moments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target or source</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial sector</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Search cost</td>
<td>$c$</td>
<td>Liquidity spread at maturity</td>
<td>1.6000</td>
<td>1.6000</td>
</tr>
<tr>
<td>Holding cost</td>
<td>$h$</td>
<td>Expected time to sell</td>
<td>2.0000</td>
<td>2.0000</td>
</tr>
<tr>
<td>Intensity of liquidity shocks</td>
<td>$\lambda^H$</td>
<td>Turnover rate</td>
<td>0.5700</td>
<td>0.5700</td>
</tr>
<tr>
<td><strong>Production sector</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFP growth</td>
<td>$\delta_z$</td>
<td>Maturity</td>
<td>12.2001</td>
<td>12.200</td>
</tr>
<tr>
<td>Returns to scale</td>
<td>$\sigma$</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure of new firms</td>
<td>$\mu^0$</td>
<td>Normalization</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment cost</td>
<td>$\kappa$</td>
<td>Normalization</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Matching</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of sellers</td>
<td>$\alpha^S$</td>
<td>Symmetry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bargaining power of sellers</td>
<td>$\gamma$</td>
<td>Symmetry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency</td>
<td>$A$</td>
<td>Normalization</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Others</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\rho$</td>
<td></td>
<td>He and Milbradt (2014)</td>
<td></td>
</tr>
<tr>
<td>Default rate</td>
<td>$\lambda^D$</td>
<td></td>
<td>Moody (2015)</td>
<td></td>
</tr>
</tbody>
</table>

6.2 Validation

In the estimation, we target the liquidity spread only at maturity of 12.2 years. Figure 9 evaluates the spread generated by the model at different maturities and shows that the model performs well in replicating the shape of the liquidity spread. Although the model overshoots a bit at short maturities, this should not be important for this paper as the object of study is long-term finance. One potential explanation for not matching short-term rates could be that markets are segmented by their maturity and secondary markets for short-term rates are more liquid. In fact, Section 9 extends the model to analyze the role of segmented markets. For example, we can use this extension to match the yields at both short and long horizons if we assume that search costs for short-term securities are lower than for long-term ones.

There is some indirect empirical evidence that is in line with the mechanism of the paper: when financial markets are more liquid, firms choose to issue at longer maturities. First, Saretto and Tookes (2013) compares issuance of firms with and without credit default swaps (CDS) and argues that securities of companies with CDS trade in more liquid financial markets. The paper finds that firms with CDS increase the maturity between 0.68 and 1.79 years relative to those without CDS. Second, Cortina Lorente, Didier, and Schmukler (2016) studies firms in developing countries issuing bonds both in domestic and in foreign markets. The paper finds that maturity increases by 1.6 years for foreign issuances relative to domestic ones. This
evidence also corroborates the mechanism of the paper, as foreign markets are more liquid than domestic ones. Hence, the empirical evidence supports that companies issue at longer maturities when secondary markets are more liquid.

The second mechanism of the paper argues that when the term premium increases, firms choose to invest in shorter-term projects. Dew-Becker (2012) use data from the US and concludes that when the term premium increases the duration of investment decreases. Yamarthy (2016) also finds that when firms shift their long-term debt ratio to longer maturities, profitability and investment rates are higher. Finally, Foley-Fisher, Ramcharan, and Yu (2016) use cross-sectional variation to show that companies with more dependence on long-term debt benefit more when the yield curve flattens.

### 6.3 Quantitative results: Model vs data

We study the effects of trading frictions in financial markets on the real economy. For sovereign debt markets, Broner, Lorenzoni, and Schmukler (2013) shows that it is more expensive to borrow long-term in developing countries than in advanced ones. However, there is very little evidence on the yield curve for corporate bonds across countries. To overcome this problem, we use the model to infer the trading frictions that are consistent with the data on maturity choices across countries presented on Figure 2.
Figure 10: Search costs, interest rates, and maturity.

Figure 10 shows how variations in search costs affect the liquidity spread and the maturity choice. For example, the implied search costs that rationalize a maturity of 7 years has a liquidity spread of about 10 percent. Table 3 shows that in the data, the average maturity for developed economies is 10.87 years and credit spreads are 5.25. Instead, for developing economies, maturity is 7.29 years, and credit spreads are 11.25. We set search costs to match these two maturities. Column three shows that the estimated credit spreads predicted by the model are close to the data counterpart. Note that we did not use any data on credit spreads for developing countries in the estimation of the model which provides confidence in the exercise performed in this section.\(^{21}\) For each country \(j\) we find the search costs \(c_j\) such that the model matches the maturity \(\tau_j\) observed in the data. Then, we evaluate the performance of the model on replicating the effects on the real economy for each country. We should interpret this exercise as answering What are the effects of bringing the financial system of country \(j\) to the US?

Table 3: Credit spreads across countries.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>Developed</td>
<td>10.87</td>
</tr>
<tr>
<td>Developing</td>
<td>7.29</td>
</tr>
</tbody>
</table>

*Note: Credit spread data is from World Bank financial structure and economic development database. Maturity data is from Cortina Lorente, Didier, and Schmukler (2016).*

Figure 11 shows that the model predicts a relationship between maturity and GDP across countries quite similar to the data counterpart. Note that the model has a high explanatory

\(^{21}\)Of course, a fraction of this spreads is due to default and a fraction to liquidity. In this exercise we keep the default component constant across countries and only vary search costs which affect the liquidity component.
power for developed countries. However, for less-developed economies trading frictions explains about half of the GDP differences. In the next section, we use the estimation of the model and the implied financial systems across countries to study how the government can improve the liquidity of financial markets.

7 How to increase the liquidity of financial markets?

While it is outside the scope of this paper to fully characterize the optimal policy in this environment, we explore the effect of a government intervention designed to increase the liquidity of financial markets. We design the policy following two well-known policies in the US. First, Government-sponsored enterprises (GSEs) such as Freddie Mac and Fannie Mae, are steady-state institutions intended to improve the credit to households (see Fieldhouse and Mertens, 2017, for a recent review of this organizations). The proposed policy will be similar to GSEs but targeting the efficiency of credit to the corporate sector instead of households. Second, during the 2008 financial crisis, the central bank perform large-scale asset purchased (known as QE1, QE2, and QE3). Gertler, Karadi, et al. (2013) argues that this policy was effective due to limits to arbitrage in private intermediation.

We propose a public institution called Government-Sponsored Intermediaries (GSIs) that intermediate in secondary markets by buying and selling bonds at potentially different prices than in private bilateral meetings. The government charges a proportional profit tax on the corporate sector to finance the policy.
There are some examples of policies implemented or proposed in developing countries that share several features of GSIs. First, the “priority-sector lending” in India requires banks to lend at least 40% of their net credit to the “priority sector” and also establishes specific targets for different sub-sectors (which includes agriculture, and small-scale industry among others).\footnote{For details see Banerjee and Duflo (2014) and Reserve Bank of India master circular of July 2015, “Priority Sector Lending- Targets and Classification” available at https://rbi.org.in/scripts/bs_viewmascirculardetails.aspx?id=9857. Importantly, most of the banks in India are in the public sector, hence they could be interpreted as the government agents in the model.}

And interesting feature of this policy is that banks have to trade to meet all the specific targets, and, therefore, increase the liquidity of the secondary market of assets in the priority sector (a similar motive for trade as in the Fed Funds Market in the US, see Afonso and Lagos (2015)). One difference between “priority-sector lending” and GSIs is that the policy in India is for short-term debt (usually below one year) while this paper argues that liquidity is relatively more important for long-term assets. Hence, we propose similar policies but for the long-end of the market.

We find another similar example on policy proposals in Brazil. The private capital markets association (Anbima) launched a project to facilitate long-term financing. Some of the measures aim to increase the secondary market liquidity by the creation of a “Liquidity Improvement Fund” in which private agents manage public resources to act as market makers which are very similar to the proposed GSIs.\footnote{See Park (2012) for further details.}

### 7.1 Government-Sponsored Intermediaries

The government agency intermediates assets in the secondary market to improve the liquidity of financial markets.\footnote{We design the government agents with several restrictions so we can claim that the results should be interpreted as a lower bound. Section 7.4 relax some of this assumptions and shows that the results of the intervention improve above this lower bound.} We assume that government agents are subject to the same idiosyncratic risk of holding costs as private agents to preserve the same technological constraints. Therefore, government agents act both as buyers and sellers in the secondary market.\footnote{The formulation of government agents is similar to Aiyagari, Wallace, and Wright (1996).} However, we let the government choose the price at which they buy and sell on the secondary market. The government will run a deficit if it chooses to buy (sell) at a high (low) price. The resources, and therefore the size of GSIs, are limited by the taxes levied to the corporate sector.

Figure 12 shows a schematic representation of the model when we introduce GSIs. Note that...
private sellers can now sell both to private and government agents. Moreover, private buyers can now buy in the secondary market from either private or government sellers. In this section, we describe the key features of the model with GSIs while Appendix C contains the additional details.

Note that both primary and secondary financial markets are competitive—i.e., participants make zero profits in expectation. However, the production sector—i.e., the borrowers—have positive profits in equilibrium. Hence, the objective of the government is to maximize the value of the corporate sector. The government chooses prices for buying and selling for their trading agents, the size of GSIs, and the corporate tax rate, subject to running a balanced budget.

**Matching**  There is random matching between sellers and buyers. In the benchmark policy, we assume that both government and private agents have the same efficiency to find a counterpart. However, for robustness exercises performed in Section 7.4, we propose a general formulation in which government and private agents can have different efficiency to find a counterpart. In particular, let $e^{i,j}$ be the efficiency for $i = P, G$ (private and government, respectively) of $j = B, S$ (buy and sell, respectively) and in the benchmark model we assume that $e^{i,j} = 1$. 
The total mass of sellers, $\mu^S$, is composed by private and government agents. Private sellers are
$$\mu^{S,P} = \int_0^T e^{P,S} \mu^{L,P}(s) ds$$
where $\mu^{L,P}(s)$ is the measure of low-valuation private agents holding an asset and willing to sell. Similarly, government sellers are
$$\mu^{S,G} = \int_0^T e^{G,S} \mu^{L,G}(s) ds,$$
where $\mu^{L,G}(s)$ is the measure of low-valuation government agents holding an asset and willing to sell.

The total measure of buyers has private buyers $\mu^{P,B}$, determined by a free-entry condition, and government buyers $\mu^{B,G}$ which is a policy instrument chosen by the government. Hence,
$$\mu^B = e^{P,B} \mu^{P,B} + e^{G,B} \mu^{G,B}.$$ 

The market tightness is $\theta = \frac{\mu^S}{\mu^B}$ and characterizes the buying and selling intensities $\lambda^B = A\theta^\alpha$, and $\lambda^S = A\theta^{\alpha-1}$, respectively. Let $\lambda^{S,s-b}$ be the intensity at which a seller of type $s = P,G$ (private and government) meets a buyer of type $b = P,G$. The matching technology implies that

$$\lambda^{S,P-P} = \lambda^S e^{P,B} e^{P,S} \frac{\mu^{B,P}}{\mu^B}, \quad \lambda^{S,P-G} = \lambda^S e^{G,B} e^{P,S} \frac{\mu^{B,P}}{\mu^B}, \quad \lambda^{S,G-P} = \lambda^S e^{P,B} e^{G,S} \frac{\mu^{B,P}}{\mu^B}.$$ 

We have to assume what happens after a government buyer and a government seller meets. We want to interpret the government as a large player and private agents as atomistic. However, for tractability, we assume that all investors can hold either zero or one unit of the asset. To bypass this restriction, assume that a government seller cannot trade with a government buyer, i.e., $\lambda^{S,P-P} = 0$. Note, that this is a conservative assumption. The cost of the policy will be smaller if we allow for this type of trade. In fact, we find similar results when we solve an alternative policy in which these trades are allowed (see Section 7.4).

Let $\lambda^{B,s-b}(y)$ be the intensity at which a buyer of type $b = P,G$ meets a seller of type $s = P,G$ with an asset of time-to-maturity $y$. Then

$$\lambda^{B,P-P}(y) = \lambda^B e^{P,B} e^{P,S} \frac{\mu^{L,P}(y)}{\mu^S}, \quad \lambda^{B,G-P}(y) = \lambda^B e^{P,B} e^{G,S} \frac{\mu^{L,G}(y)}{\mu^S},$$

Prices in secondary markets There are three types of meetings in secondary markets. Let $P^{S,s-b}(y)$ be the price when a seller of an asset with time-to-maturity $y$ of type $s = P,G$ meets a buyer of type $b = P,G$. The price is determined by Nash Bargaining when a private buyer meets a private seller. The bargaining power of the seller is $\gamma$ so

$$P^{S,P-P}(y) = D^L(y) + \gamma(D^H(y) - D^L(y)).$$
The prices that involve either a government buyer or seller are determined by the government. In the quantitative solution we restrict prices to be in the following parametric family

\[ P_{S,G}^-(y) = G^L(y) + \gamma_{G,S}(D^H(y) - D^L(y)) \]
\[ P_{S,P}^-(y) = D^L(y) + \gamma_{G,B}(D^H(y) - D^L(y)) \]

and let the government choose \( \gamma_{G,S} \) and \( \gamma_{G,B} \). Note that prices are similar to those in private meetings but we allow the government to choose a different bargaining power. As we show latter, it is optimal to set \( \gamma_{G,S} = 1 \) and \( \gamma_{G,B} = 0 \). This implies that the government gives all the bargaining power to the private sector (i.e., the government buys at a high price and sell at a low price).

Of course, this is an important restriction on government prices. For example, we can use the model with segmented markets presented in Section 9 and allow the government to set different prices according to the maturity. However, we will show that even without this flexibility, the effects of GSIs are quantitatively significant and the extension of targeting prices according to the maturity is likely to improve the results from the lower bound identified in this exercise. Importantly, note that this alternative specification would work trough the same channel than the mechanism described in the benchmark policy.

**Private valuations** The value of holding an asset for a high-valuation private agent is equivalent to the benchmark model. However, the value of a low-valuation private agent is different as now he can sell it both to private and government buyers. Under the government prices specified in (16), the price that the government offers is equivalent to that in private meetings but in which the seller has a different bargaining power. Hence, the value of a constrained agent is equal to the benchmark model with an augmented selling intensity: \( \Lambda = \lambda_{S,P}^P \gamma + \lambda_{P}^P \gamma_{G,B} \).

Let \( \Lambda^{LF} \) and \( \Lambda^{GSI} \) be the equilibrium liquidity in the laissez-faire economy and in model with GSIs, respectively. If \( \Lambda^{GSI} > \Lambda^{LF} \), then Proposition 1 implies that the liquidity spread will be lower in an economy with GSIs. However, borrowers have to pay a tax to finance the intervention. Hence, ex-ante, we don’t know if the policy increase the benefits of the borrowers.

The free entry condition for the private sector also changes as private buyers can meet both
with private and government sellers. The free entry condition is

\[
c = \lambda^B e^{P_B} e^{P_S} \int_0^\tau \frac{\mu^{L,P}(y)}{\mu^S} \left( D^H(y) - P^{S,P}(y) \right) dy \\
+ \lambda^B e^{P_B} e^{G_S} \int_0^\tau \frac{\mu^{G,L}(y)}{\mu^S} \left( D^H(y) - P^{S,G}(y) \right) dy.
\]

**Cost of GSIs** The government runs a balanced budget

\[
x^e f(\tau) \mu^F + \left[ \mu^{G,H}(0) + \mu^{G,L}(0) \right] + \lambda^{S,G-P} \int_0^\tau \mu^{L,G}(y) P^{S,G-P}(y) dy = \mu^{B,G} + \mu^{G} \int_0^\tau \lambda^{B,P-G}(y) P^{G-P}(y) dy + h \int_0^\tau \mu^{L,G}(y) dy
\]

The left-hand-side of Equation (17) represents the government’s income. First, the government charge a proportional corporate tax \( x^e \) to producing firms \( \mu^F \), where flow profits are \( f(\tau) = (1 - \sigma)\tau \). Second, some of the securities held by government agents mature. Third, some low-valuation government agents sell the securities to the private sector.

The right-hand side of Equation (17) captures the expenditures. First, a measure \( \mu^{B,G} \) of agents are searching in secondary markets, and some of them buy a bond. Moreover, some government agents are low-value and have to pay the holding cost \( h \).

**Optimal policy** In equilibrium, the financial sector is competitive—i.e., financiers make zero profits ex-ante. Only the production sector has positive profits in equilibrium. Hence, the objective of the government’s is to maximize the profit of the production sector subject to the equilibrium conditions and the budget constraint (17)

\[
\max_{x^e, \mu^{G,S}, \gamma^{S,G}, \gamma^{S,GS}} e^{-(\mu + \lambda D)T} \left( (1 - x^e) F(\tau) - I(\tau) e^{r(\tau)T} \right)
\]

Note that GSIs cause two effects. On the one hand, the direct effect implies that a larger GSIs (with higher taxes) lowers welfare. On the other hand, we have the equilibrium effect of GSIs. A larger intervention can increase the liquidity and reduce credit spreads which benefits the borrowers. The optimal policy solve the trade-off between these two effects.
7.2 Results: Optimal Policy in the US

The government chooses the size of GSIs and buying and selling prices to maximize the value of intermediate goods producers. We use the calibration of the model for the US to solve for the optimal policy.

The bargaining power when the government acts as a buyer, \( \gamma^{G,P-G} \), directly affects the value of low and high valuation private agents. The optimal policy sets \( \gamma^{G,P-G} = 1 \) so private sellers get more gains from trade when trading with the government. This generates a direct effect on increasing the value of private agents in the financial sector and reduce the financial cost for the production sector.

The bargaining power when the government acts as a seller, \( \gamma^{G,G-P} \), has a direct effect on the incentives of private agents to search in the secondary market. The optimal value for \( \gamma^{G,G-P} = 0 \). Finally, the measure of government agents searching in the secondary markets is optimally chosen to maximize the welfare gains. If \( \mu^B = 0 \) the economy is equivalent to no intervention, while as \( \mu^{G,B} \) increases, the tax rate also increases to balance the budget. Given the results in this section, in the next exercises we set \( \gamma^{G,P-G} = 1 \) and \( \gamma^{G,G-P} = 0 \) and the government chooses \( \mu^{G,B} \).

7.3 Results: GSIs for different financial systems

We now evaluate the effect of GSIs for different financial systems. From Section 6 we can map the efficiency of financial markets across countries with the value of the search costs. We set a lower bound to mimic the system of the US and the upper bound to replicate financial markets in Latin America.

The top left panel of Figure 13 shows the effect of GSIs on the liquidity of financial markets. GSIs are more efficient to improve the liquidity of financial markets when search frictions are relatively low. However, the right top figure shows that the flattening of the yield curve due to GSIs is more effective when there are larger frictions. In an advanced financial market, the marginal effect of an increase in liquidity is smaller than in a less developed financial system. Hence, even though the improvement in liquidity is lower in developing systems, the consequences might be larger.
The bottom right figure shows that GSIs can increase the maturity of corporate debt about 0.5 years in the US. Note that this effect is much larger in less developed financial markets. For example, in a system similar to Latin America, GSIs would increase the maturity of corporate debt about 1.4 years. Finally, the bottom right figure shows that GSIs generate an increase in welfare of about 1% in the US and 5% in Latin America.

7.4 Robustness: Alternative policies

The exercises considered so far should be consider as a lower bound on the effect of GSIs. In this section we explore alternative assumptions that can improve the effects of the government intervention. Table 4 present the results for two financial systems. The first panel consider search costs at the level estimated for the US while the second panel consider an economy with trading frictions such as in Latin America.
First, we explore what happens if government agents are more efficient to search a counterpart. In particular, the third and fourth rows of Table 4 show the result of increasing the search efficiency of government agents by 10% and 50%, respectively. Overall, the results show that as the efficiency of the government increases, GSIs become more effective in increasing the liquidity of the economy and the cost of the intervention reduces (lower tax rates). As a result, the yield curve flattens and firms issue at longer maturities.

Finally, recall that in the benchmark policy we assume that $\lambda^{S,G-G} = 0$, i.e., a government seller cannot trade with a government buyer. For a given size $\mu^{G,B}$, the cost of GSIs decreases if we allow the government to reallocate the securities within its trading agents. The last row of table 4 shows that GSIs are more efficient if we assume that a government seller can trade with a government buyer.

There are legitimate reasons to imagine that government agents might have more flexibility than private agents. Hence, the results of the benchmark policy should be considered as a lower bound on the effect of GSIs. The results on table 4 show that the gains from the government intervention can be larger if government agents are more efficient to find counterparts or can trade within each agents.

Table 4: GSI: Alternative policies.

<table>
<thead>
<tr>
<th>Low trading frictions (US)</th>
<th>Liquidity</th>
<th>Spread 5 years</th>
<th>Tax</th>
<th>Maturity</th>
<th>Welfare gains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laissez faire</td>
<td>13.00</td>
<td>1.23</td>
<td>0.000</td>
<td>12.20</td>
<td></td>
</tr>
<tr>
<td>Benchmark policy</td>
<td>20.00</td>
<td>0.80</td>
<td>0.002</td>
<td>12.66</td>
<td>1.12</td>
</tr>
<tr>
<td>Gov. 10% more efficient</td>
<td>20.24</td>
<td>0.80</td>
<td>0.002</td>
<td>12.67</td>
<td>1.18</td>
</tr>
<tr>
<td>Gov. 50% more efficient</td>
<td>20.97</td>
<td>0.77</td>
<td>0.001</td>
<td>12.70</td>
<td>1.37</td>
</tr>
<tr>
<td>Gov. transactions</td>
<td>22.20</td>
<td>0.73</td>
<td>0.000</td>
<td>12.75</td>
<td>1.63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High trading frictions (Latin America)</th>
<th>Liquidity</th>
<th>Spread 5 years</th>
<th>Tax</th>
<th>Maturity</th>
<th>Welfare gains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laissez faire</td>
<td>1.37</td>
<td>9.32</td>
<td>0.000</td>
<td>6.62</td>
<td></td>
</tr>
<tr>
<td>Benchmark policy</td>
<td>2.33</td>
<td>6.02</td>
<td>0.027</td>
<td>8.02</td>
<td>5.92</td>
</tr>
<tr>
<td>Gov. 10% more efficient</td>
<td>2.36</td>
<td>5.96</td>
<td>0.026</td>
<td>8.06</td>
<td>6.24</td>
</tr>
<tr>
<td>Gov. 50% more efficient</td>
<td>2.44</td>
<td>5.78</td>
<td>0.023</td>
<td>8.16</td>
<td>7.24</td>
</tr>
<tr>
<td>Gov. transactions</td>
<td>2.47</td>
<td>5.72</td>
<td>0.011</td>
<td>8.23</td>
<td>9.14</td>
</tr>
</tbody>
</table>
8 Robustness: Rollover and Default

In this section we study how borrowers choose and finance investment projects when they can roll over short-term debt to finance long-term projects. Section 4 presents the general problem of the firm and analyzed how the financial choices (number of issuances and maturity structure) is affected by issuance costs and the slope of the yield curve. In this section we analyze how the choice of the project $\tau$ is affected by the issuance costs, the liquidity of the secondary market, and the default rate. In a nutshell, the main take-away of this section is that the effect of the liquidity of the secondary market on the choice of projects is surprisingly very similar when the firm can issue bonds only one time or is allowed to rollover short-term debt.

Proposition 3 and the quantitative exercises on Section 6 show that the liquidity of the secondary market is important for investment decisions when firms match the maturity of assets and liabilities. Moving from an OTC secondary market with liquidity as in the US economy to a shut down of the secondary market reduce the project’s maturity from 12.2 to 3.8 years in the model without rollover (first panel of table 5, rows two and three). The second to fourth panel show that we find a similar drop in project’s maturity when firms are allowed to rollover short-term debt. This result is robust to issuance costs that are relatively low, and issuances are frequently, or high, and issuances are infrequently. Hence, the results on Section 6 about how investment decisions are affected by the liquidity of the secondary market do not depend on the rollover assumptions.

An increase in the issuance cost generate two effects: (i) Conditional on a project, firms issue less frequent and increase the maturity of debt; and (ii) Firms reduce the project’s maturity. Quantitatively, most of the adjustment is made by the number of issuances while the choice of projects is rather unaffected. This result also confirms that abstracting from rollover decisions in the main analysis of the paper is rather inconsequential for the real side of the economy.

One potential concern about the exercises performed in the paper is that the liquidity of secondary markets and rollover costs might be correlated. In the model, liquidity costs are endogenous while rollover costs are exogenous and fixed, so they do not respond to changes in the liquidity of secondary markets, and a potential Lucas critique may apply. However, we expect that when the secondary market becomes more liquid, both issuance and rollover costs should diminish. Hence, it is conservative to assume fixed issuance costs. As we have shown in Table 5, when liquidity of secondary markets improves, firms adopt projects of longer maturity. Similarly, when issuance costs diminish, firms choose longer-term projects. Hence, if the two
effects are present (an increase in liquidity and a reduction of issuance costs), the firm will extend the maturity of the project even more than with fixed issuance costs.

8.1 Default

Proposition 1 shows that the liquidity spread increases with the default intensity. Interestingly, this effect depends on the maturity of the security. Figure 14 shows how the liquidity spread changes with the default rate, relative to the benchmark of $\lambda^D = 3\%$, for yields at one and ten years. Both of them are increasing in $\lambda^D$. However, long-term rates react more than short-term rates to changes in default risk. When the default rate increases, the yield curve shifts upwards because both default and liquidity spread increase. As a result, firms choose to implement shorter-term projects (see row four to six on Table 5).

Figure 14: Liquidity-default interactions.

```
0 2 4 6
Default rate
0.9
1
1.1
1.2
Change in liquidity spread
1 year
10 years
```

*Note: The figure shows how the liquidity spread at 1 and 10 years changes with $\lambda_D$ with respect to the benchmark of 3%.*

When there is no default risk, and secondary markets are centralized firms have no incentive to issue short-term debt, regardless of the issuance cost. However, when default risk is positive, even if secondary markets are centralized firms choose to rollover debt when the issuance cost is not too high. Hence, both default risk and the liquidity of the secondary market shape rollover choices.
Table 5: Solution under alternative specifications.

<table>
<thead>
<tr>
<th>Default</th>
<th>Secondary market</th>
<th>Issuance cost</th>
<th>Issuances</th>
<th>Maturity</th>
<th>Interest rate</th>
<th>Credit spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Project</td>
<td>Bond</td>
<td></td>
<td>Default</td>
</tr>
<tr>
<td>No</td>
<td>Centralized</td>
<td>0.10</td>
<td>6</td>
<td>14.2</td>
<td>2.4</td>
<td>5.0%</td>
</tr>
<tr>
<td>No</td>
<td>OTC</td>
<td>0.10</td>
<td>7</td>
<td>13.7</td>
<td>2.0</td>
<td>6.1%</td>
</tr>
<tr>
<td>No</td>
<td>Shut down</td>
<td>0.10</td>
<td>9</td>
<td>5.3</td>
<td>0.6</td>
<td>8.8%</td>
</tr>
<tr>
<td>No</td>
<td>Centralized</td>
<td>0.50</td>
<td>3</td>
<td>14.1</td>
<td>4.7</td>
<td>5.0%</td>
</tr>
<tr>
<td>No</td>
<td>OTC</td>
<td>0.50</td>
<td>3</td>
<td>13.4</td>
<td>4.5</td>
<td>6.2%</td>
</tr>
<tr>
<td>No</td>
<td>Shut down</td>
<td>0.50</td>
<td>5</td>
<td>5.3</td>
<td>1.1</td>
<td>11.6%</td>
</tr>
<tr>
<td>No</td>
<td>Centralized</td>
<td>1.00</td>
<td>2</td>
<td>13.9</td>
<td>7.0</td>
<td>5.0%</td>
</tr>
<tr>
<td>No</td>
<td>OTC</td>
<td>1.00</td>
<td>2</td>
<td>13.1</td>
<td>6.6</td>
<td>6.3%</td>
</tr>
<tr>
<td>No</td>
<td>Shut down</td>
<td>1.00</td>
<td>4</td>
<td>5.3</td>
<td>1.3</td>
<td>13.0%</td>
</tr>
<tr>
<td>No</td>
<td>Centralized</td>
<td>1.00</td>
<td>1</td>
<td>35.0</td>
<td>35.0</td>
<td>2.0%</td>
</tr>
<tr>
<td>No</td>
<td>OTC</td>
<td>1.00</td>
<td>3</td>
<td>30.3</td>
<td>10.1</td>
<td>3.2%</td>
</tr>
<tr>
<td>No</td>
<td>Shut down</td>
<td>1.00</td>
<td>4</td>
<td>5.5</td>
<td>1.4</td>
<td>10.2%</td>
</tr>
</tbody>
</table>
9 Extension: Segmented Markets

In the benchmark model assets of different maturities are traded in a single secondary market. A potential concern could be that assets of short maturities, with small gains from trade, preclude the entry of buyers to the secondary market. In this section, we consider an alternative specification with secondary markets segmented by the time-to-maturity of the asset. The main takeaway is that even though the market tightness for short-term bonds increases, the market tightness for long-term assets remains similar to the case with only one market. Hence, the secondary market in the benchmark model is effectively a market for long-term assets.

This extension can be useful to study why the model over-estimates the yield curve at short horizons (recall Figure 9). For example, if the entry cost for a market with short-term assets is smaller than for a market with long-term assets, then this extension can help the model to match the data. We can also use this model to consider government interventions such as in Section 7 in which the government set different targets for short- and long-term securities.

Intuitively, in long-term markets, there are more gains from trade and therefore more entry of buyers. However, because there is Nash Bargaining over the gains from trade and buyers keep a fraction of the gains, the increase in the entry of buyers in long-term markets is not enough to compensate the increase in the importance of the secondary markets for longer securities. As a result, the yield curve increases in maturity even with segmented markets. We describe the key features of the model in the main text and relegate to Appendix D the full characterization of this extension.

Let \( \tau \) be the initial maturity and consider the case in which secondary markets are segmented in \( N \) markets. Let \( 0 = \tau_1 < \cdots < \tau_{N+1} = \tau \) so each market \( j = 1, \ldots, N \) trade assets with time to maturity \( t \in [\tau_j, \tau_{j+1}] \).

Matching and distribution of agents Let \( \mu^j(y) = [\mu^{H,j}(y), \mu^{L,j}(y)] \) be the measure of high- and low-valuation agents holding an asset with time to maturity \( t \) in market \( j \). We start by the last market and solve for the distribution of agents backwards. The boundary condition is \( \mu^N(\tau) = [\mu^0, 0] \). Next, we can iterate towards markets of shorter maturities. The boundary condition for market \( j = 1, \ldots, N-1 \) is \( \mu^j(\tau_{j+1}) = \mu^{j+1}(\tau_{j+1}) \). Lemma 4 characterize the distribution of agents in each market.

**Lemma 4.** The measure of agents for markets \( j = 1, \ldots, N \) is given by the following backward
recursion
\[
\begin{bmatrix}
\mu_{H,N+1}(\tau) \\
\mu_{L,N+1}(\tau)
\end{bmatrix} = \begin{bmatrix}
\mu_0 \\
0
\end{bmatrix}
\]
and
\[
\mu_{H,j}(y) = \frac{\lambda^H}{\lambda^H + \lambda^S,j} \left[ \lambda^S,j e^{\lambda^D(y-\tau_{j+1})} \left( \mu_{H,j+1}(\tau_{j+1}) + \mu_{L,j+1}(\tau_{j+1}) \right) - e^{(\lambda^H + \lambda^S,j + \lambda^D)(y-\tau_{j+1})} \left( -\mu_{H,j+1}(\tau_{j+1}) + \lambda^S,j \mu_{L,j+1}(\tau_{j+1}) \right) \right]
\]
\[
\mu_{L,j}(t) = \frac{\lambda^H}{\lambda^H + \lambda^S,j} \left[ e^{\lambda^D(y-\tau_{j+1})} \left( \mu_{H,j+1}(\tau_{j+1}) + \mu_{L,j+1}(\tau_{j+1}) \right) + e^{(\lambda^H + \lambda^S,j + \lambda^D)(y-\tau_{j+1})} \left( -\mu_{H,j+1}(\tau_{j+1}) + \lambda^S,j \mu_{L,j+1}(\tau_{j+1}) \right) \right].
\]

Valuations  To solve for the value functions we start with the first market. The boundary condition is that at maturity the value is equal to one. Then, we iterate towards longer-term markets. Let \( D_j(y) = [D_{H,j}(y), D_{L,j}(y)] \) be the value functions of high- and low-valuation agents holding an asset with time to maturity \( t \) in market \( j = 1, \ldots, N \). The boundary condition for market \( j = 1 \) is \( D_1(\tau_1) = [1, 1] \). Value matching for market \( j = 2, \ldots, N \) implies \( D_j(\tau_j) = D_{j-1}(\tau_j) \) and the Hamilton-Jacobi-Bellman equations are the same as in the benchmark model, Equations (6) and (7).

Free entry  Free entry in each market implies that
\[
c = (1 - \gamma) \int_{\tau_j}^{\tau_{j+1}} \lambda^{B,j}(y) \left( D_{H,j}(y) - D_{L,j}(y) \right) dy
\]

Appendix D.2 provides the analytical solution for the value functions and the free entry condition.

9.1 Results

The first panel of Figure 15 shows the market tightness, relative to the case of only one market, when \( N = 2 \) and \( N = 3 \). With segmentation, markets for short-term assets are tighter (more sellers to buyers) as there are fewer gains from trade in short-term markets. However, for
long-term bonds, we find a tightness similar to the case of no segmentation. The second panel repeats the exercise under different degrees of segmentation ($N = 1, \ldots, 50$). Note that even with 50 different markets, the tightness for markets with maturity above four years is almost identical to the case of no segmentation.

The third panel of Figure 15 shows the liquidity spread for different models with $N = 1$ to $N = 50$. As the market tightness after four years is identical in all these models, the implied liquidity spread is also the same. For short-term assets (maturities up to 4 years), there are some differences in the market tightness. However, they generate small variation in the yield curve. Therefore, we conclude that the secondary market in the benchmark model with $N = 1$ is effectively a market for long-term assets.

10 Conclusion

This paper studies the linkages between the maturity of corporate debt, the liquidity of financial markets and the real economy. When corporate debt markets are more liquid, the financing cost for firms diminish, and entrepreneurs choose to enter into projects of longer maturities. Hence, firms invest in better technologies causing economic development.
References


European Comission (2013): “Long-Term Financing of the European Economy.”


Gertler, M., P. Karadi, et al. (2013): “Qe 1 vs. 2 vs. 3...: A framework for analyzing large-scale asset purchases as a monetary policy tool,” international Journal of central Banking, 9(1), 5–53.


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A Omitted proofs: Lenders

In this section we provide the proofs of the results for lenders studied in Section 3.

A.1 Distribution of financiers

Proof of Lemma 1. Let \( \mu(y) = [\mu^H(y), \mu^L(y)] \) and note that matching implies \( \mu^B \lambda^B \mu^L(y) = \lambda^S \mu^L(y) \). Then, Equations (1)-(2) imply that \( \dot{\mu}(y) = A \mu(y) \) where

\[
A = \begin{bmatrix}
\lambda^D + \lambda^H & -\lambda^S \\
-\lambda^H & \lambda^D + \lambda^S
\end{bmatrix}
\]

with boundary condition \( \mu(\tau) = [\mu^0, 0] \). Note that \( A \) has two real and distinct eigenvalues, let \( R \) the vector with the eigenvalues and \( V \) be the matrix with eigenvectors of \( A \). Also, use the boundary condition to define \( B = (V)^{-1} \mu(\tau) \)

\[
V = \begin{bmatrix}
-1 & \lambda^S \\
1 & 1
\end{bmatrix}, \quad R = \begin{bmatrix}
\lambda^H + \lambda^S + \lambda^D \\
\lambda^D
\end{bmatrix}, \quad B = \frac{\lambda^H \mu^0}{\lambda^H + \lambda^S} \begin{bmatrix}
-1 \\
1
\end{bmatrix}
\]

Then it is standard to show that

\[
\mu^H(y) = \sum_{i=1}^{2} e^{R_i(y-\tau)} B_i V (1, i) \quad \mu^L(y) = \sum_{i=1}^{2} e^{R_i(y-\tau)} B_i V (2, i)
\]

Finally, a few lines of algebra deliver

\[
\mu^H(y) = \frac{\mu^0 \lambda^H}{\lambda^H + \lambda^S} \left( e^{\lambda^D(y-\tau)} \frac{\lambda^S}{\lambda^H} + e^{(\lambda^H + \lambda^S + \lambda^D)(y-\tau)} \right)
\]

\[
\mu^L(y) = \frac{\mu^0 \lambda^H}{\lambda^H + \lambda^S} \left( e^{\lambda^D(y-\tau)} - e^{(\lambda^H + \lambda^S + \lambda^D)(y-\tau)} \right)
\]
A.2 Value functions

Proof of Lemma 2. Replace the price of the asset in the secondary market $P^S(t)$ in (6)-(7) so

$$(\rho + \lambda^D) D^H(y) = -\dot{D}^H(y) + \lambda^H (D^L(y) - D^H(y))$$

$$(\rho + \lambda^D) D^L(y) = -h - D^L(y) + \lambda^S \gamma (D^H(y) - D^L(y))$$

Let $H(y) = D^H(y) - D^L(y)$ then

$$(\rho + \lambda^D + \lambda^H + \lambda^S \gamma) H(y) = h - \dot{H}(y)$$

with $H(0) = 0$. It is straight forward to see that $H(y) = h \frac{1 - e^{-c_1 y}}{c_1}$ where $c_1 = \rho + \lambda^D + \lambda^H + \lambda^S \gamma$.

Next, we can solve for $D^H(y)$ as

$$(\rho + \lambda^D) D^H(y) = -\dot{D}^H(y) - \lambda^H h \frac{1 - e^{-c_1 y}}{c_1}$$

with boundary $D^H(0) = 1$. The solution is $D^H(y) = A e^{-\left(\rho + \lambda^D\right) y} + Ce^{-c_1 t}$ with constants

$$A = -\frac{1}{\rho + \lambda^D} \frac{\lambda^H h}{c_1}, \quad C = -\frac{1}{\lambda^H + \lambda^S \gamma} \frac{\lambda^H h}{c_1}, \quad B = 1 + \frac{\lambda^H h}{(\lambda^H + \lambda^S \gamma)(\rho + \lambda^D)}$$

Finally, a few lines of algebra deliver

$$D^U(y) = e^{-\left(\rho + \lambda^D\right) y} - \mathcal{L}(y, \lambda^S)$$

$$\mathcal{L}(y, \Lambda) = \frac{\lambda^H h}{\lambda^H + \Lambda} \left( \frac{1 - e^{-\left(\rho + \lambda^D\right) y}}{\rho + \lambda^D} - \frac{1 - e^{-\left(\rho + \lambda^D + \lambda^H + \Lambda\right) y}}{\rho + \lambda^D + \lambda^H + \Lambda} \right)$$

$$\mathcal{L}(\tau, \Lambda) = h \frac{\lambda^H}{\lambda^H + \Lambda} \int_0^\tau e^{-\left(\rho + \lambda^D\right) y} \left( 1 - e^{-\left(\lambda^H + \Lambda\right) y} \right) dy$$

where $\Lambda = \lambda^S \gamma$. Finally, we can compute the value of a low-valuation agent as $D^L(y) = D^H(y) - H(y)$, that is

$$D^L(y, \Lambda) = e^{-\left(\rho + \lambda^D\right) y} - h \frac{1 - e^{-\left(\rho + \lambda^D\right) y}}{\rho + \lambda^D} + \frac{\Lambda}{\lambda^H} \mathcal{L}(y, \Lambda).$$

Properties of the illiquidity cost:

1. **Positive**: $\mathcal{L}(\tau, \Lambda)$ is positive as $\rho + \lambda^D + \lambda^H + \Lambda \geq \rho + \lambda^D$. 

4
2. Sensitivity with respect to maturity $\tau$:

(a) Increasing in $\tau$:

$$\frac{\partial L(\tau, \Lambda)}{\partial \tau} = h \frac{\lambda^H}{\lambda^H + \Lambda} e^{-(\rho + \lambda^D)\tau} \left(1 - e^{-(\lambda^H + \Lambda)\tau}\right) \geq 0$$

(b) Limit:

$$\lim_{\tau \to \infty} L(\tau, \Lambda) = h \frac{\lambda^H}{\lambda^H + \Lambda} \frac{1}{\rho + \lambda^D - \rho + \lambda^D + \lambda^H + \Lambda} = h \frac{\lambda^H}{(\rho + \lambda^D)(\rho + \lambda^D + \lambda^H + \Lambda)}$$

3. Sensitivity with respect to liquidity shocks $\lambda^H$:

(a) If there are no liquidity shocks ($\lambda^H = 0$), then $L(\tau, \Lambda) = 0$.

(b) If $\lambda^H \to \infty$ (i.e., always has to pay the cost $h$) then

$$\lim_{\lambda^H \to \infty} L(\tau, \Lambda) = h \frac{1 - e^{-(\rho + \lambda^D)\tau}}{\rho + \lambda^D}$$

4. Sensitivity with respect to liquidity of the secondary market $\Lambda$:

(a) It is decreasing in $\Lambda$. Note that the illiquidity cost is

$$L(\tau, \Lambda) = \lambda^H h \left(\frac{1}{(\rho + \lambda^D)(\rho + \lambda^D + \lambda^H + \Lambda)} - \frac{e^{-(\rho + \lambda^D)\tau}}{(\lambda^H + \Lambda)(\rho + \lambda^D)}\right)$$

$$\quad \quad + \lambda^H h \frac{e^{-(\rho + \lambda^D + \lambda^H + \Lambda)\tau}}{(\lambda^H + \Lambda)(\rho + \lambda^D + \lambda^H + \Lambda)}$$

so

$$\frac{\partial L(\tau, \Lambda)}{\partial \Lambda} = \lambda^H h \left(-\frac{1}{(\rho + \lambda^D)(\rho + \lambda^D + \lambda^H + \Lambda)^2} + \frac{e^{-(\rho + \lambda^D)\tau}}{(\rho + \lambda^D)(\lambda^H + \Lambda)^2}\right)$$

$$\quad \quad - \lambda^H h \left(\frac{\tau e^{-(\rho + \lambda^D + \lambda^H + \Lambda)\tau}}{(\lambda^H + \Lambda)(\rho + \lambda^D + \lambda^H + \Lambda)} + \frac{e^{-(\rho + \lambda^D + \lambda^H + \Lambda)\tau}}{(\lambda^H + \Lambda)^2(\rho + \lambda^D + \lambda^H + \Lambda)}\right)$$

$$\quad \quad - \lambda^H h \frac{e^{-(\rho + \lambda^D + \lambda^H + \Lambda)\tau}}{(\lambda^H + \Lambda)(\rho + \lambda^D + \lambda^H + \Lambda)^2}$$
Let \( a = \lambda^H + \Lambda \) and \( b = \rho + \lambda^D \) so

\[
\frac{\partial L(\tau, \Lambda)}{\partial \Lambda} = \lambda^H h \left( -\frac{1}{b(a+b)^2} + \frac{e^{-b\tau}}{ba^2} \right) - \lambda^H h \frac{e^{-(a+b)\tau}}{a(a+b)} \left( \tau + \frac{2a+b}{a(a+b)} \right)
\]

We want to show that

\[
\frac{e^{-b\tau}}{ba^2} \leq \frac{1}{b(a+b)^2} + \frac{e^{-(a+b)\tau}}{a(a+b)} \left( \tau + \frac{2a+b}{a(a+b)} \right)
\]

Define \( L(\tau) \) and \( R(\tau) \) to the left- and right-hand-side of (18), respectively. Note that \( R(0) = L(0) = \frac{1}{ba^2} \). Hence, it is sufficient to show that the slope of \( L(\tau) \) is lower than the slope of \( R(\tau) \) for all \( \tau \). Note that

\[
\frac{\partial L(\tau)}{\partial \tau} = -\frac{e^{-b\tau}}{a^2}, \quad \frac{\partial R(\tau)}{\partial \tau} = -\frac{e^{-(a+b)\tau}}{a} \left( \tau + \frac{1}{a} \right)
\]

Hence, the slope of \( L \) is lower than the slope of \( R \) because \( a\tau \geq \log(a\tau + 1) \).

(b) If there are no secondary markets, i.e., \( \Lambda = 0 \), then the illiquidity cost represents the expected holding costs, i.e.,

\[
L(\tau, 0) = h \int_0^\tau e^{-(\rho+\lambda^D)y} \left( 1 - e^{-\lambda^H y} \right) dy
\]

(c) If secondary markets are totally liquid (i.e., \( \Lambda \to \infty \)) then \( L(\tau, \Lambda) = 0 \).

5. **Liquidity is more important for long-term assets:** Recall that

\[
\frac{\partial L(\tau, \Lambda)}{\partial \tau} = h \frac{\lambda^H}{\lambda^H + \Lambda} e^{-(\rho+\lambda^H)\tau} \left( 1 - e^{-(\lambda^H + \Lambda)\tau} \right)
\]

\[
\frac{\partial L(\tau, \Lambda)}{\partial \tau} = \lambda^H h e^{-(\rho+\lambda^D)\tau} \int_0^\tau e^{-(\lambda^H + \Lambda)y} dy
\]

therefore

\[
\frac{\partial L(\tau, \Lambda)}{\partial \tau \Lambda} = -\lambda^H h e^{-(\rho+\lambda^D)\tau} \int_0^\tau y e^{-(\lambda^H + \Lambda)y} dy \leq 0
\]

\[\square\]

### A.3 Liquidity spread

*Proof of Proposition 1.* We show that:
1. The liquidity spread \( cs^{liq}(\tau, \Lambda) \) is increasing in maturity \( \tau \):

\[
\frac{\partial cs^{liq}(t, \Lambda)}{\partial t} = \frac{1}{t^2} \log \left( 1 - e^{(\rho + \lambda^D)t} \mathcal{L}(t, \Lambda) \right) + \frac{e^{(\rho + \lambda^D)t}(\rho + \lambda^D)\mathcal{L}(t, \Lambda)}{1 - e^{(\rho + \lambda^D)t}\mathcal{L}(t, \Lambda)} + \frac{\partial \mathcal{L}(t, \Lambda)}{\partial t}
\]

Recall that \( \log(x) \geq \frac{x-1}{x} \). Hence

\[
\log \left( 1 - e^{(\rho + \lambda^D)t} \mathcal{L}(t, \Lambda) \right) \geq \frac{-e^{(\rho + \lambda^D)t}\mathcal{L}(t, \Lambda)}{1 - e^{(\rho + \lambda^D)t}\mathcal{L}(t, \Lambda)}
\]

Which implies that

\[
\frac{\partial cs^{liq}(t, \Lambda)}{\partial t} \geq \frac{1}{t^2} e^{(\rho + \lambda^D)t} \mathcal{L}(t, \Lambda) \left( t(\rho + \lambda^D) + \frac{\partial \mathcal{L}(t, \Lambda)}{\partial t} \frac{t}{\mathcal{L}(t, \Lambda)} - 1 \right)
\]

Let \( \varepsilon_{\mathcal{L}, t} = \frac{\partial \mathcal{L}(t, \Lambda)}{\partial t} \frac{t}{\mathcal{L}(t, \Lambda)} \) and note that

\[
\varepsilon_{\mathcal{L}, t} = t \left[ e^{-(\rho + \lambda^D)\tau} - e^{-(\rho + \lambda^D + \lambda^H + \lambda)t} \right] \left[ \frac{1 - e^{-(\rho + \lambda^D)t}}{\rho + \lambda^D} - \frac{1 - e^{-(\rho + \lambda^D + \lambda^H + \lambda)t}}{\rho + \lambda^D + \lambda^H + \lambda} \right]^{-1}
\]

A sufficient condition is \( t(\rho + \lambda^D) + \varepsilon_{\mathcal{L}, t} - 1 \geq 0 \). Let \( a = \rho + \lambda^D \) and \( b = \lambda^H + \lambda \) and define

\[
E(t, a, b) = t \left( a + \left[ e^{-at} - e^{-(a+b)t} \right] \left[ \frac{1 - e^{-at}}{a} - \frac{1 - e^{-(a+b)t}}{a+b} \right]^{-1} \right) - 1
\]

It is easy to show numerically that \( E(t, a, b) \geq 0 \) for all \( t, a, b \geq 0 \). Hence, the liquidity spread is increasing in maturity. Finally, it is straightforward to see that the liquidity spread is decreasing in liquidity \( \Lambda \).

2. The liquidity spread is increasing in the default intensity \( \lambda^D \): Note that

\[
e^{(\rho + \lambda^D)\tau} \mathcal{L}(\tau, \Lambda) = \frac{\lambda^H}{\lambda^H + \Lambda} \int_0^\tau e^{(\rho + \lambda^D)(\tau-t)} \left( 1 - e^{-(\lambda^H + \lambda)t} \right) dt
\]

\[
\frac{\partial \left( e^{(\rho + \lambda^D)\tau} \mathcal{L}(\tau, \Lambda) \right)}{\partial \lambda^D} = \frac{\lambda^H}{\lambda^H + \Lambda} \int_0^\tau (\tau-t) e^{(\rho + \lambda^D)(\tau-t)} \left( 1 - e^{-(\lambda^H + \lambda)t} \right) dt > 0
\]

3. The proportional bid-ask spread is increasing in maturity: The mid-price is

\[
\frac{1}{2} \left( D^H(y) + D^L(y) \right) = e^{-(\rho + \lambda^D)y} - \frac{1}{2} \left( h \frac{1 - e^{-(\rho + \lambda^D)y}}{\rho + \lambda^D} + \left( \frac{\lambda^H - \Lambda}{\lambda^H} \right) \mathcal{L}(y, \Lambda) \right)
\]
where
\[
\left( \frac{\lambda^H - \Lambda}{\lambda^H} \right) \mathcal{L}(y, \Lambda) = \frac{\lambda^H - \Lambda}{\lambda^H + \Lambda} \left( \frac{1 - e^{-(\rho + \lambda^D)y}}{\rho + \lambda^D} - \frac{1 - e^{-(\rho + \lambda^D + \lambda^H + \Lambda)y}}{\rho + \lambda^D + \lambda^H + \Lambda} \right)
\]

The mid-price is
\[
e^{-(\rho + \lambda^D)y} - h \frac{\lambda^H - \Lambda}{\lambda^H + \Lambda} \left( \frac{\lambda^H 1 - e^{-(\rho + \lambda^D)y}}{\rho + \lambda^D} - \frac{(\lambda^H - \Lambda) 1 - e^{-(\rho + \lambda^D + \lambda^H + \Lambda)y}}{2} \right)
\]

Define the gains from trade as
\[
GT(y) = h \frac{1 - e^{-(\rho + \lambda^D + \lambda^H + \Lambda)y}}{\rho + \lambda^D + \lambda^H + \Lambda}
\]

so
\[
BA(y) = GT(y) \left[ e^{-(\rho + \lambda^D)y} - \frac{1}{\lambda^H + \Lambda} \left( h\lambda^H 1 - e^{-(\rho + \lambda^D)y} \frac{1 - e^{-(\rho + \lambda^D)y}}{\rho + \lambda^D} - \frac{(\lambda^H - \Lambda) GT(y)}{2} \right) \right]^{-1}
\]

Note that \(\frac{e^{-(\rho + \lambda^D)y}}{GT(y)}\) is decreasing in \(y\) as \(e^{-(\rho + \lambda^D)y}\) is decreasing and \(GT(y)\) is increasing in \(y\).

Note that \(\frac{\rho + \lambda^D}{GT(y)}\) is increasing in \(y\) as the discount in \(GT\) is larger than in the numerator.

Hence, with the negative sign it is decreasing. Therefore, all the square bracket is decreasing in \(y\), and as it is to the power of \(-1\), the \(BA(y)\) is increasing in \(y\).

\[\Box\]

### A.4 Free entry

**Proof of Proposition 2.** We start with the free entry of financiers. Recall that the gains from trade are
\[
D^H(y) - D^L(y) = h \frac{1 - e^{-c_1y}}{c_1} \quad c_1 = \rho + \lambda^D + \lambda^H + \lambda S \gamma
\]
The buyer gets \((1 - \gamma)\) of the gains from trade. Hence, the free entry condition reads
\[
c = (1 - \gamma) \int_0^\tau \lambda B \frac{\mu_L(y)}{\mu^S} h \frac{1 - e^{-c_1 y}}{c_1} dy
\]
And \(\theta = \frac{\mu^S}{\mu^H}\). Also, recall that \(\mu^S = \int_0^\tau \mu_L(y) dy\)

Hence, the free entry condition is
\[
c = (1 - \gamma) \frac{h}{c_1} A \theta^* \int_0^\tau \frac{\mu_L(y)}{\mu^S} (1 - e^{-c_1 y}) dy
\]
\[
c = (1 - \gamma) \frac{h}{c_1} A \theta^* \left(1 - \frac{\int_0^\tau e^{-c_1 y} \mu_L(y) dy}{\int_0^\tau \mu_L(y) dy}\right)
\]

Define \(c_2 = \lambda^H + \lambda^D + \lambda^S\) and note that
\[
\int_0^\tau e^{-c_1 y} \mu_L(y) dy = \mu^0 \frac{\lambda^H}{\lambda^H + \lambda^S} \left(\frac{e^{-c_1 \tau} - e^{-\lambda^D \tau}}{\lambda^D - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1}\right)
\]
As a result, the ratio of integrals in the free entry condition reads
\[
\left(\frac{e^{-c_1 \tau} - e^{-\lambda^D \tau}}{\lambda^D - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1}\right) \left(1 - e^{-\tau \lambda^D} - \frac{1 - e^{-(\lambda^H + \lambda^D + \lambda^S) \tau}}{\lambda^H + \lambda^D + \lambda^S}\right)^{-1}
\]
(19)

And the free-entry condition boils down to
\[
c = (1 - \gamma) \frac{h}{c_1} A \theta^* \left(1 - \left(\frac{e^{-c_1 \tau} - e^{-\lambda^D \tau}}{\lambda^D - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1}\right) \left(1 - e^{-\tau \lambda^D} - \frac{1 - e^{-c_2 \tau}}{c_2}\right)^{-1}\right)
\]

First, note that it is easy to show that Equation (19) is increasing in \(\tau\). Next, consider \(\tau = 0\). Note
that the ratio of integrals in the free entry condition is equal to one as

$$\lim_{\tau \to 0} \left( \frac{e^{-c_1 \tau} - e^{-\lambda^D \tau}}{\lambda^D - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1} \right) \left( \frac{1 - e^{-\lambda^D \tau}}{\lambda^D} - \frac{1 - e^{-c_2 \tau}}{c_2} \right)^{-1}$$

$$= \lim_{\tau \to 0} \left( \frac{-c_1 e^{-c_1 \tau} + \lambda^D e^{-\lambda^D \tau}}{\lambda^D - c_1} - \frac{-c_1 e^{-c_1 \tau} + c_2 e^{-c_2 \tau}}{c_2 - c_1} \right) \left( \frac{\lambda^D e^{-\tau\lambda^D}}{\lambda^D} - \frac{c_2 e^{-c_2 \tau}}{c_2} \right)^{-1}$$

$$= \lim_{\tau \to 0} \left( \frac{(c_1)^2 e^{-c_1 \tau} - (\lambda^D)^2 e^{-\lambda^D \tau}}{\lambda^D - c_1} - \frac{(c_1)^2 e^{-c_1 \tau} - (c_2)^2 e^{-c_2 \tau}}{c_2 - c_1} \right) \left( \frac{-(\lambda^D)^2 e^{-\tau \lambda^D}}{\lambda^D} - \frac{-(c_2)^2 e^{-c_2 \tau}}{c_2} \right)^{-1}$$

$$= \left( \frac{(c_1)^2 - (\lambda^D)^2}{\lambda^D - c_1} - \frac{(c_1)^2 - (c_2)^2}{c_2 - c_1} \right) \left( \frac{-(\lambda^D)^2}{\lambda^D} - \frac{-(c_2)^2}{c_2} \right)^{-1}$$

$$= \left( \frac{(c_1 + \lambda^D)(c_1 - \lambda^D)}{\lambda^D - c_1} - \frac{(c_1 + c_2)(c_1 - c_2)}{c_2 - c_1} \right) (c_2 - \lambda^D)^{-1}$$

$$= \left( - (c_1 + \lambda^D) + (c_1 + c_2) \right) (c_2 - \lambda^D)^{-1} = (c_2 - \lambda^D) (c_2 - \lambda^D)^{-1} = 1$$

where we applied L’Hôpital’s rule in the second and third line. As a result, the free-entry condition is satisfied if and only if \(\lim_{\tau \to 0} \theta = \infty\). Hence, \(\lim_{\tau \to 0} \lambda^S = 0\). That is, \(\lambda^S (0) = 0\).

Next, consider the case of \(\tau \to \infty\). The ratio of integrals in the free entry condition is equal to zero. Hence \(c = \frac{h}{c_1} (1 - \gamma) A\theta^\alpha\). Recall that \(c_1 = \rho + \lambda^D + \lambda^H + \lambda^S \gamma\) and \(\lambda^S = A\theta^{\alpha-1}\). Hence,

$$\rho + \lambda^D + \lambda^H + \gamma A\theta^{\alpha-1} = \frac{h (1 - \gamma)}{c} A\theta^\alpha$$

As \(\alpha \in (0, 1)\) the left hand side is decreasing in \(\theta\) and the right hand side is increasing in \(\theta\). As a result, there exists a unique \(\theta \in \mathbb{R}_+\). That is, \(\lim_{\tau \to \infty} \lambda^S (\tau) = \overline{\lambda^S} \in \mathbb{R}_+\). \(\square\)

**B Omitted proofs: Borrowers**

In this section we provide the proofs of the results for borrowers.

**B.1 Rollover**

*Proof of Lemma 3.* Consider a sequence \(\{y_i\}_{i=1}^J\) such that \(\sum_{i=1}^J y_i = \tau\). Then

$$E (\tau, 0, 0) = e^{-(\rho + \lambda^D) \tau} F (\tau) - B_J$$
where for \( j = 1, \ldots, J \) \( B_j = e^{r(y_j)y_j} (\Phi + B_{j-1} + I(y_j)) \) and \( B_0 = 0 \). We define the financial cost as

\[
\text{FINCOST}(\tau) = \min_{J,\{y_j\}_{j=1}^J} e^{-(\rho + \lambda D)\tau} B_j
\]

and

\[
E(\tau, 0, 0) = \max_{\tau} e^{-(\rho + \lambda D)\tau} F(\tau) - \text{FINCOST}(\tau)
\]

Finally, iterate on \( B_j \) forward to get

\[
B_j = \sum_{i=1}^j (\Phi + I(y_i)) e^{\sum_{s=i}^j r_s y_s}
\]

### B.2 Maturity and liquidity

#### Proof of Proposition 3

Let \( J(\tau, \Lambda) \) be the value of the firm with maturity \( \tau \) and liquidity \( \Lambda \) and let \( A = \delta_x(1 - \sigma) \). The first order condition is

\[
J(\tau, \Lambda) = e^{-(\rho + \lambda D)\tau} A (1 - (\rho + \lambda D) \tau)
- e^{cs_{liq}(\Lambda, \tau)\tau} \left[ \frac{\partial I(\tau)}{\partial \tau} + (\Phi + I(\tau)) cs_{liq}(\Lambda, \tau) \right]
\]

Note that \( \tau \) is increasing in \( \Lambda \) if the derivative of the first order condition with respect to \( \Lambda \) is positive

\[
J(\tau, \Lambda) = -e^{cs_{liq}(\Lambda, \tau)\tau} \left( \frac{\partial cs_{liq}(\Lambda, \tau)}{\partial \Lambda} \right) \left[ \frac{\partial I(\tau)}{\partial \tau} + (\Phi + I(\tau)) cs_{liq}(\Lambda, \tau) \right]
- e^{cs_{liq}(\Lambda, \tau)\tau} \left( \Phi + I(\tau) \right) cs_{liq}(\Lambda, \tau) \frac{\partial cs_{liq}(\Lambda, \tau)}{\partial \Lambda}
\]

Recall that \( \frac{\partial cs_{liq}(\Lambda, \tau)}{\partial \Lambda} \leq 0 \), so the first and second terms are positive. However, the last term involves \( \frac{\partial cs_{liq}(\Lambda, \tau)}{\partial \Lambda} \) for which we do not know the sign. We can write \( J(\tau, \Lambda) \) as

\[
J(\tau, \Lambda) = -e^{cs_{liq}(\Lambda, \tau)\tau} \left( \frac{\partial cs_{liq}(\Lambda, \tau)}{\partial \Lambda} \right) \left( \frac{\partial I(\tau)}{\partial \tau} + (\Phi + I(\tau)) cs_{liq}(\Lambda, \tau) \right)
- e^{cs_{liq}(\Lambda, \tau)\tau} \left( \Phi + I(\tau) \right) cs_{liq}(\Lambda, \tau) \frac{\partial cs_{liq}(\Lambda, \tau)}{\partial \Lambda}
\]
The first term is positive. A sufficient condition for $J_{\tau \Lambda}(\tau, \Lambda) \geq 0$ is that the second term is also positive. This implies
\[
\frac{\partial cs^{\text{liq}}(\Lambda, \tau)}{\partial \Lambda} (1 + \varepsilon_{cs^{\text{liq}}, \tau}(\Lambda, \tau)) \left(\tau cs^{\text{liq}}(\Lambda, \tau) + 1\right) \leq -cs^{\text{liq}}(\Lambda, \tau) \frac{\partial \varepsilon_{cs^{\text{liq}}, \tau}(\Lambda, \tau)}{\partial \Lambda}
\]

This expression depends only on $cs^{\text{liq}}(\Lambda, \tau)$. By Proposition 1 we can approximate the liquidity spread as a linear function increasing in $\tau$ and decreasing in $\Lambda$. Let $cs^{\text{liq}}(\Lambda, \tau) = c_\tau \tau + c_\Lambda \Lambda$ with $c_\tau \geq 0$ and $c_\Lambda \leq 0$. Then $\varepsilon_{cs^{\text{liq}}, \tau}(\Lambda, \tau) = \frac{c_\tau \tau}{c_\tau \tau + c_\Lambda \Lambda}$ and $\frac{\partial \varepsilon_{cs^{\text{liq}}, \tau}(\Lambda, \tau)}{\partial \Lambda} = -\frac{c_\tau \tau c_\Lambda}{(c_\tau \tau + c_\Lambda \Lambda)^2}$. The sufficient condition reads
\[
\left(c_\tau \tau + cs^{\text{liq}}(\Lambda, \tau)\right) cs^{\text{liq}}(\Lambda, \tau) \tau + cs^{\text{liq}}(\Lambda, \tau) \geq 0
\]
which is satisfied. Therefore, $J_{\tau \Lambda}(\tau, \Lambda) \geq 0$ and $\frac{\partial \tau(\Lambda)}{\partial \Lambda} \geq 0$. Finally it is straightforward to see that $\tau = \tau(0) \leq \lim_{\Lambda \to \infty} \tau(\Lambda) = \tau^* < \infty$. \qed

B.3 Alternative specifications for demand of long-term finance

B.3.1 Quality ladder model

We follow the standard quality-ladder model with a continuum of intermediate goods producers with monopolistic power and final goods producers in perfect competition (e.g., Akcigit and Kerr, 2017). The final good is produced with labor and a continuum of intermediate goods $j \in [0, 1]$ with the production technology
\[
Y = \frac{L^\beta}{1 - \beta} \int_0^1 q_j^\beta k_j^{1 - \beta} dj
\]
where $L$ is the labor input, $k_j$ is the quantity of intermediate good $j$, and $q_j$ is its quality. We normalize the price of the final good $Y$ to be one in every period without loss of generality. The final good is produced competitively with input prices taken as given. The inverse demand of intermediate inputs reads $p_j = L^\beta q_j^\beta k_j^{-\beta}$ and the labor demand is $L = \frac{\partial Y}{w}$.

Each intermediate good $j$ is produced with a linear technology $k_j = q l_j$ where $q = \int_0^1 q_j dj$ is the average quality, and $l_j$ is the labor input. Intermediate producers take the inverse demand and maximize static profits
\[
\pi(q) = \max_{k_j \geq 0} p_j k_j - \frac{w}{q} k_j \quad \text{s.t.} \quad p_j = L^\beta q_j^\beta k_j^{-\beta}
\]

There is an exogenous labor supply normalized to one. Labor market clearing implies $L + \int_0^1 l_j = 1$. 

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Lemma 5 shows that static profits are linear in quality and aggregate output is linear in the average quality of the economy.

**Lemma 5.** *Static profits are linear in quality and output is linear in average quality*

\[ \pi(q) = \pi q \]

where the constants \( \pi \) and \( Y \) are

\[ \pi = \beta L \left( \frac{\bar{q}}{w} \right)^{\frac{1-\beta}{\bar{\sigma}}} (1 - \beta)^{\frac{1-\beta}{\bar{\sigma}}} \]

\[ L = \frac{\beta}{(1 - \beta)^2 + \beta} \]

\[ Y = \frac{\beta^3 (1 - \beta)^{1-2\beta}}{(1 - \beta)^2 + \beta} \]

The proof is standard, see Akcigit and Kerr (2017).

**Life cycle of intermediate goods producers**  There are two important empirical facts which motivate assumptions about the back-loaded profile of investment projects. First, the *Arrow’s replacement effect* establish that small and young firms are more innovative than large and old firms (e.g., Arrow, 1962; Itenberg, 2015; Akcigit and Kerr, 2017). Second, small firms are more financially constrained, and in particular for research and development (Midrigan and Xu, 2014; Itenberg, 2015). Based on these facts, we assume that a newborn firm chooses a project maturity \( \tau \) such that she is young for \( t \leq \tau \) and mature otherwise.

A young firm invest in research and development to improve the quality of the product. The evolution of quality is given by \( \dot{q}(t) = \delta_1 + \lambda Q \delta_2 \). The first component, \( \delta_1 \), captures a deterministic growth on quality. We calibrate \( \delta_1 \) such that at maturity the firm wants to repay the debt.\( ^{26} \) Second, as it is standard in the literature, we assume that quality makes jumps of size \( \delta_2 \) that arrive at Poisson rate \( \lambda Q \). Doing research is costly. The firm pays \( \kappa \) per unit of time doing research. Moreover, the firm is financially constrained when young. She borrows to finance investment which generates a demand for long-term finance.

At age \( \tau \) the firm becomes mature, stop R&D and start production with quality \( q(\tau, N) \), where \( N \) is a counting process with the number of jumps on quality before \( \tau \). Static profits are linear in quality \( \pi(q) = \pi q \). Hence, the net present value of a mature firm with quality \( q \) is given by \( F(q) = \frac{\pi(q)}{\rho + \lambda^E} \) where \( \lambda^E \) is the Poisson arrival rate of an exogenous exit shock.

\(^{26}\)If \( \delta_1 = 0 \) some firms will prefer to default at maturity. This might be an interesting setup to study corporate default. However, as we want to study the role of liquidity we calibrate \( \delta_1 \) such that we can abstract from strategic default.
B.3.2 Time-to-build capital

Rioja and Valev (2004) finds that finance boosts growth in rich countries primarily by speeding-up productivity growth, while finance encourages growth in poorer countries primarily by accelerating capital accumulation. In this section we propose an alternative microfoundation for back-loaded projects based on time-to-build capital.

At every moment a new cohort of identical firms $\mu^0$ enters the economy and choose a project to implement. There is a continuum of projects differentiated by the time-to-build $\tau \geq 0$. For $t \in [0, \tau)$ the firm is young and is investing. For $t \geq \tau$ the firm is mature and is producing.

A young firm starts with no capital, $K(0) = 0$ and investment is subject to a time-to-build constraint à la Majd and Pindyck (1987) such that $dK = idt$ and $i \leq \kappa$. The investment rate $i$ per unit of time cannot exceed $\kappa$. Given the linearity, it is optimal to build at maximum capacity. Hence, a firm with a project of duration $\tau$ concludes its investment stage with capital equal to $K = \tau \kappa$. This firm is subject to an exogenous exit shock that arrives at Poisson intensity $\lambda^D$ in which case the project is destroyed and the firm defaults.

A mature firm use the capital to produce and profits are given by $\pi = zK^\sigma$, where $z$ is the productivity and $\sigma$ are the returns to scale. This firm is subject to an exogenous exit shock that arrives at Poisson intensity $\lambda^E$. The net present value for a mature firm with project $\tau$ is

$$F(\tau) = \frac{zK^\sigma}{\rho + \lambda^E}. \quad (21)$$

Note that the return on the project $F$ is increasing in the time spent on investment. Hence, time-to-build is an alternative formulation for back-loaded projects.

B.3.3 General model of back-loaded projects

We can derive a demand for long-term debt under a general specification of back-loaded projects. Consider a reduced-form formulation for the value of the project $F(\tau)$ with $F$ increasing and either linear or concave. Then, the demand for long-term debt will be analogous to the one derived in this section. The advantage of the quality ladder model is that we can easily take it to the data using moments of the R&D literature.
B.4 Existence of equilibrium

Proof of Proposition 4. First, Proposition 2 define a schedule for the lenders $\tau^L(\Lambda)$. Note that $\tau^L(0) = 0$ and there exists $\Lambda$ such that $\tau^L(\Lambda) = \infty$.

Second, Proposition 3 define $\tau^B(\Lambda)$ and notice that $\tau^B(0) = \tau > 0$ and $\tau^B(\Lambda) \geq 0$ for all $\Lambda$.

Finally, define $F(\Lambda) = \tau^L(\Lambda) - \tau^B(\Lambda)$ and note that $F(0) = -\tau < 0$ and $F(\Lambda) = \infty$. Hence, as $F$ is continuous, Bolzano’s theorem implies that there exits $\Lambda^*$ such that $F(\Lambda^*) = 0$ which defines the equilibrium.

C Government-Sponsored Intermediaries

C.1 Distribution of financiers

Total assets with time to maturity $t$ are $\mu(t) = \mu^0 e^{-\lambda^D t}$. These assets are hold by 4 type of agents: $\mu(t) = \mu^{H,P}(t) + \mu^{L,P}(t) + \mu^{H,G}(t) + \mu^{L,G}(t)$. The law of motions for private sector are

$$
-\dot{\mu}^{H,P}(t) = -\left(\lambda^H + \lambda^D\right) \mu^{H,P}(t) + \left(\lambda^{B,P-G}(t) + \lambda^{B,G-P}(t)\right) \mu^{B,P}
$$

$$
-\dot{\mu}^{L,P}(t) = \lambda^H \mu^{H,P}(t) - \left(\lambda^D + \lambda^{S,P-P} + \lambda^{S,G-G}\right) \mu^{L,P}(t)
$$

with boundary conditions $\mu^{H,P}(0) = \mu^0$ and $\mu^{L,P}(0) = 0$. The law of motions for Government are

$$
-\dot{\mu}^{H,G}(t) = -\left(\lambda^H + \lambda^D\right) \mu^{H,G}(t) + \left(\lambda^{B,P-G}(t) + \lambda^{B,G-G}(t)\right) \mu^{B,G}
$$

$$
-\dot{\mu}^{L,G}(t) = \lambda^H \mu^{H,G}(t) - \left(\lambda^D + \lambda^{S,P-G} + \lambda^{S,G-G}\right) \mu^{L,G}(t)
$$

with boundary conditions $\mu^{H,G}(0) = \mu^{L,G}(0) = 0$. Matching implies

$$
\mu^{B,P} \lambda^{B,P-G}(t) = \mu^{L,P}(t) \lambda^{S,P-P}
$$

$$
\mu^{B,P} \lambda^{B,G-P}(t) = \mu^{L,G}(t) \lambda^{S,G-P}
$$

$$
\mu^{B,G} \lambda^{B,P-G}(t) = \mu^{L,P}(t) \lambda^{S,P-G}
$$

$$
\mu^{B,G} \lambda^{B,G-G}(t) = \mu^{L,G}(t) \lambda^{S,G-G}
$$

Define $\mu(t) = [\mu^{H,P}(t), \mu^{L,P}(t), \mu^{H,G}(t), \mu^{L,G}(t)]$. The boundary condition is $\mu(0) = [\mu^0, 0, 0, 0]$.
and the system is $\dot{\mu}(t) = A\mu(t)$ where

$$A = \begin{bmatrix}
\lambda^H + \lambda^D & -\lambda^{S,P-P} & 0 & -\lambda^{S,G-P} \\
-\lambda^H & \lambda^D + \lambda^{S,P-P} + \lambda^{S,P-G} & 0 & 0 \\
0 & -\lambda^{S,P-G} & \lambda^H + \lambda^D & -\lambda^{S,G-G} \\
0 & 0 & -\lambda^H & \lambda^D + \lambda^{S,G-P} + \lambda^{S,G-G}
\end{bmatrix}$$

The solution of this system is standard. The only caveat is that we should pay attention to how many real and complex eigenvalues has the matrix $A$.

## D Segmented markets

### D.1 Distributions of financiers

**Proof of Lemma 4.** Total assets are $\mu^j(t) = e^{(t-\tau)\lambda^D} \mu^0$. The evolution for unconstrained and constrained agents in market $j$ are

$$-\dot{\mu}^{H,j}(t) = - (\lambda^H + \lambda^D) \mu^{H,j}(t) + \mu^{B,j} \lambda^{B,j}(t)$$

$$-\dot{\mu}^{L,j}(t) = \lambda^H \mu^{H,j}(t) - (\lambda^D + \lambda^{S,j}) \mu^{L,j}(\tau_j)$$

Recall that matching implies that $\mu^{B,j} \lambda^{B,j}(t) = \mu^{L,j}(t) \lambda^{S,j}$. Hence the system is

$$\dot{\mu}^j(t) = \begin{bmatrix}
\lambda^H + \lambda^D & -\lambda^{S,j} \\
-\lambda^H & \lambda^D + \lambda^{S,j}
\end{bmatrix}
\begin{bmatrix}
\mu^{H,j}(t) \\
\mu^{L,j}(t)
\end{bmatrix}$$

With eigenvalues $\lambda^D$ and $\lambda^H + \lambda^{S,j} + \lambda^D$. Define $V^j$ to be the matrix with the eigenvectors and $R^j$ the diagonal matrix with the eigenvalues and $B^j = (V^j)^{-1} \mu^{j+1}(\tau_{j+1})$. Then

$$\mu^{H,j}(t) = \sum_{i=1}^{2} e^{R^j(i)(t-\tau_{j+1})} B^j(i) V^j(1, i)$$

$$\mu^{L,j}(t) = \sum_{i=1}^{2} e^{R^j(i)(t-\tau_{j+1})} B^j(i) V^j(2, i)$$
For $j = 1, \ldots, N - 1$ we have that

$$
\mu^{H,j} (t) = \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[ \frac{\lambda^{S,j} \mu^{D} (t - \tau_{j+1})}{\lambda^H} \left( \mu^{H,j+1} (\tau_{j+1}) + \mu^{L,j+1} (\tau_{j+1}) \right) \right] - \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[ -e^{(\lambda^H + \lambda^{S,j} + \lambda^D)(t - \tau_{j+1})} ( -\mu^{H,j+1} (\tau_{j+1}) + \lambda^{S,j} \mu^{L,j+1} (\tau_{j+1}) ) \right]
$$

$$
\mu^{L,j} (t) = \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[ e^{\lambda^D (t - \tau_{j+1})} \left( \mu^{H,j+1} (\tau_{j+1}) + \mu^{L,j+1} (\tau_{j+1}) \right) \right] + \frac{\lambda^H}{\lambda^H + \lambda^{S,j}} \left[ e^{(\lambda^H + \lambda^{S,j} + \lambda^D)(t - \tau_{j+1})} ( -\mu^{H,j+1} (\tau_{j+1}) + \lambda^{S,j} \mu^{L,j+1} (\tau_{j+1}) ) \right]
$$

\[ \Box \]

### D.2 Value functions

Let $Z^j (t) = D^{H,j} (t) - D^{L,j} (t)$ and $c_j = \rho + \lambda^D + \lambda^H + \gamma \lambda^{S,j}$, then $c_j Z^j (t) = h - Z^j (t)$. The solution is $Z^j (t) = A Z^j e^{-c_j t} + B Z^j$ with $B Z^j = \frac{h}{c_j}$ and the boundary condition pin downs $A Z^j$. For $j = 1$ the boundary condition is $D^{H,1} (\tau_1) = D^{L,1} (\tau_1) = 1$ so $A Z^j = -\frac{h}{c_j}$. For $j = 2, \ldots, N$ we have that $Z^j (\tau_j) = Z^{j-1} (\tau_j)$ which implies $A Z^j = e^{c_j \tau_j} \left( A Z^{j-1} e^{-c_j \tau_j} + \frac{h}{c_j - 1} \right)$.

Next, we can solve for the value of high and low valuation agents using $Z^j$ and the initial conditions. For high valuation agents

$$
(\rho + \lambda^D) D^{H,j} (t) = -D^{H,j} (t) - \lambda^H \left( A Z^j e^{-c_j t} + \frac{h}{c_j} \right)
$$

The solution is $D^{H,j} (t) = A^{H,j} + B^{H,j} e^{-(\rho + \lambda^D) t} + C^{H,j} e^{-c_j t}$, with $A^{H,j} = -\frac{\lambda^H h}{(\rho + \lambda^D) c_j}, C^{H,j} = \frac{\lambda^H A Z^j}{\lambda^H + \gamma \lambda^{S,j}}$, and the boundary condition pin down $B^{H,j}$. And the boundary condition pin down $B^{H,j}$. For $j = 1$ we have that $D^{H,1} (\tau_1) = 1$ and $\tau_1 = 0$ so $B^{U,1} = 1 - A^{U,j} - C^{U,j}$. For $j = 2, \ldots, N$ we have that $D^{H,j} (\tau_j) = D^{H,j-1} (\tau_j)$ so

$$
B^{H,j} = e^{(\rho + \lambda^D) \tau_j} \left( A^{H,j-1} - A^{H,j} + B^{H,j-1} e^{-(\rho + \lambda^D) \tau_j} + C^{H,j-1} e^{-c_j \tau_j} - C^{H,j} e^{-c_j \tau_j} \right)
$$

which defines a recursion in $B^{H,j}$.

**Free entry** The free entry condition in each market is

$$
c_j = (1 - \gamma) \int_{\tau_j}^{\tau_{j+1}} \lambda^B j (t) \left( D^{H,j} (t) - D^{L,j} (t) \right) dt
$$

where $\lambda^B j (t) = A (\theta^j) \frac{\mu^C j (t)}{\mu^L j}$, and both the measures and value functions are sum of exponential
functions. Hence, it is easy to solve for the integrals on the free entry condition in each market.

E  Numerical solution

E.1  Rollover

To solve the model with rollover use the following algorithm

1. Guess \( \tau \)

   (a) Solve \( \text{FIN}^{\text{COST}} \):

   i. Start with \( J = 1 \) and solve for \( \text{FIN}^{\text{COST}}(\tau, J) \)

   ii. For \( J = J + 1 \):

   A. Solve \( \text{FIN}^{\text{COST}}(\tau, J) \). Initial guess \( y = \frac{\tau}{J} \), + fminsearch

   B. Compute \( \text{FIN}^{\text{COST}} \), if lower, iterate, otherwise, stop.

   iii. Compute \( \text{valfun} \) at \( \tau \), and fminsearch

E.2  GSIs

Let \( \mu^{B,G} = \xi \mu^{B,P} \) so \( \mu^{B} = \mu^{B,P}(1 + \xi) \), \( \frac{\mu^{B,P}}{\mu^{B}} = \frac{1}{1 + \xi} \) and \( \frac{\mu^{B,G}}{\mu^{B}} = \frac{\xi}{1 + \xi} \). Algorithm:

1. Guess tax rate

2. Solve equilibrium:

   (a) Guess market tightness \( \theta \)

   (b) Given \( \theta \), and \( \xi \), solve for optimal maturity \( \tau \)

   (c) Given \( \theta \), \( \xi \), and \( \tau \): Solve for measure of agents, coefficient of value functions, and value of entry for private agents

   (d) Check free-entry condition, adjust \( \theta \) and iterate

3. Compute the value of entry for government \( G^S \)

4. Compute gov budget constraint, adjust taxes and iterate
E.3 Segmented markets

1. Guess $\{\theta^j\}_{j=1}^N$
2. Solve for measures and respective coefficients
3. Solve for value functions and respective coefficients
4. Check the free-entry conditions
5. Update $\{\theta^j\}_{j=1}^N$ and iterate until convergence.

E.4 Identification

Although the model is highly nonlinear, so that (almost) all parameters affect all outcomes, the
identification of some parameters relies on some key moments in the data. We keep all parameters
fixed at their estimated value and change only one parameter to study the numerical comparative
statics of the moment with respect to the parameter in the neighborhood of the estimated parameter.
First, Figure 16 shows that as the intensity of liquidity shock increases there are more needs for trading
and the turnover increases. Second, as the search cost increases, there is less entry in secondary markets
and the expected time to sell increases. Third, as the holding cost increases, the secondary market
becomes more relevant and the credit spread due to liquidity increases. Finally, as the returns on
investment increases—larger $\delta_z$—the firm issue at longer maturity.

F Data

F.1 Corporate debt Maturity across countries

The data for maturities in Figure 2 was shared by Cortina Lorente, Didier, and Schmukler (2016). The
source is Thompson Reuters Security Data Corporation Platinum database and details about the data
can be found in Cortina Lorente, Didier, and Schmukler (2016). Table 6 reports the list of countries
along with each region, income classification and average maturity of corporate debt. For GDP we use
GDP per capita relative to US for 2014 downloaded from the World Bank database (GDP per capita
PPP, constant 2011 international $, series NY.GDP.PCAP.PP.KD).

Table 6: List of countries

<table>
<thead>
<tr>
<th>Region</th>
<th>Country</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

19
<table>
<thead>
<tr>
<th>Region</th>
<th>Country</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>Egypt</td>
<td>5.6</td>
</tr>
<tr>
<td>Africa</td>
<td>South Africa</td>
<td>6.3</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Australia *</td>
<td>10.0</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Bangladesh</td>
<td>52.5</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>China</td>
<td>5.7</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Hong Kong *</td>
<td>6.1</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>India</td>
<td>8.5</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Indonesia</td>
<td>6.0</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Japan</td>
<td>7.8</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Malaysia</td>
<td>8.8</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>New Zealand *</td>
<td>7.3</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Pakistan</td>
<td>5.7</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Philippines *</td>
<td>6.8</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Singapore *</td>
<td>6.1</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>South Korea *</td>
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</tr>
<tr>
<td>Asia Pacific</td>
<td>Taiwan *</td>
<td>5.9</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Thailand</td>
<td>5.9</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>Vietnam</td>
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</tr>
<tr>
<td>Canada USA</td>
<td>Canada *</td>
<td>12.3</td>
</tr>
<tr>
<td>Canada USA</td>
<td>United States *</td>
<td>11.9</td>
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<tr>
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</tr>
<tr>
<td>Eastern Europe</td>
<td>Czech Republic *</td>
<td>5.7</td>
</tr>
<tr>
<td>Eastern Europe</td>
<td>Hungary *</td>
<td>2.0</td>
</tr>
<tr>
<td>Eastern Europe</td>
<td>Poland *</td>
<td>7.1</td>
</tr>
<tr>
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<td>Romania</td>
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<td>Eastern Europe</td>
<td>Russia</td>
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<tr>
<td>Latin America</td>
<td>Argentina</td>
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<tr>
<td>Latin America</td>
<td>Bolivia</td>
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<td>Latin America</td>
<td>Brazil</td>
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<tr>
<td>Latin America</td>
<td>Colombia</td>
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<td>Costa Rica</td>
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<tr>
<td>Latin America</td>
<td>Ecuador</td>
<td>2.5</td>
</tr>
<tr>
<td>Latin America</td>
<td>El Salvador</td>
<td>2.4</td>
</tr>
<tr>
<td>Latin America</td>
<td>Mexico</td>
<td>6.5</td>
</tr>
<tr>
<td>Latin America</td>
<td>Panama</td>
<td>6.8</td>
</tr>
<tr>
<td>Latin America</td>
<td>Peru</td>
<td>6.1</td>
</tr>
</tbody>
</table>
Latin America  Venezuela  2.6
Middle East  Israel *  9.6
Middle East  Oman *  4.0
Middle East  Saudi Arabia *  17.0
Middle East  United Arab Emirates *  5.4
Western Europe  Austria *  7.7
Western Europe  Belgium *  9.7
Western Europe  Denmark *  5.2
Western Europe  Finland *  6.4
Western Europe  France *  8.9
Western Europe  Germany *  8.5
Western Europe  Greece *  3.8
Western Europe  Ireland-Rep *  7.0
Western Europe  Italy *  7.1
Western Europe  Luxembourg *  6.9
Western Europe  Netherlands *  8.9
Western Europe  Norway *  9.1
Western Europe  Portugal *  8.3
Western Europe  Spain *  7.1
Western Europe  Sweden *  4.9
Western Europe  Switzerland *  7.7
Western Europe  United Kingdom *  10.8

Note: This table presents the list of countries that constitute the different regions, their classification by income level and the average maturity. Countries are classified as developed or developing based on the World Bank income level classification of 2012 * means the country is classified as developed.
Source: Cortina Lorente, Didier, and Schmukler (2016).

F.2 OTC trading data

For the US we use data from Trade Reporting and Compliance Engine (TRACE) that provides transaction-level information of corporate bonds in secondary markets and has been widely used to study the liquidity of this market (e.g. Edwards, Harris, and Piwowar, 2007; Bao, Pan, and Wang, 2011, among others). I clean the data following Dick-Nielsen (2009) and merge with bond characteristics provided by Mergent Fixed Income Securities Database (FISD). We access to this database through Wharton Research Data Services.
For Argentina, we use a novel data set borrowed from Mercado Abierto Electronico (MAE, the over-the-counter exchange in Argentina). We got access to transaction-level information of corporate bonds (Obligaciones Negociables) which provides similar information than the TRACE database.\footnote{I thanks Argentina’s Central Bank to provide access to the data. To the best of my knowledge, this is the first paper to use this database. Borensztein, Cowan, Eichengreen, and Panniza (2008) documents some aggregate facts about the corporate debt market in Argentina using an aggregate version of this data.} We compare the liquidity of the US and Argentinean markets during the year 2012.\footnote{Results are similar to different time periods.}

### F.3 Treasury data

For interest rates we use estimates from the US Treasury for the synthetic zero-coupon bonds yield curve. The daily treasury data was retrieved from https://www.treasury.gov/resource-center/data-chart-center/interest-rates. The corporate yield curve corresponds to the High quality
market (bonds with rating above A) and it is available at https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/. We define the treasury and corporate yield curve as the daily average for the year 2016.